# Optimal Banking Regulation with Endogenous Liquidity Provision*

Tai-Wei Hu[†] and Yiting Li[‡]

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**Abstract**

In a money-search model, banks costly manage assets to obtain dividends. Banks have limited commitment but a proportion of asset holdings is pledgeable for deposit issuance. Optimal regulation features reserve requirements, entry restriction, and leverage regulation. Chartered banks enjoy profits with leverages above free market. Optimal leverages rise when aggregate demand or inflation increases, and fall when assets are more productive. With heterogeneous banks, optimal regulations feature a more concentrated banking sector. Proportional capital requirements deter banks from gambling and they become stricter as moral hazard worsens, but it is optimal to have higher bank profits and unsecured deposits.

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[†]University of Bristol

[‡]National Taiwan University

# 1 Introduction

The serious interruption of the real economy from the global financial crisis of 2008 has given rise to a renewed interest in understanding the role of financial intermediaries and how to regulate them. Two particular issues have surfaced both in mass media and in policy debates: The banking sector seems too concentrated with seemingly unjustified profits, and bankers seem too well protected by limited liability from prosecution of misbehavior.[1] We contribute to this debate by studying optimal banking regulations and their implications on bank size and profits when bankers face limited liability. Our model explicitly accounts for essential banking activities—banks perform asset transformation in that their liabilities serve as a means of payment, providing liquidity to depositors, and banks' equity is crucial for operations.

This asset-transformation process, as the financial crisis revealed, involves many credit market frictions, such as limited commitment and potential moral hazard in banks' asset management. These frictions, together with the externality from the use of deposits as a means of payment, a special feature of deposits as argued by many,[2] provide a rationale for potential regulations. By taking these frictions seriously, our model demonstrates that two banking regulations are essential for welfare—a liquidity requirement and a leverage requirement. Both regulations are designed to induce adequate liquidity provision by the banking sector, which would be otherwise constrained by limited commitment and moral hazard. Considering liquidity provision from the perspective of welfare also has novel implications for how these regulations should optimally respond to changes in aggregate economic conditions and to individual banks' idiosyncratic circumstances.

Our model of financial intermediaries features endogenous liquidity provision by introducing *banks* into a standard monetary model à la Lagos and Wright (2005) to maintain tractability. In our economy, banks are the only agents with the necessary expertise to manage and monitor loans (modeled as one-period Lucas trees) to earn returns. However, banks' asset holdings require

[1] Commenting on the Dodd-Frank reform, James Surowiecki claimed in his *New Yorker* article ("Banking's New Normal," May 16, 2016) that "Bankers still make absurd amounts of money."For a popular view on the difficulty of such prosecutions, see "Why Corrupt Bankers Avoid Jail," Patrick Radden Keefe, July 31, 2017, *New Yorker*.

[2] See, for example, Benston and Kaufman (1996) and Dow (1996), who argue that banking is special because banks produce "money," or assets that can be used as a means of payment.

external financing, and banks may obtain this financing by issuing deposits which can serve as a means of payment for households. The usual frictions (lack of commitment and monitoring) in the Lagos-Wright environment render a means of payment essential, and this gives rise to an endogenous demand for bank liabilities.

We consider two main frictions in the banking sector. First, it is costly for banks to manage and monitor loans to entrepreneurs. There is an economy of scale in the sector by way of a fixed cost of bank operations and an increasing marginal cost of managing assets, which determine the efficient size of banks. Second, banks cannot fully commit to honor their future obligations; instead, they can only credibly pledge a fraction of their assets that the court can seize upon bankruptcy. This friction constrains the amount of liquidity banks can provide and may prevent the first-best level of consumption for households to be achieved. As an application, we also consider the scenario where banks may gamble their assets to receive private returns without being observed, and how this moral hazard issue may also hinder banks' liquidity provision.

We first consider the implication of limited commitment in a free market. Under free entry banks make zero profits in equilibrium, and the amount of deposits a bank can issue is constrained by limited pledgeability of assets through market discipline via a capital requirement imposed by depositors; no one would deposit in a bank unless it can credibly repay. Bank size is then determined by a zero-profit condition that balances the operational profit from asset management and the fixed cost of bank operation, which coincides with the efficient level of asset holdings. When the pledgeability constraint is binding, aggregate liquidity cannot achieve the first-best level of consumption, and the asset price exhibits a liquidity premium.

Against this free market arrangement, we show that a charter system with a banking regulator can improve social welfare. We first impose a liquidity requirement so that banks have to hold reserves, which are simply outside money that can settle interbank payments, against their deposit issuance while maintaining free entry. We show that both reserves and reserve requirements are essential: whenever the first-best allocation is not achievable under laissez faire, the optimal reserve requirement can strictly improve social welfare. Since reserves are fully pledgeable, mandatory holding can effectively expand banks' pledgeable assets while inducing banks to pay positive interest on deposits. In contrast, since reserves are costly to hold, in the absence of a reserve requirement, either reserves are not valued in equilibrium, or bank deposits pay no interest. Nevertheless, even under the optimal reserve requirement the banking sector may not be able to provide sufficient liquidity and further regulation may be called for.

We then introduce limited entry in the charter system, wherein only chartered banks can issue deposits and the regulator sets the pledgeability constraint beyond the limited pledgeability of assets (which can be interpreted as a leverage requirement), and can shut down a bank when it does not honor its obligations. This scheme allows for a dynamic incentive to relax the pledgeability constraint imposed by market discipline and improve liquidity. For this dynamic incentive to be effective, however, it is necessary to limit the number of charters relative to the efficient number under free entry, a condition that allows banks to earn economic profits. This scheme makes it incentive feasible for banks to issue unsecured deposits beyond the pledgeable assets, and can thus increase bank leverage. The optimal policy thus faces a trade-off: a smaller number of charters increases bank profits and hence helps increase liquidity, which improves households' welfare, but it also increases the overall cost of banking operations as each bank gets inefficiently large. Our main result demonstrates that, whenever liquidity is tight under free entry, it is optimal to restrict the number of charters relative to the efficient level that minimizes the costs of banking and to raise the leverage ratio above the laissez-faire level.

Our model is sufficiently tractable so that we can study how optimal banking regulation varies with both individual bank characteristics and aggregate economic conditions. For the former, we extend the model by allowing for heterogeneous efficiency in managing assets across banks. In the absence of regulation, more efficient banks end up being larger in terms of asset holdings, an efficient outcome in terms of asset management by itself. The optimal charter system consists of two components in capital regulation: the first is a proportional capital requirement that depends only on the pledgeability of assets and hence is constant across banks, while the second is an overall leverage requirement that differs across banks. We demonstrate that under the optimal charter system, more efficient banks face less stringent leverage requirements and end up with an even larger balance sheet. The intuition is simple: when the number of charters is limited, large banks make higher profits and it is more efficient to incentivize them to repay unsecured deposits. The optimal arrangement results in a positive correlation between bank size and the leverage ratio, and a higher concentration above the laissez-faire arrangement.

To study how aggregate economic conditions affect optimal banking regulations, we introduce two relevant aggregate parameters—a shock that affects aggregate demand, and another that affects asset productivity. A positive shock from either category will increase aggregate output. The proportional capital requirements are not affected by changes in either parameters; hence, they belong to the category of microprudential regulations. The leverage requirement is affected by both individual bank characteristics and macroeconomic conditions, which captures features of macroprudential regulations.

The two kinds of shocks, however, have very different implications for the leverage requirements. A positive aggregate demand shock creates higher demand for bank deposits, which in turn suppresses interests on deposits. As a result, it is optimal to relax the leverage requirement; it is also incentive compatible to do so as lower interest on deposits increases bank profits and makes it feasible to increase bank unsecured borrowing. In contrast, a positive shock on asset productivity causes a higher supply of bank deposits in real terms, which in turn raises the deposit rate. This then calls for a tighter leverage requirement, which makes it incentive compatible for banks to repay unsecured deposits. Thus, optimal regulation features a procyclical leverage requirement for shocks to aggregate demands, while it features a countercyclical leverage requirement for shocks to liquidity supply.

Finally, we apply our model to two issues often raised in banking regulations. The first is the moral hazard issue with asset management; banks may engage in socially suboptimal gambling for a private gain. Because banks have limited liability, they may increase their profits from the private gain while households have to suffer (most of) the consequences if the gamble fails. Under the charter system, the optimal proportional capital requirement is designed to control the incentive to gamble only. In contrast, the leverage requirement is affected by this incentive issue as well as aggregate economic conditions. As moral hazard becomes worse, it is optimal to tighten the proportional capital requirement to mitigate the incentive to gamble, but it also hurts liquidity provision. As a result, it is optimal to increase unsecured borrowing by making banks larger and more profitable, due to the welfare role of banks' liquidity provision. Moreover, for the same reason, when the moral hazard issue is very severe, it can be optimal to adopt regulations under which banks gamble in equilibrium.

Our second application considers the effects of inflation on optimal banking regulations. In our model, the real effects of inflation come from its effects on banks' cost of holding reserves. When inflation rises, this cost becomes higher and hinders banks' ability to provide liquidity. To alleviate this adverse effect, it is optimal to relax the overall leverage requirement, which is feasible because inflation decreases the real deposit rate and thus, raises bank profits. As a result, higher inflation is associated with higher bank profits and lower production in the frictional market. Unlike the monetary economics literature where the cost of inflation comes about through households' lower demand for money, here it is through the banking channel and its impacts on bank liquidity provision.

**Related Literature**

Our framework inherits features from two strands of the literature, and here we focus on the key differences and the novelty of our model. The first includes studies on financial intermediation and credit market frictions which build on, e.g., Kiyotaki and Moore (1997, 2001), and the second includes those that explicitly model the transaction role of means of payment using models that are based on, e.g., Lagos and Wright (2005) and Rocheteau and Wright (2005). Regarding the first strand, our pledgeability constraint is similar to that in Gertler and Kiyotaki (2010). There are two key differences, however. First, we explicitly model deposits as a means of payment and focus on banks' liquidity role for depositors. In particular, our model features an essential reserve requirement where reserves are fiat objects. Second, we focus on banks' limited commitment and moral hazard problem and optimal regulations to deal with both. Our approach to modeling bank assets as (one-period) Lucas trees effectively assumes that all agency issue between the banks and the end borrowers is captured by the management and monitoring cost, an approach shared by some recent papers such as Begenau and Landvoigt (2017).

Regarding the second strand, in Williamson (2012, 2016) both fiat money and deposits are used as means of payment, but he assumes that in some transactions deposits cannot be used and hence, money is valued even though it is dominated in the return. Our model integrates banking and fiat money (reserves) via reserve requirements, and demonstrates the essential role of both institutions for welfare. In addition, Williamson (2016) considers that bank deposits are backed by different fractions of long- and short-maturity assets and equity, subject to pledgeability constraints in the style of Kiyotaki and Moore (1997, 2001). Gu, Mattesini, Monnet, and Wright (2013) consider an environment where bank deposits enable intertemporal exchange, but banks have limited commitment and deposits are unsecured. Monnet and Sanches (2015) study the incentive problem when banks' deposit issuance is not observable. In both papers, like ours, charter value (or future bank profit) is the crucial instrument proposed for dealing with limited commitment.[3] In contrast to papers in this strand, our model with efficiency concerns in bank asset management gives rise to a new trade-off between bank profits and inefficient bank size, with implications for the secured and unsecured components in optimal leverage regulations, as well as optimal concentration in the banking sector.[4] Moreover, our framework also generates novel implications of monetary policy to optimal bank regulations and bank profits.

Our paper is also related to those that study the moral hazard issue in banking. Hellmann, Murdock, and Stiglitz (2000), in a model where banks have market power and face moral hazard, show that it is optimal to use a combination of capital requirements and deposit-rate ceilings, and the latter instrument is important to allow for bank profits and deter gambling. In our model with

[3]Keeley (1990) provides some evidence that charter value restricts banks' risk-taking behavior.

[4]Phelan (2016) considers optimal leverage regulation in a model with deposit-in-utility function. The focus in that paper, however, is very different from ours. In his model, bank leverage increases asset price volatility, and he shows that limiting leverage decreases the likelihood that the financial sector is under-capitalized. In addition, he finds that when banks are subject to endogenous borrowing constraints (e.g. procyclical leverage), countercyclical leverage regulation can improve welfare.

endogenous bank entry, imposing an interest ceiling will be suboptimal. Christiano and Ikeda (2016) study a similar moral hazard issue, and they show that imposing a leverage regulation that limits banks' borrowing may induce banks to exert proper efforts. As in our model, restricting bank leverage also imposes a restriction on bank size. In contrast to these two papers, however, our model features endogenous liquidity provision from banks, which enters welfare explicitly through the transactional role of deposits. This role gives a novel policy implication: while severe moral hazard would impose a stringent proportional capital requirement, the unsecured component in the leverage regulation should be relaxed to improve liquidity by way of higher bank profits and larger bank balance sheets.

# 2 The Environment

The environment is borrowed from Lagos and Wright (2005). Time is discrete and has an infinite horizon, $t \in \mathbb{N}_0$. The economy is populated by three sets of agents, and each set has a unit measure of them. The first set consists of *households*, the second consists of *banks*, and the third consists of *entrepreneurs*. Each date has two stages: the first stage has random pairwise meetings between households in a decentralized market (called the DM), and the second has a centralized market (called the CM) where all agents meet. In each DM, a household with a successful meeting can either be a *consumer* or a *producer*, and the probability that a household has a successful meeting and becomes a consumer is $\sigma \leq 1/2$, and the probability that a household has a successful meeting and becomes a producer is also $\sigma$. There is a single perishable good produced in each stage, with the CM good taken as the numéraire. Households' labels as consumers and producers depend on their roles in the DM, where only producers are able to produce and only consumers wish to consume. All households can produce and consume in the CM. Banks do not consume or produce in the DM but do so in the CM, and entrepreneurs consume only in the CM and do not produce.

A household's preference is represented by the following utility function

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[u(q_t)-c(q_t)+x_t-h_t\right],$$

where $\beta\equiv(1+r)^{-1}\in(0,1)$ is the discount factor, $q_t$ is DM consumption or production (depending on the household's role), $x_t$ is CM consumption, and $h_t$ is the supply of hours in the period-$t$ CM. The first-stage utility functions, $u(q)$ and $-c(q)$, are increasing and concave, twice differentiable, and $u(0)=c(0)=0$. The surplus function, $u(q)-c(q)$, is strictly concave, with $q^{*}=\arg\max\left[u(q)-c(q)\right]$. Moreover, $u'(0)=c'(\infty)=\infty$ and $c'(0)=u'(\infty)=0$. All agents have access to a linear technology to produce the CM good from their own labor, $x=h$. Banks, however, have limited capacity to produce in the CM, and each can produce up to $\bar{h}$ units.[5] In the DM there are limitations in commitment, enforcement and record keeping, and except for banks, agents are anonymous. Therefore, means of payment are necessary to facilitate trades in the DM.

There is only one real asset, projects from entrepreneurs. Each entrepreneur is endowed with $\bar{A}$ projects, which, for simplicity, are assumed to materialize within a single period, and each unit has a gross return $\tau$ (in terms of the CM good) that pays off in the next CM. However, only banks have the required expertise to perform costly monitoring and management to receive the return from these projects, and, to do so, a bank has to become active and purchase these projects from a competitive market. For simplicity we assume that even the entrepreneurs themselves cannot obtain the returns without banks, and that they simply sell their projects to banks.[6]

On the liability side, each bank can issue deposit certificates to finance its asset purchase in the open market. Since banks have limited capacity of CM production, this borrowing is crucial. We assume that these certificates are perfectly divisible and cannot be counterfeited. Such liabilities are payable to the bearer. Thus, households may use such certificates to finance

[5] We can introduce a similar capacity constraint on households, but sufficiently large not to bind.

[6] As in Gertler and Kiyotaki (2010), one can also think of the bank's claim on these projects as equity. We could have assumed that the projects require input from CM goods that can only come from external financing and then considered formal loan contracts between banks and entrepreneurs. However, this will complicate the exposition without adding any insights.

their consumptions in the DM. There is a public record of banks' liabilities and asset holdings, but there is no record keeping of households' deposit holdings and their transaction records.[7]

There are two frictions associated with this financial intermediation. The first friction is the cost associated with managing and monitoring the projects.[8] Only active banks can hold assets and issue deposits; to become active, a bank has to pay a fixed cost of $\gamma$ each period. There is also a cost of asset management: for an active banker to hold $a$ projects, he needs to pay $\psi(a)$ to monitor and manage them.[9] We assume that $\psi(0) = 0$, $\psi(a)$ is twice differentiable, strictly increasing, and strictly convex.

Second, banks have limited liability and cannot commit to future actions. If a bank files for bankruptcy, the court could seize up to a fraction of $\rho < 1$ of the value of the bank's project holdings. Thus, a bank can credibly pledge up to $\rho$ fraction of the returns from the projects it has invested in but it can run away with $1 - \rho$ fraction of the returns. Banks maximize their life-time profits with discount factor $\beta$.

Finally, we define social welfare in our economy. An allocation consists of both the quantity of goods traded in each successful DM meeting, denoted by $q$, and the number of active banks, denoted by $m$ (and hence the amount of asset holding for each bank is $\bar{A}/m$). Given an allocation $(q, m)$, the total welfare is given by

$$\mathcal{W}(q,m) = \underbrace{\sigma[u(q) - c(q)]}_{(a)} - \underbrace{\big[m\psi(\bar{A}/m) + m\gamma\big]}_{(b)}, \tag{1}$$

in which term (a) evaluates efficiency in DM production, $q$, and term (b) evaluates efficiency in asset management that depends on $m$. The first-best allocation, defined as the allocation $(q, m)$ that maximizes $\mathcal{W}(q, m)$, is denoted by $(q^*, m^*)$. The allocation $q^*$ maximizes term (a) and $m^*$

[7] A historical example of this deposit claim is banknotes, and a modern counterpart is stored-value cards issued by banks. We can also introduce record keeping of households' deposits holdings and there will be room for credit card issuance, but that would complicate the analysis without affecting our main results.

[8] This assumption is consistent with the delegated monitoring model of Diamond (1983) and Williamson (1986). However, given our focus on liquidity provision by banks, we do not model the agency problem between banks and entrepreneurs in detail.

[9] This cost of asset management should be interpreted as the cost of labor that a bank incurs in addition to that which it expends in its CM production; here we assume that there is no bound on this cost but since $\bar{h}$ limits the asset holding, it also implicitly limits this cost.

minimizes term (b), and they satisfy the following FOC's: $u'(q^*) - c'(q^*) = 0$ and $\Pi(\bar{A}/m^*) = \gamma$, where

$$\Pi(a) = \psi'(a)a - \psi(a). \tag{2}$$

To ensure that $m^* < 1$, we assume

$$\Pi(\bar{A}) < \gamma < \Pi\left[(\psi')^{-1}\left(\frac{\tau}{1+r}\right)\right]. \tag{3}$$

We also assume that $\Pi(a)$ is convex (this is satisfied if $\psi(a) = \lambda a^x/x$, $x > 1$, for example).

We remark here that $m^*$ can be implemented if banks cannot issue debt and if there has been no limit on banks' capacity to produce CM goods to finance their asset holdings. Given the asset price, $\phi$, a bank chooses the asset holdings to maximize profits given by

$$\pi(a;\phi) = -\phi a - \gamma - \psi(a) + \beta\tau a.$$

The optimal asset holding then solves $-\phi + \beta\tau = \psi'(a)$. By substituting this FOC back into $\pi(a;\phi)$, we obtain banks' profits under optimal asset holding of $a$ projects as $\Pi(a) - \gamma$. For any given measure of active banks, $m$, market clearing requires $\phi = \beta\tau - \psi'(\bar{A}/m)$, and each active bank's profit is given by $\Pi(\bar{A}/m) - \gamma$. The equilibrium number of banks, $m^*$, is pinned down by free entry, which requires $\Pi(A/m^*) = \gamma$, where $m^*$ is also the first-best number of active banks. Note that assumption (3) ensures that a unit measure of banks is sufficient to provide free entry to the banking sector. The equilibrium asset price is then

$$\phi^* = \frac{\tau}{1+r} - \psi'\left(\frac{\bar{A}}{m^*}\right), \tag{4}$$

which can be regarded as the *fundamental value* of projects. Note that $\phi^* > 0$ by (3).

We make the following assumption about $\bar{h}$, the capacity of a bank's CM production:

$$(1-\rho)\phi^*\frac{\bar{A}}{m^*} < \bar{h} < \phi^*\frac{\bar{A}}{m^*}. \tag{5}$$

The first inequality in (5) ensures that each bank's capacity is sufficiently large enough to furnish equity to finance the fraction, $1-\rho$, of the efficient asset holdings that cannot be credibly financed by issuing debt; the second inequality implies that each bank's capacity to raise equity is limited and external financing is necessary for the efficient asset holdings.

# 3 Bank contracts

In this section we consider equilibrium bank contracts with households. We begin with free entry without any regulations, and highlight the potential inefficiency under this free-market arrangement. We then introduce regulations in two steps: first we consider the optimal reserve requirement in Section 3.2, and then we consider an optimal charter system in Section 4; in both cases we maximize the social welfare but respect voluntary participation and incentive compatibility due to the limited commitment of all agents and the anonymity of households.

We first describe the time line and the general characteristics of the bank contracts.

**The course of events.** In the period-$t$ CM, the course of events is as follows:

1. banks settle obligations to holders of deposit certificates issued in period $t-1$;
2. banks buy projects in the competitive market at price $\phi_t$ (in terms of CM good);
3. banks issue deposit contracts, promising a gross return $R_t$ (in exchange for CM good).

We use $d$ to denote the total amount of deposits that the bank promises to give out in the next CM (and hence it will receive $d/R_t$ in the current period-$t$ CM). Note that there are two different spot markets in the CM—one for assets and the other for deposits. Because only banks can monitor and manage entrepreneurial projects, with no loss of generality we assume that households do not participate in the asset market.

In the DM, upon a successful meeting with a producer, the consumer makes a take-it-or-leave-it offer, $(q, p)$, where $q$ is the DM production from the producer and $p$ is the amount of deposit transfer (in terms of the coming CM goods).

## 3.1 Static bank contracts

Here we consider free entry of banks. We call this situation "static" since free entry implies a zero-profit condition, which in turn implies that banks cannot credibly promise any repayment beyond what could be seized by the court due to limited commitment. This then gives rise to the following pledgeability constraint,

$$d \leq \rho\tau a, \tag{6}$$

which limits the amount of a bank's deposit issuance. In this sense, the bank deposits are collateralized liabilities.

Given $R_t$ and $\phi_t$, a bank's profit from holding $a$ projects and issuing $d$ deposits at period-$t$ CM is given by

$$\begin{aligned} \pi(a,d;\phi_t,R_t) &= \frac{d}{R_t} - \phi_t a - \gamma - \psi(a) + \beta\{\tau a - d\} \qquad (7) \\ &= \beta\left\{\iota_t d + [\tau - (1+r)\phi_t]a - (1+r)[\psi(a)+\gamma]\right\}, \\ \text{where} \qquad & \iota_t \equiv \frac{1+r}{R_t} - 1. \end{aligned}$$

The variable $\iota_t$ will play a prominent role in our analysis. For banks, it measures the wedge between the gain to banks from deposit issuance and the return banks promise to pay depositors; a positive $\iota_t$ means a positive margin for banks' profits from issuing deposits. As we will see below, $\iota_t$ also measures the cost of holding deposits for households across periods.

A bank chooses $a$ and $d$ to maximize (7) subject to (6). Note that whenever $\iota_t > 0$, the constraint (6) is binding, and the optimal level of asset holdings, denoted by $A(\phi_t, \iota_t)$, is determined by the following FOC when $\bar{h}$ is not binding:

$$-(1+r)\phi_t + (1+\iota_t\rho)\tau = (1+r)\psi'(a); \tag{8}$$

Otherwise, $A(\phi_t, \iota_t) = \bar{h}/(\phi_t - \rho\tau/R_t)$. In general, because of the pledgeability constraint, banks need to produce in the CM to finance part of their asset holdings. Thus, the pledgeability constraint works as a proportional capital requirement imposed by market discipline.

Now we turn to households' behavior. We use $V_t(d)$ to denote a household's continuation value upon entering period-$t$ DM with deposit $d$, and use $W_t(d)$ to denote a household's continuation value upon entering period-$t$ CM with deposit $d$, facing a sequence of bank returns $\{R_t\}_{t=0}^{\infty}$. Note that $d$ is the *promised value* of the deposit in the coming CM. Assuming that banks repay their deposit obligations (which will be the case in equilibrium), standard Lagos-Wright (2005) arguments show that $W_t(d) = d + W_t(0)$; that is, $W_t$ is linear in $d$ (see Appendix B1 for details). Now we consider the household's DM problem. The consumer with deposit $d$ facing a producer with deposit $d'$ solves the following problem:

$$\begin{aligned} &\max_{(p,q)} u(q) + W_t(d-p), \\ \text{subject to} \quad & p \leq d, \ -c(q) + W_t(d'+p) \geq W_t(d'), \end{aligned} \tag{9}$$

in which $p$ is the transfer of deposits and $q$ is the DM production for the consumer. Given the linearity of $W_t$, it is straightforward to see the solution to (9) is given by

$$\begin{aligned} q(d) = c^{-1}(d) \quad &\text{and} \quad p(d) = d \text{ if } d < c(q^*); \\ q(d) = q^* \quad &\text{and} \quad p(d) = c(q^*) \text{ otherwise.} \end{aligned} \tag{10}$$

Given $R_t$, a household's deposit choice in the CM is given by

$$\max_{d\geq 0} -\frac{d}{R_t} + \beta\{\sigma\{u[q(d)] + W_{t+1}[d-p(d)]\} + (1-\sigma)W_{t+1}(d)\},$$

where $[q(d), p(d)]$ is given by (10). By linearity of $W_{t+1}$, we can simplify the problem into

$$\max_{d\geq 0} -\frac{d}{R_t} + \beta\left\{\sigma[u[q(d)] - c[q(d)]] + d\right\}, \tag{11}$$

with the FOC

$$\iota_t = \frac{\sigma\{u'[q(d)] - c'[q(d)]\}}{c'[q(d)]}. \tag{12}$$

Let $D(\iota_t)$, the deposit demand per household, be the solution to (12). Note that since $u(q) - c(q)$ is strictly concave, for any $\iota > 0$, $D(\iota)$ is uniquely determined; when $\iota = 0$, $D(\iota)$ is not pinned down but $D(\iota) \geq c(q^*)$. Without loss of generality we set $D(0) = c(q^*)$, the minimum value of $D(\iota)$ when $\iota = 0$. Then, $D(\iota)$ is continuous and strictly decreasing in $\iota$.

We restrict our attention to stationary equilibria, where goods exchanged in the DM and the real value of projects are constant over time; i.e., $q_t = q_{t+1}$ and $\phi_t = \phi_{t+1}$, for all $t$, and in what follows we drop the time subscript $t$. Equilibrium requires market clearing conditions for deposits and assets:

$$D(\iota) \begin{cases} \leq \rho\tau\bar{A} \text{ if } \iota = 0, \\ = \rho\tau\bar{A} \text{ if } \iota > 0, \end{cases} \tag{13}$$

$$mA(\phi, \iota) = \bar{A}. \tag{14}$$

The right side of (13), $\rho\tau\bar{A}$, is derived from binding (6) and the market-clearing condition (14). In (13), we have inequality because (6) does not have to bind when $\iota = 0$. Finally, free entry implies that all active banks earn zero profits.

**Lemma 3.1.** *There is a unique equilibrium allocation, $(m, \phi, \iota, q, d)$, in which $m = m^*$ and $\bar{h}$ is not binding, and $(\phi, \iota, q, d)$ is characterized as follows.*
*(a) In equilibrium $\phi = \phi^*$, $q = q^*$, and $\iota = 0$ if*

$$\rho\tau\bar{A} \geq c(q^*). \tag{15}$$

*(b) Otherwise, $\iota > 0$, $q = c^{-1}[D(\iota)] < q^*$, and*

$$\phi = \frac{(\iota\rho + 1)\tau}{1 + r} - \psi'\left(\frac{\bar{A}}{m^*}\right). \tag{16}$$

In Lemma 3.1, the equilibrium $\iota$ is uniquely determined by (13) with $A(\phi, \iota) = \bar{A}/m^*$, which follows from (14) and free entry. When the real pledgeable value of the assets, $\rho\tau\bar{A}$, is small (e.g., pledgeability is low, or returns on projects are small), $\iota$ can be large and possibly larger than $r$; hence, the deposit contract has a negative net return. However, in a system with fiat money and no banks, a monetary equilibrium with zero net return exists and would dominate such an equilibrium with banking. In the next section we show that a banking authority, by imposing a reserve requirement under free entry, would make the banking system essential, which dominates fiat money in terms of social welfare.

## 3.2 Reserve requirements

Here we introduce a banking regulator who imposes a reserve requirement, but we maintain free entry. The bank reserve is a nominal liability of the regulator. As such, banks can fully pledge their reserve holdings. We assume that the reserve is of constant supply, bears no interest, and has a zero net return. The reserve thus may be regarded as outside money for the banking system, and can be used as an instrument to settle interbank payments.

As before, we focus on stationary equilibria where the reserves are traded at a constant price over time. Consider a bank that holds $a$ units of projects and $z$ units of reserve (measured in terms of coming CM good), and issue $d$ units of deposits. Given the return on deposits, $R$, and the price for projects, $\phi$, such a bank's profit is given by

$$\begin{aligned} \pi(a, d, z; \phi, R) &= \frac{d}{R} - \phi a - z - \gamma - \psi(a) + \beta\{\tau a + z - d\} \\ &= \beta\left\{\iota d + [\tau - (1+r)\phi]a - rz - (1+r)[\psi(a) + \gamma]\right\}, \end{aligned} \tag{17}$$

and is subject to the pledgeability constraint and reserve requirement,

$$d \leq \rho\tau a + z, \tag{18}$$

$$z \geq \eta d, \tag{19}$$

and the bound on CM production, $\bar{h}$. According to (18), reserves are fully pledgeable. Constraint (19) is the reserve requirement, with $\eta$ as the fraction of deposits required to be backed by reserves.

The solution to the bank problem depends on the spread, $r - \iota$, and whether the constraints (18) and/or (19) are binding. Assuming that (18) is binding, the profit can be written as

$$\pi(a, d, z; \phi, R) = \beta \left\{ [(\iota\rho + 1)\tau - (1 + r)\phi]a - (r - \iota)z - (1 + r)[\psi(a) + \gamma] \right\}.$$

Thus, when $\iota > r$, there is infinite demand for reserves, which cannot be an equilibrium. When $\iota = r$, banks are indifferent between holding any amount of reserves satisfying (19). Now consider $\iota < r$. In that case, (19) is binding and the profit becomes

$$\pi(a, d, z; \phi, R) = \beta \left\{ (\iota - \eta r)d + [\tau - (1 + r)\phi]a - (1 + r)[\psi(a) + \gamma] \right\}.$$

Thus, it is optimal to have $d > 0$ only if

$$\iota - \eta r \geq 0, \tag{20}$$

which states that the profit margin of issuing deposits, $\iota$, must be greater than the marginal cost of holding reserves for issuing deposits, $\eta r$.

In sum, for $\iota \in [0, r]$, the bank profit can be written as

$$\pi(a, d, z; \phi, R) = \beta \left\{ \frac{\max\{\iota - \eta r, 0\}}{1 - \eta} \rho\tau a + [\tau - (1 + r)\phi]a - (1 + r)[\psi(a) + \gamma] \right\}. \tag{21}$$

We use $A^r(\phi, \iota)$ to denote the asset demand that maximizes (21) subject to the bound on CM production, $\bar{h}$. The demand for reserves, denoted by $\bar{z}(\phi, \iota)$, is determined as follows. If $\iota < \eta r$, it is optimal not to issue any deposits and hold any reserves, i.e., $\bar{z}(\phi, \iota) = 0$. If $\iota \in (\eta r, r)$, both (18) and (19) are binding, and

$$\bar{z}(\phi, \iota) = \frac{\eta}{1 - \eta} \rho\tau A^r(\phi, \iota). \tag{22}$$

If $\iota = r$, any level of reserve holdings above the right side of (22) is optimal, and we should think of $\bar{z}(\phi, r)$ as a correspondence. A household's problem is still given by (11), and the deposit demand, $D(\iota)$, remains the same. Finally, for a given $\eta$ and $m$, we have the following market-clearing conditions:

$$mA^r(\phi, \iota) = \bar{A}, \tag{23}$$

$$D(\iota) \begin{cases} \le \frac{1}{1-\eta}\rho\tau\bar{A} \text{ if } \iota = 0 \text{ and } \eta = 0, \\ = \frac{1}{1-\eta}\rho\tau\bar{A} \text{ if } \iota \in (\eta r, r) \text{ or } \iota = \eta r > 0, \\ \ge \frac{1}{1-\eta}\rho\tau\bar{A} \text{ if } \iota = r, \end{cases} \tag{24}$$

where the right side of (24) is derived from binding (18), which requires $\iota \ge \eta r$, and (22) and (23); the last inequality appears in (24) because (18) binds while (19) may not when $\iota = r$.

**Lemma 3.2.** *Suppose that (15) does not hold. There exists a threshold $\overline{\eta} \in (0, 1)$ such that there is an equilibrium in which active banks issue deposits if and only if $\eta \le \overline{\eta}$. In that case, $m = m^*$ and $\bar{h}$ is not binding. We have two subcases.*
*(a) If $\eta \le \underline{\eta} \equiv 1 - \rho\tau\bar{A}/D(r) < \overline{\eta}$, then equilibrium $\iota = r$.*
*(b) If $\underline{\eta} < \eta \le \overline{\eta}$, equilibrium $\iota = \iota(\eta) \in [0, r)$ satisfies (20) and*

$$\phi = \frac{\left(1 + \rho\frac{\iota - r\eta}{1-\eta}\right)\tau - \psi'\left(\frac{\bar{A}}{m^*}\right)(1+r)}{1+r}; \tag{25}$$

*moreover, equilibrium $\iota$ is determined by (24) with equality, and it is strictly decreasing in $\eta$.*

By Lemma 3.1 (a), the reserve requirement can potentially improve welfare only when (15) does not hold. In this case, according to Lemma 3.2, there are three regions of equilibria depending on the value of $\eta$. When the reserve requirement is too stringent ($\eta > \overline{\eta}$), banks find it unprofitable to issue deposits and hence the reserve requirement is not effective. Otherwise, all active banks issue deposits, and the equilibrium achieves full efficiency in terms of asset management as $m = m^*$. When the reserve requirement is too loose ($\eta \le \underline{\eta}$), Lemma 3.2 (a) states that $\iota = r$, and the equilibrium quantity of output traded in the DM would be the same as that in a system with fiat money as the only means of payment. Note that it is possible that $\underline{\eta} < 0$ and hence this region may not exit.

Finally, when $\eta$ is in an intermediate range, Lemma 3.2 (b) states that $\iota < r$ and the reserve requirement can implement a better allocation than fiat money. In this case, equilibrium $\iota$ is strictly decreasing in $\eta$. Intuitively, since the right side of (24) is strictly increasing in $\eta$, raising $\eta$ helps increase liquidity provision, but it is also constrained by (20). Indeed, the threshold $\overline{\eta}$ is obtained by substituting $\eta$ with $\iota/r$ into (24); that is, $\overline{\eta} = \iota/r$ and $\iota$ solves

$$D(\iota) = \frac{r}{r-\iota}\rho\tau\bar{A}. \tag{26}$$

The fact that (15) does not hold implies that $\overline{\eta} > 0$. We have the following theorem.

**Theorem 3.1.** *Suppose that (15) does not hold. The optimal $\eta = \overline{\eta} > 0$.*

According to Theorem 3.1, when (15) does not hold, the optimal reserve requirement is strictly positive. The reason is as follows. For all $\eta \in [0, \overline{\eta}]$, $m = m^*$ and hence term (b) in the welfare expressed by equation (1) is always at the optimum. Because $\iota$ is strictly decreasing in $\eta$ and equilibrium $q = c^{-1}[D(\iota)]$ is decreasing in $\iota$, term (a) in (1) is maximized at $\eta = \overline{\eta}$. To implement $\overline{\eta}$, the regulator has to first anticipate the equilibrium $\iota$ given by (26) to set the optimal reserve requirement given by $\overline{\eta} = \iota/r$. The market participants will take that as given, and (26) ensures that the equilibrium $\iota$ is as anticipated.

Theorem 3.1 also shows that both reserves and reserve requirement are essential for welfare. Indeed, since $\overline{\eta} > 0$ and (26) implies that under $\overline{\eta}$ equilibrium $\iota < r$, the DM allocation under the optimal reserve requirement is better than in a system with fiat money as the only means of payment. Moreover, since a reserve system without a reserve requirement is equivalent to setting $\eta = 0$, Theorem 3.1 shows that it is essential for welfare to have a reserve requirement. The reserve requirement improves welfare for two reasons: reserve money is fully pledgeable, and mandatory holding ensures that banks hold the fully pledgeable reserves proportional to their asset holdings. The reserve requirement in fact performs asset transformation for the economy with banking—by choosing the requirement optimally, the regulator can ensure that banks compete to guarantee a positive return on deposits.[10]

Now we discuss bank leverage, defined as the ratio between the value of debt and the value of assets a bank has. Since reserves are completely secured in our model, we exclude the value of reserves in our definition.[11] An active bank's balance sheet has *book value* of assets (in terms of coming CM good) equal to $\tau\frac{\bar{A}}{m^*}$, and deposits (excluding portions backed by reserves) equal to $\rho\tau\frac{\bar{A}}{m^*}$. Thus, the leverage ratio is given by $\rho < 1$, the same as the one without reserve requirement.

Finally, despite the optimal use of bank reserves, the amount of liquidity banks can provide is still bounded by the pledgeable asset supply. In fact, if the first-best is not implementable without the reserve system, it is still not implementable with it. In the next section we introduce a charter system that can relax further the pledgeability constraint.

*Remark* 3.1. One may wonder whether there exists an equilibrium where households trade expired deposit certificates, and we argue this would not occur. We assume one-period deposit contracts, and banks do not pay further interest for expired deposit certificates. In a stationary equilibrium, the redemption value of a deposit certificate is $d$, regardless of the issuing date. Therefore, if anyone ever holds the expired certificates and tries to trade, sellers do not produce more output for them than for the unexpired ones. Also, sellers will not delay the redemption of certificates due to discounting. Moreover, since in equilibrium $i < r$, new deposit contracts pay positive interest, and hence, no household would buy expired deposit certificates in the CM, and no one would hold expired certificates.

[10] Our result that reserve requirements are essential echoes the findings in Hu and Rocheteau (2013), in which coexistence of money and assets with higher rates of returns is a feature of optimal mechanism. The underlying economics is very different, however. In that paper the optimal mechanism uses fiat money to correct overaccumulation of capital.

[11] This definition is consistent with the view that leverage regulation should be separated from bank reserve management and monetary policy, which motivated Prudential Regulation Authority in UK to modify its regulation on leverage ratio to exclude claims on the central bank such as reserves (see policy statement "UK Leverage Ratio treatment of claims on central banks – PS21/17" published on 3rd Oct 2017). Moreover, most of our results are robust to the introduction of reserves into the calculation, but that makes both conceptual and technical analysis more complicated.

# 4 Charter system

Modern banks are highly regulated and entry is restricted through permission from a banking regulator, as in the US charter system. Under a charter system, only chartered banks have the privilege to issue deposits, but all active banks, chartered or unchartered, can hold assets. Here we show that such a system, by restricting entry to allow for bank profits and by potentially depriving banking privilege to discipline possible opportunistic behavior, can relax the pledgeability constraint and improve the efficiency of liquidity provision by banks.

## 4.1 Optimal leverage regulation

Under the charter system, we introduce two new policy instruments. The first is the number of banking licenses, denoted by $m$, and only banks with licenses can issue deposits. The second is the amount of unsecured deposits each chartered bank can issue, and it is formulated with the following more relaxed pledgeability constraint and reserve requirement:

$$d \leq \rho\tau a + z + \kappa, \tag{27}$$

$$z \geq \max\{0, \eta(d - \kappa)\}. \tag{28}$$

Compared to (18), (27) implies that the regulator allows each chartered bank to issue $\kappa$ units of additional deposits not backed by pledged assets. Compared to (19), (28) allows the unsecured component, $\kappa$, to be exempted from the reserve requirement as well.[12]

Bank profit is still given by (17); thus, it is always optimal to choose $d \geq \kappa$. As before, we can focus on the case where $\iota \in [\eta r, r)$ and hence both (27) and (28) are binding, and the profit is given by

$$\pi(a, d, z; \phi, R) = \beta\left\{\frac{\iota - \eta r}{1 - \eta}\rho\tau a + [\tau - (1 + r)\phi]a - (1 + r)[\psi(a) + \gamma] + \iota\kappa\right\}. \tag{29}$$

[12]In principle, under the charter system the regulator could also choose a different (and presumably larger) $\rho$. In Appendix B2 we show that (27) and (28) indeed describe the optimal regulations under a mild sufficient condition, and show that our results are robust to a more general setup.

Compared with (21), (29) has an additional term, $\beta\iota\kappa$, that reflects additional profit from unsecured deposit issuance. This term does not affect optimal asset holdings, and asset demand is still given by $A^r(\phi, \iota)$ and reserve demand $\bar{z}(\phi, \iota)$.

Here we assume that only chartered banks are active and there are measure $m$ of them; later on we verify that unchartered banks have no incentives to hold assets. We also focus only on policies under which equilibrium $\iota < r$; this is with no loss of generality to study optimal policies. Given $\kappa$ and $m$, the market-clearing conditions for deposits and assets are

$$D(\iota) \le \frac{\rho\tau}{1-\eta}\bar{A} + m\kappa \tag{30}$$

with equality whenever $\iota > 0$, and (23). The first term of the right side of (30) is the same as the right side of (24); the second term comes from the unsecured deposits that each chartered bank can issue.

**Lemma 4.1.** *Let $m$ be a given measure of active banks, and let $\kappa$ be given. Assuming that all active banks issue deposits and that $\bar{h}$ does not bind in equilibrium, there exist thresholds $\underline{\eta}(\kappa, m)$ and $\overline{\eta}(\kappa, m)$ such that for all $\eta \in (\underline{\eta}(\kappa, m), \overline{\eta}(\kappa, m)]$, there is a unique allocation $(\phi, \iota, q, d)$ that satisfies the market-clearing conditions, in which $\phi$ is given by (25) with $m^*$ replaced by $m$, and equilibrium $\iota \in [\eta r, r)$. Moreover, the profit for each bank is given by*

$$\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota\kappa}{1+r}.$$

The general equilibrium analysis requires verification of three incentive compatibility constraints and that banks can furnish necessary capital under the bound on CM production, $\bar{h}$. First, we replace the upper bound $\overline{\eta}(\kappa, m)$ for $\eta$ by the constraint (20), which ensures that banks are willing to issue deposits beyond $\kappa$. Second, banks have to be willing to repay the unsecured component, $\kappa$. Since the court can seize the reserves and only $\rho$ proportion of the bank's projects, the bank faces a temptation to not repay the $\kappa$ component of its liability in (27). To deter this temptation, the regulator can remove the charter and stop the bank from conducting future business if the bank fails to honor its deposit obligations. Thus, if a bank defaults, it loses the pledged assets, $\rho\tau\bar{A}/m$, and the reserves, $z$, as well as the charter to run the business, beginning from the period when it fails to repay. As a result, a bank is willing to repay deposits if and only if

$$-\kappa - \rho\tau\bar{A}/m - z + \sum_{t=0}^{\infty}\beta^t\left\{\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota\kappa}{1+r}\right\} \geq -\rho\tau\bar{A}/m - z.$$

This constraint can be simplified to

$$-(r-\iota)\kappa + (1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right] \geq 0. \tag{31}$$

Finally we need to consider incentives for unchartered banks to become active and to hold assets and the feasibility of chartered banks furnishing necessary capital. We restrict our attention to equilibria in which only chartered banks are active for a given measure of charters, $m$, and discuss for which $m$ such an equilibrium exists. Giving more charters than $m^*$ is ineffective; it is equivalent to free entry. For $m$ smaller but close to $m^*$, the second inequality in (5) ensures that unchartered banks cannot finance the unconstrained optimal asset holding and the fixed cost $\gamma$ would deter them from entry. The first inequality in (5) ensures that chartered banks have sufficient capacity to furnish the necessary capital.

However, when $m$ becomes too small, chartered banks' asset holdings become inefficiently large and that depresses asset prices according to (25). A low $\phi$ would attract unchartered banks to hold assets, even though due to $\bar{h}$ they cannot hold the unconstrained optimal amount. Moreover, for very small $m$ and hence very large asset holdings per chartered bank, each has to furnish more capital for its asset holdings, although the need for capital may be offset by higher unsecured borrowing. In Appendix B3 we show that, for any given $\iota$ and $\eta$, there exists the smallest threshold $\bar{m}(\iota,\eta) < m^*$ such that unchartered banks will not hold assets and active banks can furnish the necessary capital under $\bar{h}$ for all $m \geq \bar{m}(\iota,\eta)$ (in fact, for $r$ not too large, chartered banks can always furnish the necessary capital under condition (5)). We also show there that without loss of generality we need not consider equilibria in which unchartered banks hold assets; for any $m$ with such an equilibrium, a slightly larger $m$ has a better allocation. In sum, we have another constraint for the regulator's problem:

$$\bar{m}(\iota, \eta) \leq m \leq m^*. \tag{32}$$

We use $\bar{m}$ to denote the maximum of $\bar{m}(\iota, \eta)$ among $\eta$'s that satisfy (20) and $\iota \in [0, r]$. Under condition (5) $\bar{m} < m^*$. We then summarize all the incentive compatibility conditions in the following definition.

**Definition 4.1.** A policy $(m, \eta, \kappa)$ is *implementable* if there exists an $\iota$ such that (20), (30), (31), and (32) hold, and, if implementable, the corresponding allocation is given by $q = D(\iota)$ and $m$.

Our strategy for solving for the optimal policy is first to assume that (32) does not bind to simplify the problem, and then to consider the case when it does. Assuming that (32) does not bind, we follow a similar methodology in Theorem 3.1 to solve for optimal $\eta$ and optimal $\kappa$: we maximize the right side of (30) subject to the incentive constraints, (20) and (31), for a given $m$ and $\iota$. Note that the optimal $m$ cannot be solved in the same way, as it enters the term (b) in welfare defined in (1) directly, while $\eta$ and $\kappa$ affects only $q$ in term (a) through equilibrium $\iota$. We have the following lemma.

**Lemma 4.2.** *Let $m \leq m^*$ and $\iota < r$ be given. The optimal $(\eta, \kappa)$ that solves*

$$\mathcal{S}(\iota, m) = \max_{\eta, \kappa} \frac{\rho\tau}{1-\eta} \bar{A} + m\kappa, \tag{33}$$

*subject to (20) and (31) is given by $\eta = \iota/r$ and*

$$\kappa = \frac{(1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right]}{r - \iota}. \tag{34}$$

Given Lemma 4.2, we solve for optimal policy parameters as follows. First, for any given

$(\iota, m)$, we set $\kappa$ and $\eta$ according to Lemma 4.2 and use the following equilibrium condition:

$$D(\iota) \leq \mathcal{S}(\iota, m) = \frac{r\rho\tau}{r-\iota}\bar{A} + m\frac{(1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right]}{r-\iota}, \tag{35}$$

with equality whenever $\iota > 0$. Thus, for any $m$, there is a unique corresponding $\iota$. Note that such an $\iota$ determines $q$ and is increasing in $m$. The optimal policy involves a trade-off between term (a) and term (b) in welfare (1): a lower $m$ would raise term (b) and inefficiency in asset management, but it allows for a lower $\iota$ and increases efficiency in DM production. As before, the regulator has to set the optimal $m$ and to anticipate equilibrium $\iota$, and then to set $\kappa$ and $\eta$ according to (34) and $\eta = \iota/r$ using the anticipated $\iota$. The equilibrium condition (35) ensures that under those policy parameters the anticipated $\iota$ coincides with the actual equilibrium $\iota$.

**Theorem 4.1.** *There exists an optimal policy $(m, \eta, \kappa)$ that maximizes welfare subject to implementability.*
*(a) If (15) holds, then $(m, \eta, \kappa) = (m^*, 0, 0)$ is an optimal policy.*
*(b) Suppose that (15) does not hold. Then, any optimal policy has $m < m^*$ and $\kappa > 0$.*

In the proof of Theorem 4.1 we have to consider the case when (32) is binding, but that does not change the fundamental trade-offs between $\iota$ (and hence $q$) and $m$ mentioned above. If (32) is not binding, then optimal $\eta = \iota/r$; otherwise, it can be even smaller.

Theorem 4.1 shows that, when designing an optimal charter system, the regulator has to balance efficiency in asset management and efficiency in liquidity provision. When there is abundant pledgeable assets so that (15) holds, full efficiency can be achieved on both dimensions, according to Theorem 4.1 (a). In this case, reserves are not needed either. Otherwise, according to Theorem 4.1 (b), the constrained efficient arrangement has to sacrifice full efficiency on both dimensions, and reserve requirements alone cannot implement the constrained efficient outcome. Restricting the number of charters reduces competition and increases banks' profits; this is suboptimal regarding efficiency in asset management. However, higher profits relax banks' incentive constraint to repay unsecured deposits, (31), and allow for a positive $\kappa$ without default. Thus, financial stability in our framework is possible because of positive profits bank enjoy, which also enhances social welfare because banks provides liquidity services as their liabilities are used as a means of payment. Compared with Theorem 3.1, this result also shows that whenever a reserve requirement is essential, the charter system and unsecured borrowing are also essential to improve welfare.

We now discuss bank size and leverage under a charter system. Consider the balance sheet of a bank in equilibrium, where the value of the asset is $\tau\frac{\bar{A}}{m}$. The leverage ratio, defined as the ratio of liabilities to assets (excluding the reserves), is given by

$$\mathcal{L} = \frac{\rho\tau\frac{\bar{A}}{m} + \kappa}{\tau\frac{\bar{A}}{m}} = \rho + \frac{m\kappa}{\tau\bar{A}}. \tag{36}$$

Theorem 4.1 (b) then implies that, when liquidity is tight, the optimal bank size is larger than under free entry. Moreover, (36) shows that regulations on $\kappa$ and $m$ can be interpreted as a *leverage requirement* beyond what would be imposed by free market, $\rho$, and Theorem 4.1 (b) shows that an optimal charter system allows for a higher leverage than free market. The optimal $\kappa$ regulates the amount of deposits a bank can issue beyond the pledgeable asset, and hence it is the most crucial component in the leverage requirement. While Theorem 4.1 does not depend on whether (32) is binding or not, the characterization of optimal policy is indeed much easier when it is not binding. A sufficient condition for this is that $r$ is not too large (see Appendix B3), and in later sections we shall focus on this case.

We emphasize that in our charter system, the regulator sets a limit on deposit issuance for each bank according to the appropriate leverage requirement, and terminates banks that fail to repay depositors. This scheme can be interpreted as a deposit insurance scheme to the extent that the depositors rely upon the regulator to monitor deposit issuance and repayment.[13] This scheme ensures equilibrium uniqueness: for any incentive compatible level of $\kappa$, it is optimal for banks to issue unsecured deposits up to $\kappa$ and depositors are willing to take them. Alternatively,

[13] In our model there is no uncertainty on the return to bank assets, and hence no need for the scheme to pay for insured deposits of failed banks on the equilibrium path. However, one can introduce that and the scheme would require premium from banks and pay for defaults. These additions will not change the main results though.

the regulator can simply restrict the number of banking licences but leave the pledgeability constraint to be determined by market discipline. In that case, however, there would be multiple equilibria: any $\kappa$ below the highest level consistent with the incentive compatibility constraint (31) can be an equilibrium, including $\kappa = 0$.

## 4.2 Liquidity supply and demand shocks

Here we consider how optimal leverage regulation responds to changes in aggregate economic conditions. For simplicity we assume that (32) does not bind throughout the section. The first shock we consider is a change in the productivity of entrepreneurial projects, measured by $\tau$. Intuitively, a positive shock to the returns of projects increases pledgeable assets, which enables banks to issue more deposits, and we call this a *liquidity supply shock*. An increase in $\tau$ has a direct effect on the overall output in the CM, but it also has an indirect effect on the DM. Under the static contract and optimal reserve requirement, (26) implies that an increase in $\tau$ results in a decrease in equilibrium $\iota$, and a decrease in the optimal reserve requirement, which in turn leads to an increase in DM output, provided that $q^*$ is not implementable. This can be interpreted as an amplification mechanism: a beneficial shock to entrepreneurial projects has a positive spillover effect on liquidity provision that results in efficiency gains in the DM. Under the charter system, however, this spillover effect is also constrained by the optimal leverage regulation. The following proposition shows that the optimal leverage regulation should be changed in a way that buffers the liquidity supply shocks. For comparative statics we need a monotonicity assumption:

$$\iota + \frac{D(\iota)}{D'(\iota)} \text{ is strictly increasing.} \tag{37}$$

A sufficient condition for (37) is $u(q) = \theta q^\alpha/\alpha$ and $c(q) = q^b$ for some $\theta > 0$, $\alpha \in (0, 1)$, and $b \geq 1$.

**Proposition 4.1.** *Suppose that (15) does not hold.*

*(a) Let $m < m^*$ be given and fixed under which $q^*$ is not implementable. An increase in $\tau$ leads*

*to a decrease in both the optimal $\kappa$ and optimal $\eta$, a decrease in the equilibrium $\iota$ and a decrease in $\mathcal{L}$ under new optimal $\kappa$ and $\eta$.*

*(b) Suppose that $\psi(a) = \lambda a^x/x$ for some $\lambda > 0$ and $x > 1$ and that (37) holds. Then, an increase in $\tau$ leads to an increase in optimal $m$, a decrease in both the optimal $\kappa$ and optimal $\eta$, and a decrease in the equilibrium $\iota$ and a decrease in $\mathcal{L}$ under new optimal $\kappa$ and $\eta$.*

Proposition 4.1 (a) considers the short-run policy response without changing $m$. It shows that when there is a positive shock to $\tau$, the optimal policy reduces banks' leverages and tightens the reserve requirement. A higher $\tau$ leads to a lower $\iota$ according to (35), which leads to a lower optimal $\kappa$, $\eta$, and $\mathcal{L}$. Proposition 4.1 (b) considers the long-term policy adjustment to an increase in $\tau$ that reoptimizes all policy parameters including $m$. The proof is more complicated as optimal $\kappa$ and hence $\mathcal{L}$ depends on optimal $m$, which takes efficiency in asset management into account. Changes in $\kappa$ and $\eta$ are consistent with the short-run response. It is optimal to increase $m$ as a higher $\tau$ increase liquidity provision and hence, at the margin, it is optimal to increase efficiency in asset-management. These results suggest that the optimal policy response to a liquidity supply shock is countercyclical in the leverage requirement.

Now we turn to shocks to aggregate demand. We parameterize the utility function $u$ by

$$u(q;\theta) = \theta q^{\alpha}/\alpha,$$

where $\theta > 0$ and $\alpha \in (0, 1)$. An increase in $\theta$ is interpreted as an increase in aggregate demand; consumers desire more DM consumption and will increase their demand for bank deposits. As a result, under the static contract and using (26), an increase in $\theta$ leads to a higher optimal reserve requirement and an increase in equilibrium $\iota$ and $q$, provided that $q^*$ is not implementable. Although both the liquidity supply shock and the demand shock lead to higher DM consumption (and hence higher output) under the static contract, they have very different implications for the optimal policy response under the optimal charter system, as the following proposition shows.

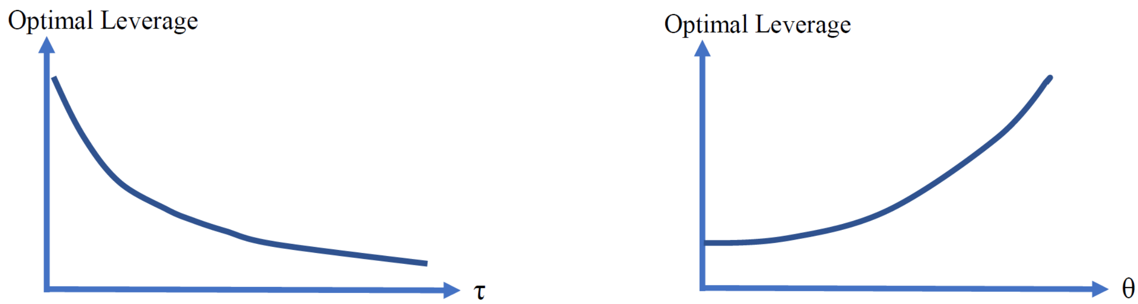


Figure 1: Optimal Leverage under Supply Shocks (left) and Demand Shocks (right)

**Proposition 4.2.** *Suppose that (15) does not hold.*

*(a) Let $m < m^*$ be given and fixed under which $q^*$ is not implementable. An increase in $\theta$ leads to an increase in both the optimal $\kappa$ and optimal $\eta$, and an increase in the equilibrium $\iota$ and an increase in $\mathcal{L}$ under new optimal $\kappa$ and $\eta$.*

*(b) Suppose that $\psi(a) = \lambda a^x/x$ for some $\lambda > 0$ and $x > 1$, and that $c(q) = q$. Then, an increase in $\theta$ leads to a decrease in optimal $m$, an increase in both the optimal $\kappa$ and optimal $\eta$, and an increase in the equilibrium $\iota$ and an increase in $\mathcal{L}$ under new optimal $\kappa$ and $\eta$.*

Proposition 4.2 suggests that the optimal policy response to an aggregate demand shock is procyclical in leverage requirement. In contrast to the liquidity supply shock, a beneficial demand shock increases the demand for liquidity in the economy and a higher deposit demand leads to a higher $\iota$ according to (35). Thus, the short-run optimal policy response is to increase $\kappa$ and hence $\mathcal{L}$. Similar policy response holds even when $m$ is adjustable in the long-run. Thus, as illustrated in Figure 1, optimal leverage decreases with a beneficial liquidity supply shock, while optimal leverage increases with a beneficial demand shock. Our results show that it is crucial to have a good understanding of the nature of the underlying economic shock when implementing the so-called macroprudential banking regulation.

## 4.3 Heterogenous bank sizes and profits

This section considers how optimal leverage varies with different banks, and whether concentration is a feature of the optimal arrangement. We introduce heterogenous banks that differ in terms of efficiency in their asset management. Specifically, for each $n \in \{1, ..., N\}$, the economy has measure $\mu_n$ of type-$n$ banks with $\sum_{n=1}^{N} \mu_n = 1$. The cost function for a bank of type-$n$ is $\lambda_n \psi(a) + \gamma$, and the lower the $\lambda_n$, the more efficient type-$n$ banks are. With no loss of generality, we assume that $\lambda_n \in [1, \bar{\lambda}]$ is strictly increasing in $n$, with type-1 banks the most efficient ones and type-$N$ the least.

**Efficient asset management**

We begin with efficient asset holdings and bank size in this environment. Without deposit issuance, efficient asset management requires the measures of type-$n$ active banks, denoted by $m_n$, to solve

$$\min_{m_n \in [0,\mu_n], A_n \geq 0, n=1,...,N} \sum_{n=1}^{N} [m_n \gamma + m_n \lambda_n \psi(A_n)] \text{ s.t. } \sum_{n=1}^{N} m_n A_n = \bar{A}. \tag{38}$$

Parallel to (3), to ensure that there is sufficient entry we assume that

$$\sum_{n=1}^{N} \mu_n \Pi^{-1}(\gamma/\lambda_n) > \bar{A}. \tag{39}$$

To characterize the solution to (38), for each $\mathbf{m} = (m_1, ..., m_N)$ with $m_1 > 0$, define $\{A_n(\mathbf{m})\}_{n=1}^{N}$ as the solution to

$$\sum_{n=1}^{N} m_n A_n = \bar{A}, \;\; \lambda_1 \psi'(A_1) = \lambda_n \psi'(A_n) \text{ for all } n \text{ with } m_n > 0. \tag{40}$$

We have the following lemma.

**Lemma 4.3.** *Assume (39). The solution to (38) is unique, denoted by* $\mathbf{m}^*$*, and is characterized* *by $\bar{n} \in \{1, ..., N\}$ and $0 < m^*_{\bar{n}} \leq \mu_{\bar{n}}$ such that*

$$\mathbf{m}^* = (\mu_1, .., \mu_{\bar{n}-1}, m^*_{\bar{n}}, 0..., 0), \tag{41}$$

$$\lambda_n \Pi(A_n(\mathbf{m}^*)) \geq \gamma, \text{ for all } n = 1, ..., \bar{n}, \tag{42}$$

$$\lambda_n \Pi(A_{\bar{n}}(\mathbf{m}^*)) = \gamma \text{ if } m_{\bar{n}} < \mu_{\bar{n}}, \tag{43}$$

$$\lambda_n \Pi(A_n(\mathbf{m}^*)) < \gamma, \text{ for all } n = \bar{n}+1, ..., N. \tag{44}$$

*Banks of type-$n$ hold $A_n(\mathbf{m}^*)$ projects for all $n \leq \bar{n}$.*

The FOC (40) implies that $A_n(\mathbf{m}^*)$ is strictly decreasing in $n$, and hence more efficient banks hold more assets. Similar to the homogenous case, the level of efficient asset holdings can be implementable with free entry if banks can finance those holdings by their own labour and cannot issue deposits. Indeed, conditions (41)-(44) also characterize free entry conditions; as we will show later, the profit for holding $a$ projects optimally for type-$n$ bank is $\lambda_n \Pi(a) - \gamma$ if active. Also, the fundamental value of projects can be defined as

$$\phi^* = \frac{\tau}{1+r} - \lambda_1 \psi' \left(A_1(\mathbf{m}^*)\right). \tag{45}$$

By (40) we can replace $\lambda_1 \psi' \left(A_1(\mathbf{m}^*)\right)$ by $\lambda_n \psi' \left(A_n(\mathbf{m}^*)\right)$ for any other $n \leq \bar{n}$ in (45). To ensure $\phi^* > 0$, we assume

$$\lambda_n \Pi \left[ (\psi')^{-1} \left( \frac{\tau}{\lambda_n(1+r)} \right) \right] > \gamma \text{ for all } n. \tag{46}$$

Note that (39) and (46) generalize (3) as they coincide when $\lambda_n = 1$ for all $n$. For similar reasons as before, we impose bounds on the capacity of CM production for type-$n$ banks, $\bar{h}_n$:

$$(1-\rho)\phi^* A_n(\mathbf{m}^*) < \bar{h}_n < \phi^* A_n(\mathbf{m}^*). \tag{47}$$

Finally, we discuss the welfare function under heterogenous banks. In general, an allocation now includes DM production, $q$, measures of active banks, $\mathbf{m}$, and asset holdings, $A_n$, for each active bank of type-$n$. For a given $\mathbf{m}$, the asset holdings described by $A_n(\mathbf{m})$ are optimal, and, as will be seen later, this is always implementable. Thus, we can consider only allocation in terms of $q$ and $\mathbf{m}$, and define

$$\mathcal{W}(q, \mathbf{m}) = \sigma[u(q) - c(q)] - \sum_{n=1}^{N} m_n \left[\lambda_n \psi(A_n(\mathbf{m})) + \gamma\right]. \tag{48}$$

**Heterogenous bank leverage under the charter system**

Now we consider the optimal charter system. We assume that bank efficiency, $\lambda_n$, is observable, and policy parameters also include a measure of active banks for each type, $\mathbf{m} = (m_1, ..., m_N)$. The pledgeability and reserve requirements for type-$n$ banks are

$$d \leq \rho\tau a + z + \kappa_n, \tag{49}$$

$$z \geq \max\{0, \eta(d - \kappa_n)\}. \tag{50}$$

Given $R$ (and hence $\iota$) and $\phi$, the profit of a type-$n$ bank by holding $a$ projects and reserves $z$, and issuing $d$ deposits is

$$\pi_n(a, z, d; \phi, R) = \beta \left\{\iota d - [(1+r)\phi - \tau]a - rz - (1+r)[\lambda_n \psi(a) + \gamma]\right\},$$

which is subject to the pledgeability constraint, (49), and the reserve requirement, (50),[14] together with the feasibility constraint given by $\bar{h}_n$. As before, we focus on the case $\iota \in [\eta r, r)$ and hence both (49) and (50) are binding, and

$$\pi_n(a, d, z; \phi, R) = \beta \left\{\frac{\iota - \eta r}{1 - \eta}\rho\tau a + [\tau - (1+r)\phi]a + \iota\kappa_n - (1+r)[\lambda_n \psi(a) + \gamma]\right\}.$$

[14]Here we assume that the reserve requirement is not size-dependent—$\eta$ is the same across banks. This is without loss of generality; making the reserve requirement size-dependent will not improve welfare.

When $\bar{h}_n$ is not binding, optimal asset holding for type-$n$ banks solves

$$\psi'(a) = \frac{-(1+r)\phi + \left[1 + \frac{\rho(\iota - r\eta)}{1-\eta}\right]\tau}{(1+r)\lambda_n}. \tag{51}$$

We use $A_n(\phi, \iota)$ to denote the asset demand from a type-$n$ bank, and its reserve demand, $\bar{z}_n(\phi, \iota)$, can be derived analogously to (22).

Note that $D(\iota)$ is still given by (12). Again, to analyze optimal policies, we can focus on policies under which equilibrium $\iota < r$. Hence, for given $\mathbf{m}$ and $\{\kappa_n\}$, the market-clearing conditions can be written as

$$\begin{aligned} D(\iota) &\leq \frac{\rho\tau}{1-\eta}\bar{A} + \sum_{n=1}^{N} m_n \kappa_n; && (52) \\ \sum_{n=1}^{N} m_n A_n(\phi, \iota) &= \bar{A}. && (53) \end{aligned}$$

In Appendix B4 we show that, for any given $\mathbf{m}$, $\eta$, and $\{\kappa_n\}$, there is a unique allocation that satisfies the above market clearing conditions. In particular, the asset holdings for type-$n$ banks with $m_n > 0$ are given by $A_n(\mathbf{m})$ as the solution to (40) and hence are independent of $\eta$ and $\{\kappa_n\}$. Moreover, the equilibrium profit for a type-$n$ bank with $m_n > 0$ is

$$\lambda_n \Pi(A_n(\mathbf{m})) - \gamma + \frac{\iota \kappa_n}{1+r}, \tag{54}$$

which is strictly decreasing in $n$. As a result, one can show that any optimal charter has the form $\mathbf{m} = (\mu_1, .., \mu_{\tilde{n}-1}, m_{\tilde{n}}, 0, ....0)$ for some $\tilde{n} \in \{1, ..., \bar{n}\}$, and $0 < m_{\tilde{n}} \leq \mu_{\tilde{n}}$. Let $M$ denote the set of all such $\mathbf{m}$'s, and each is characterized by $\tilde{n}$ and $m_{\tilde{n}}$. For $\mathbf{m}, \mathbf{m}' \in M$ with $\mathbf{m}$ characterized by $\tilde{n}$ and $m_{\tilde{n}}$ and $\mathbf{m}'$ by $\tilde{n}'$ and $m'_{\tilde{n}}$, we say that $\mathbf{m} < \mathbf{m}'$ iff $\tilde{n} < \tilde{n}'$ or $\tilde{n} = \tilde{n}'$ but $m_{\tilde{n}} < m'_{\tilde{n}}$.

Now, for any given $\mathbf{m}$, $\eta$, and $\{\kappa_n\}$, the corresponding equilibrium asset price is given by

$$\phi(\mathbf{m}) = \frac{\left(1 + \frac{\rho(\iota - r\eta)}{1-\eta}\right)\tau}{1+r} - \lambda_1 \psi'[A_1(\mathbf{m})]. \tag{55}$$

As in the homogenous case, if $\mathbf{m}' < \mathbf{m} \in M$, then $\phi(\mathbf{m}') < \phi(\mathbf{m})$, and, for $\phi(\mathbf{m})$ sufficiently small, it would make it profitable for some unchartered banks to hold assets. Using the same logic as before, to ensure unchartered banks to not enter and to ensure feasibility for chartered banks, there exists a minimum $\bar{\mathbf{m}}(\iota, \eta) < \mathbf{m}^*$ such that the regulator has to choose the measure of active banks from

$$\bar{\mathbf{m}}(\iota, \eta) \leq \mathbf{m} \leq \mathbf{m}^*. \tag{56}$$

Now, we still have to discuss incentive compatibility regarding $\eta$ and $\{\kappa_n\}$. For $\eta$, (20) still applies. For $\{\kappa_n\}$, the incentive constraint for a type-$n$ bank to repay $\kappa_n$ is

$$-(r - \iota)\kappa_n + (1 + r)[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma] \geq 0. \tag{57}$$

The derivation of this constraint follows the same logic as in the homogenous case; we give the full details in Appendix B4. We have the following theorem.

**Theorem 4.2.** *There exists an optimal policy* $(\mathbf{m}, \eta, \{\kappa_n\})$*; in any optimal policy, we have that* $\mathbf{m} \leq \mathbf{m}^*$*, and* $\mathbf{m} \in M$*, and that* $\eta$ *is the maximal* $\eta$ *that satisfies (20). Let* $\hat{\kappa}_n(\mathbf{m}) = \frac{1+r}{r}[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma]$ *for each* $n = 1, ..., N$*.*

*(a) Suppose that*

$$c(q^*) \leq \rho \frac{\tau}{1+r} \bar{A} + \sum_{n=1}^{N} m_n \hat{\kappa}_n(\mathbf{m}), \tag{58}$$

*holds for* $\mathbf{m} = \mathbf{m}^*$*, then optimal policy has* $\mathbf{m} = \mathbf{m}^*$*,* $\kappa_n = \hat{\kappa}_n(\mathbf{m}^*)$ *for all* $n \leq \bar{n}$*, and* $\eta = 0$*.*

*(b) Suppose that (58) does not hold for* $\mathbf{m} = \mathbf{m}^*$*.*

*(b.1) Any optimal policy* $(\mathbf{m}, \eta, \{\bar{\kappa}_n(\mathbf{m})\})$ *has* $\mathbf{m} < \mathbf{m}^*$*.*

*(b.2) Suppose that* $\psi(a) = \lambda a^x / x$ *for some* $x > 1$*. Then, for any optimal policy* $(\mathbf{m}, \{\bar{\kappa}_n(\mathbf{m})\})$*,*

$$\mathcal{L}_n = \frac{\rho \tau A_n(\mathbf{m}) + \bar{\kappa}_n(\mathbf{m})}{\tau A_n(\mathbf{m})}$$

*is strictly decreasing in* $n$*.*

Theorem 4.2 (b.1) shows that unless the first-best is implementable, the restriction of banking

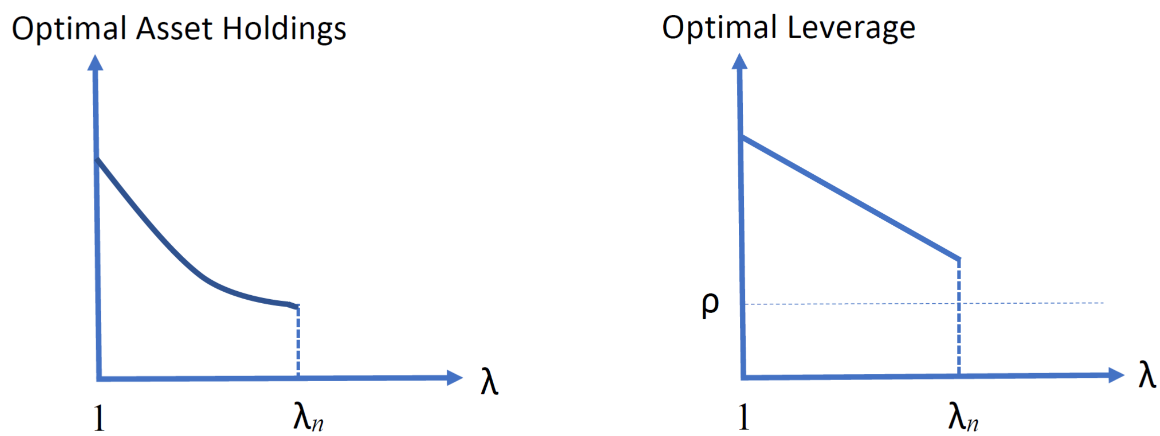


Figure 2: Heterogenous Asset Holdings and Leverages under Optimal Charter System

licences is optimal. This generalizes Theorem 4.1. Moreover, (b.2) shows that under the optimal arrangement, not only would the regulator allow higher unsecured deposit issuance for larger banks, but the leverage would also increase with the bank size; that is, it is optimal to allow for higher leverage ratio requirements for larger banks, as illustrated in Figure 2. The underlying intuition is this. When liquidity is tight, it is desirable to allow for more unsecured deposit issuance at the expense of inefficiency of asset management. Since larger banks are also more efficient in managing assets, it is desirable to give them higher profits and to allow them to issue more unsecured deposits.

# 5 Applications

We give two applications. The first applies our model to a moral hazard problem considered in the banking literature and discusses how it affects optimal bank size and leverage regulations. The second considers implications of monetary policy to optimal banking regulations.

## 5.1 Moral hazard

Here we apply our framework to a conventional moral hazard issue discussed in the literature. Suppose that the returns to a bank's asset holdings are subject to moral hazard and the bank may *gamble* with the assets. There are two benefits when banks gamble. First, by gambling, the cost of managing assets is lower: managing $a$ projects now costs $\psi(a) - ea + \gamma$ with $e > 0$. Second, while by gambling the return is stochastic and lower on average, banks receive private returns when it succeeds. Specifically, with probability $\nu \in (0, 1)$ the return from gambling will be $\tau_h > \tau$, and with probability $1 - \nu$ it will be $\tau_\ell < \tau$. When $\tau_h$ is realized, the difference $\tau_h - \tau$ is not observable (and hence gambling behavior is not detected) and it is a private gain to the bank. The decision to gamble is not observable, but when return $\tau_\ell$ occurs, it is observable to all (and gambling behavior is detected). We assume that

$$(1 + r)e < \tau - [\nu\tau_h + (1 - \nu)\tau_\ell]. \tag{59}$$

Condition (59) ensures that not gambling, or *prudent behavior*, is socially beneficial (without considerations of liquidity provision). For simplicity, throughout the section we assume the constraint analogous to (32) does not bind, and only consider $\eta$'s so that $\iota < r$ in equilibrium.

**Reserve requirement with free entry**

We begin with static contracts and analyze the optimal reserve requirement. We shall impose free entry later, but for now assume that the number of active banks is given by a fixed $m$. To induce prudent behavior, the constraint (6) may no longer be appropriate, and a more general pledgeability constraint is required:

$$d \leq \rho\tau_\ell a + \omega(\tau - \tau_\ell)a + z \tag{60}$$

for some $\omega \in [0, \rho]$. We still keep the reserve requirement as in (19). Note that when $\omega = \rho$, (60) coincides with (6). When $\omega = 0$, the bank has to suffer the full consequences of gambling and (59) ensures that the bank has no incentive to gamble in this case. Otherwise, depositors have to share the consequences, and banks may gain from gambling because of that.

Here we give a remark about what we mean by an equilibrium that induces prudent behavior under moral hazard, or a *prudent equilibrium*. Depositors would discipline banks by not depositing in those banks that do not satisfy the appropriate pledgeability constraints. Thus, in equilibrium the pledgeability constraint (60) must satisfy two conditions: first, it has to ensure that banks are willing to be prudent; second, no bank can credibly issue more deposits than what the constraint requires. Equivalently, equilibrium requires the *highest* $\omega$ under which no bank has an incentive to gamble. It has to be the highest as otherwise banks can credibly deviate to issue more deposits than the constraint (60) requires.[15]

We modify our previous analysis and obtain market-clearing conditions. Recall that we assume a fixed number of active banks, $m$. The profit of a prudent bank can be obtained by substituting (60) at equality into (7) (as before, we focus on equilibria with $\iota \in [\eta r, r)$, and hence both (19) and (60) are binding):

$$\pi(a, z, d; \phi, R) = \beta \left\{ \frac{\iota - r\eta}{1-\eta}[\rho\tau_\ell + \omega(\tau - \tau_\ell)]a + [\tau - (1+r)\phi]a - (1+r)[\psi(a) + \gamma] \right\}. \tag{61}$$

We use $A^{mh}(\phi, \iota)$ to denote the optimal asset holding of a prudent bank, and market clearing requires $A^{mh}(\phi, \iota) = \bar{A}/m$, which determines equilibrium $\phi$. Thus, equilibrium bank profit is given by $\Pi(\bar{A}/m) - \gamma$. Focusing on policies such that $\iota < r$, equilibrium $\iota$ solves

$$D(\iota) \leq \frac{\rho\tau_\ell + \omega(\tau - \tau_\ell)}{1-\eta}\bar{A}, \tag{62}$$

with equality whenever $\iota > 0$. As before, the right side of (62) is derived from (60) and (19) at equality with $a = \bar{A}/m$ and aggregation of $m$ active banks; note that it is increasing in $\omega$ and hence a lower $\omega$ decreases liquidity. Note that (20) is still necessary.

Finally, we need to consider the incentive to gamble, and study the profit to a bank if it gambles, taking the equilibrium $\phi$ as given. Though a gambling bank may issue deposits constrained by (60) (since the gambling decision is not observable), it pays only $d_\ell = \rho\tau_\ell a + z$ to depositors once $\tau_\ell$ is realized; hence, the profit to a gambling bank is given by (with (19) and

[15]Obviously, here we assume monotonicity in terms of incetivizing banks to be prudent in terms of $\omega$, a fact that will be confirmed in our equilibrium analysis.

(60) binding)

$$\begin{aligned}\pi^s(a,z,d;\phi,R) &= \frac{d}{R} - z - \phi a - [\psi(a) - ea + \gamma] + \beta\{[\nu\tau_h + (1-\nu)\tau_\ell]a + z - \nu d - (1-\nu)d_\ell\} \\ &= \beta\left\{\begin{array}{c} \frac{\iota - r\eta}{1-\eta}[\rho\tau_\ell + \omega(\tau - \tau_\ell)]a + [\tau - (1+r)\phi]a - (1+r)[\psi(a)+\gamma] \\ +[\nu\tau_h + (1-\nu)\tau_\ell - \tau + (1-\nu)\omega(\tau-\tau_\ell)]a + (1+r)ea] \end{array}\right\}. \qquad (63)\end{aligned}$$

Let $A^s$ be the optimal level of asset holdings under gambling, and bank profits under gambling are given by $\Pi(A^s) - \gamma$. Comparing the two FOCs for (61) and (63), we have

$$\psi'(A^s) - \psi'\left(\frac{\bar{A}}{m}\right) = \beta\{-\underbrace{[\tau - (\nu\tau_h + (1-\nu)\tau_\ell)]}_{(a)} + \underbrace{\omega(1-\nu)(\tau-\tau_\ell)}_{(b)} + \underbrace{(1+r)e}_{(c)}\}. \qquad (64)$$

The trade-offs from being prudent and gambling are revealed in three terms of the right side of (64): The cost of gambling is (a) a lower average rate of return; the benefits of gambling include savings from (b) having to pay less when the return is $\tau_\ell$, and (c) lower effort.

To ensure that banks have no incentive to gamble, we need the following condition:

$$\Pi(\bar{A}/m) - \Pi(A^s) \geq 0. \qquad (65)$$

To summarize, equilibrium conditions then consist of (20), (62), and (65). We have the following lemma.

**Lemma 5.1.** *Consider the static contract. In equilibrium $m = m^*$. For any given $\eta$, the highest $\omega$ under which no bank gambles in equilibrium is given by* $\min\{\omega_1, \rho\}$ *with*

$$\omega_1 \equiv 1 - \frac{(1+r)e + \nu(\tau_h - \tau)}{(1-\nu)(\tau - \tau_\ell)}. \qquad (66)$$

*The optimal $\eta$ is given by $\eta = \iota/r$ with $\iota$ that solves*

$$D(\iota) \leq \frac{r\left[\rho\tau_\ell + \omega(\tau - \tau_\ell)\right]}{r - \iota}\bar{A}, \qquad (67)$$

*with equality whenever* $\iota > 0$.

Note that $\omega_1$ is solved by equating term (a) to the sum of terms (b) and (c) in (64). It is then straightforward to verify from the proof of Lemma 5.1 that banks have no incentive to gamble if and only if $\omega \le \min\{\omega_1, \rho\}$ in equilibrium, and hence the highest $\omega$ is also the one consistent with the incentive to issue deposits. Moreover, since equilibrium $\iota$ is decreasing in $\omega$, the highest $\omega$ is also optimal from the depositors' perspective.

**Charter system with moral hazard**

Now we turn to the charter system under moral hazard. Relative to the literature, the novelty here is to study the two capital regulations together, one parameterized by $\omega$ and the other by $\kappa$. Under moral hazard, the general pledgeability constraint is given by:

$$d \le \rho\tau_\ell a + \omega(\tau - \tau_\ell)a + z + \kappa. \tag{68}$$

Again, here we consider only policies that induce prudent behavior and $\iota < r$. The policy parameter now becomes $(m, \eta, \kappa, \omega)$.

First we analyze equilibrium behavior for a given policy parameter. By being prudent, the bank profit is given by (assuming (20) so that all constraints are binding):

$$\pi(a, z, d; \phi, R) = \beta \left\{ \frac{\iota - r\eta}{1 - \eta}[\rho\tau_\ell + \omega(\tau - \tau_\ell)]a + \iota\kappa + [\tau - (1 + r)\phi]a - (1 + r)[\psi(a) + \gamma] \right\}. \tag{69}$$

Asset demand is still given by $A^{mh}(\phi, \iota)$, and the equilibrium profit to each active bank is $\Pi(\bar{A}/m) - \gamma + \beta\iota\kappa$. Given the policy parameter $(m, \eta, \kappa, \omega)$, equilibrium $\iota$ is the unique solution to

$$D(\iota) \le \frac{\rho\tau_\ell + \omega(\tau - \tau_\ell)}{1 - \eta}\bar{A} + m\kappa, \tag{70}$$

with equality whenever $\iota > 0$.

Now we turn to the incentive compatibility of banks to be prudent and to repay $\kappa$. One can verify that it is optimal to terminate the bank's charter when the realized return is $\tau_\ell$. Thus, a gambling bank with asset holdings $a$ pays only $d_\ell$ given by

$$d_\ell = \rho \tau_\ell a + z \tag{71}$$

to depositors under returns $\tau_\ell$. Thus, the profit to a gambling bank in this case is given by (63) plus a term $\beta(\iota + 1 - \nu)\kappa$, and hence the optimal asset holding is still given by $A^s$ and the profit is $\Pi(A^s) - \gamma + \beta(1 - \nu + \iota)\kappa$.

The incentive constraint for repaying $\kappa$ is the same as before and still given by (31). The incentive for prudent behavior is given by

$$\begin{aligned} & [\Pi(A^s) - \gamma] + \beta\iota\kappa + \nu\frac{\beta}{1-\beta}\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota \cdot \kappa}{1+r}\right] + (1-\nu)\beta\kappa \\ \leq \; & [\Pi(\bar{A}/m) - \gamma] + \beta\iota\kappa + \frac{\beta}{1-\beta}\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota \cdot \kappa}{1+r}\right]. \end{aligned}$$

The above condition is obtained by checking the one-shot deviation. After some algebra, the condition is equivalent to

$$\Pi(A^s) - \Pi(\bar{A}/m) + \underbrace{\beta(1-\nu)\kappa}_{(d)} \leq \underbrace{\frac{\beta(1-\nu)}{1-\beta}\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota \cdot \kappa}{1+r}\right]}_{(e)}, \tag{72}$$

Relative to the trade-offs identified in (64), (72) reveals two new trade-offs between prudent and gambling behavior under a charter system: the additional benefits of gambling now include savings from (d) not paying $\kappa$ when the gamble fails; the additional cost includes (e) future profits lost when the gamble fails.

**Theorem 5.1.** *Let $m \leq m^*$ be given. Suppose that $\rho \in (0,1)$. The optimal capital requirement is such that $\omega = \min\{\rho, \omega_1\}$ given by (66), and $\kappa$ is the highest $\kappa$ that satisfies (31) and $\eta$ by $\eta = \iota/r$ with $\iota$ determined by (70) and with $\omega = \min\{\rho, \omega_1\}$.*

Theorem 5.1 shows that under the charter system the optimal $\omega$ is the same as the one imposed by market discipline, and it is optimal to use the dynamic incentive to increase $\kappa$ and $\kappa$ only. Note, however, that the optimal $\kappa$ is indeed affected by moral hazard, since the choice of $\omega$ does affect the amount of liquidity banks can provide through asset prices and returns to deposits. Moreover, since Theorem 5.1 holds for any given $m$, it follows that we can solve for the optimal $m$ as in Theorem 4.1, and we will have $m < m^*$ and $\kappa > 0$ unless the first-best is implementable under $m = m^*$ and $\kappa = 0$, and $\omega = \omega_1$. But how would the optimal $m$, and hence the optimal $\kappa$ and profits, vary with the moral hazard issue? The following theorem gives a characterization.

**Theorem 5.2.** *Suppose that $\psi(a) = \lambda a^x/x$, $x > 1$, that (37) holds, and that $\omega_1 < \rho$. If the first-best is not implementable under market discipline and the optimal $m$ has an interior solution, then, as $\tau_h$ or $e$ increases, optimal $m$ decreases, and the optimal level of profits increases.*

Theorem 5.1 implies that, as $\tau_h$ or $e$ increases and hence the moral hazard issue becomes more serious, $\omega_1$ decreases. This directly decreases the amount of liquidity banks can provide. However, Theorem 5.2 shows that the optimal response to such change is to decrease $m$, which allows banks to become larger and to enjoy higher profits, and, therefore, permits the regulator to set a higher incentive-feasible $\kappa$. This implies a nontrivial interaction between the conventional capital requirement designed to counter moral hazard and the leverage requirement in our charter system, with the aim to balance stability and liquidity. Crucially, this result follows from our explicit treatment of liquidity provision from banks. The threat to removing charters is used to control for banks' incentive to be prudent, as well as to provide sufficient liquidity that is otherwise tightened up by preventing banks from gambling.

Finally, while we have focused on the equilibrium with regulations that induce banks to be prudent, we can also consider regulations that allow banks to gamble in equilibrium. In Appendix B5 we show that it can be optimal to have banks gamble, and we give a full characterization for when such a gambling equilibrium is better than imposing additional capital requirements to induce prudent behavior. Intuitively, this would be the case when the additional capital requirement is too stringent.

## 5.2 Monetary policy and banking regulations

In this application we study the effects of monetary policy on banking regulations. Suppose that, instead of a constant money supply, reserves grow at a constant net rate $\zeta \geq 0$. Similar to the monetary economics literature, we define the cost of holding money as

$$\iota_m = \frac{1+\zeta-\beta}{\beta}. \tag{73}$$

For simplicity, we assume that money creation is implemented through lump-sum transfers to households, who can then sell the reserves to banks.[16] We assume the same pledgeability and reserve constraints as in (27) and (28). We focus on stationary equilibrium in which the inflation rate equals the money creation rate, and still use $R$ to denote the *real* gross return on bank deposits and $\iota$ defined in the same way. (The nominal gross return on bank deposits is then $R(1+\zeta)$.) This changes the profit of a bank from (29) to

$$\pi(a,d,z;\phi,R) = \beta\left\{\frac{\iota-\eta\iota_m}{1-\eta}\rho\tau a + [\tau-(1+r)\phi]a - (1+r)[\psi(a)+\gamma] + \iota\kappa\right\}, \tag{74}$$

and the equilibrium condition (20) to

$$\iota - \eta\iota_m \geq 0, \tag{75}$$

and the previous equilibrium condition $\iota \leq r$ is now changed to $\iota \leq \iota_m$ (which is equivalent to $R(1+\zeta) \geq 1$). For the same reasons as before, we focus only on $\eta$'s so that in equilibrium $\iota < \iota_m$. This change does not affect the incentive compatibility constraint for banks to repay the unsecured deposits, (31), nor does it affect the incentive constraint related to restrictive banking licenses and the feasibility of financing asset holdings, (32); for simplicity we also assume that (32) does not bind. We have the following lemma.

[16]With lump-sum transfers, it does not matter who the recipients are.

**Lemma 5.2.** *Let $m \leq m^*$ and $\iota < \iota_m$ be given. The optimal $(\eta, \kappa)$ that solves*

$$\mathcal{S}(\iota, m) = \max_{\eta,\kappa} \frac{\rho\tau}{1-\eta}\bar{A} + m\kappa, \tag{76}$$

*subject to (75) and (31) is given by $\eta = \iota/\iota_m$ and (34).*

Given the lemma, for any given $m$, under the optimal policy the equilibrium $\iota$ solves

$$D(\iota) \leq \frac{\iota_m}{\iota_m - \iota}\rho\tau\bar{A} + m\frac{(1+r)[\Pi(\bar{A}/m) - \gamma]}{r - \iota}, \tag{77}$$

with equality whenever $\iota > 0$. We can then solve for the optimal $m$ as before.

**Theorem 5.3.** *There exists an optimal policy $(m, \eta, \kappa)$ that maximizes welfare subject to implementability.*

*(a) If (15) holds, then $(m, \eta, \kappa) = (m^*, 0, 0)$ is an optimal policy.*

*(b) Suppose that (15) does not hold. Then, any optimal policy has $m < m^*$ and $\kappa > 0$.*

*(c) For a given $m < m^*$ under which $q^*$ is not implementable, an increase in $\zeta$ leads to an increase in optimal $\kappa$ and a decrease in equilibrium $q$ under optimal $\eta$ and $\kappa$.*

Theorem 5.3 extends Theorem 4.1 to allow for inflation. When (15) holds and hence reserves are not needed for constrained efficiency, no regulation is needed and monetary policy has no effect in our pure deposit economy, as stated in Theorem 5.3 (a). In contrast, when (15) does not hold, inflation has real effects and is a determinant of optimal banking regulation. According to Theorem 5.3 (c), inflation is costly to welfare. Indeed, for any given $m$, (77) implies that a higher $\iota_m$ leads to a higher $\iota$, and hence lower equilibrium DM production. As a result, the optimal response to higher inflation is to relax the leverage requirement by increasing bank profits. Note that in contrast to most of the literature, wherein inflation is costly because it directly discourages households from holding cash and hence drains liquidity, in our model the cost of inflation arises through the banking channel and by lowering the real interest rate paid on deposits.[17]

[17]Here we consider only inflation. If the regulator has access to resources outside the banking sector to finance deflation, then our results also imply that this can be welfare-improving.

Theorem 5.3 also shows that the optimal monetary policy (among all policies implemented with lump-sum transfers) in this economy is a constant money supply. However, one can imagine alternative schemes to implement money creation or withdrawal. On the one hand, newly created money can be transferred to chartered banks alone to expand their pledgeable assets. On the other hand, money withdrawal can be implemented through a tax on chartered banks. To be incentive compatible, the only punishment for tax evasion is removal of the charter. It turns out that, as we show in Appendix B6, either scheme cannot improve welfare. The intuition is simple: the added liquidity under an inflationary scheme is offset by the higher cost of holding reserves, and the increased rate of return of reserves under a deflationary scheme is offset by lower sustainable unsecured borrowing because the tax decreases profits.

# 6 Concluding remarks

We have taken the liquidity role of banks seriously and derived optimal banking regulations. Our results demonstrate that when banks are subject to limited commitment, a leverage requirement with restricted banking privilege is optimal for welfare, and deposit insurance can guarantee uniqueness of the optimal equilibrium. Under this arrangement, banks have higher profits and higher leverage ratios than under the laissez-faire economy with market discipline. With heterogeneous banks, optimal concentration is higher than in the free market. This is broadly consistent with the contrast in bank profits and leverage for the US banking industry entering the Great Depression (an era when market discipline plays a larger role than regulations) and the industry entering the recent financial crisis (an era with more regulations and government insurance on deposits), as documented by Koch et al. (2016).

Compared to most of the literature, we have shown that considerations of liquidity provision can change some conventional wisdom about banking regulation. First, our model features three sets of policy instruments, a liquidity requirement, a proportional capital requirement, and a leverage requirement. We have shown that all three are essential to implement constrained efficient allocations. Second, our model emphasizes the welfare role of banks' liquidity provision and its implications for the optimal response to economic shocks. We show that the key to determining the optimal response is how a particular shock affects overall liquidity needs. In particular, when the moral hazard issue worsens, it is optimal to tighten the proportional capital requirement, but the leverage ratio should not be proportionally decreased.

Our results are robust to a few assumptions that we made. First, we can introduce heterogeneous assets, both in terms of pledgeability and in terms of maturities, and our main conclusions would remain the same. In such an extension, one can apply our methodology to study optimal banking regulations and how they affect the optimal security design of bank portfolios. Second, the take-it-or-leave-it offer in the DM can be replaced by Kalai bargaining or even the optimal mechanism as in Hu, Kennan, and Wallace (2009), without affecting any of the results qualitatively. Third, while households are anonymous in our model to simplify the analysis, we can allow banks to keep track of households' deposit holding and spending. This would not change our main results on banking regulation, but adding the ability to track the households' trade histories would make our framework a natural model for credit card issuance and welfare analysis. Indeed, in such a world, it would be optimal to allow households with extra liquidity needs to issue unsecured credit, with future exclusion from banking services as a threat to induce repayment.

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# Appendix A: Proofs of Lemmas and Theorems

**Proof of Lemma 3.1** In both cases, we assume that $\bar{h}$ does not bind in equilibrium, an assumption that we verify at the end. Given this assumption, a bank's optimal asset holding is determined by (8), and, if $a$ is the optimal holding, then the bank profit is given by

$$\begin{aligned} & \beta\{\iota_t d + [\tau - (1+r)\phi_t]a\} - \psi(a) - \gamma \\ = \ & \beta[(\iota_t \rho + 1)\tau - (1+r)\phi_t]a - \psi(a) - \gamma = \psi'(a)a - \psi(a) - \gamma = \Pi(a) - \gamma, \end{aligned} \tag{78}$$

where the first line is from (7), the first equality is from (6) binding, and the second equality is from (8).

(a) Since for any given measure of active banks, $m$, bank profit in equilibrium is given by $\Pi(\bar{A}/m) - \gamma$, free entry implies that banks earn zero profit in equilibrium and hence equilibrium $m = m^*$. Since (15) holds, the first line of (13) is satisfied with $D(0) = c(q^*)$, and hence equilibrium $\iota = 0$. At $m = m^*$, market clearing requires $A(\phi, \iota) = \bar{A}/m^*$. Taking $\iota = 0$ and $a = \bar{A}/m^*$ into (8), this implies that equilibrium $\phi$ is given by (4).

(b) Again, in equilibrium $m = m^*$ and $A(\phi, \iota) = \bar{A}/m^*$, and substituting $a = \bar{A}/m^*$ into (8) we obtain $\phi$ given by (16). Since $D(\iota)$ is strictly decreasing in $\iota$, there is a unique solution to (13) with equality: when $\iota = 0$, since (15) does not hold, the left-side of (13) is strictly greater than the right-side; when $\iota \to \infty$, $D(\iota) \to 0$ by Inada conditions. Thus, equilibrium $\iota > 0$.

Finally, for both cases (a) and (b), we show that under the assumption (5), $\bar{h}$ is not binding in equilibrium; that is, $\bar{h}$ is sufficient to finance the difference between the CM good needed to purchase assets and that coming from deposits. In equilibrium, each active bank needs to pay $\phi\bar{A}/m^*$ for assets, and have $(\rho\tau\bar{A}/m^*)/R$ from deposits. The required CM good is thus given by

$$\begin{aligned} & \left[\phi - \frac{\rho\tau}{R}\right]\frac{\bar{A}}{m^*} = \left[\frac{\tau(1+\iota\rho)}{1+r} - \psi'(\bar{A}/m^*) - \frac{1+\iota}{1+r}\rho\tau\right]\frac{\bar{A}}{m^*} \\ = \ & (1-\rho)\phi^*\frac{\bar{A}}{m^*} - \rho\psi'(\bar{A}/m^*)\frac{\bar{A}}{m^*} < \bar{h}, \end{aligned}$$

where we used $R = \frac{1+r}{1+\iota}$ and $\phi^* = \frac{\tau}{1+r} - \psi'(\bar{A}/m^*)$. Note that uniqueness follows immediately from the fact that in equilibrium $\phi > 0$ (for otherwise there will be infinite demand on the projects) and hence all projects are acquired by banks, so equilibrium is pinned down by (13) and $\phi$ given by (16) is the unique price that clears the market.

**Proof of Lemma 3.2** We shall first assume that $\bar{h}$ does not bind and that (20) holds, and verify that both hold in equilibrium later. Given those assumptions, the solution to the bank problem is determined by the FOC of the profit given by (21); that is, optimal $a$ solves

$$\frac{\iota - \eta r}{1-\eta}\rho\tau + [\tau - (1+r)\phi] = (1+r)\psi'(a). \tag{79}$$

For any given $\iota$ and $m$, (23) and (79) pin down equilibrium $\phi$ as

$$\phi = \frac{\left(1 + \rho\frac{\iota - r\eta}{1-\eta}\right)\tau - \psi'\left(\frac{\bar{A}}{m}\right)(1+r)}{1+r}, \tag{80}$$

and, following a similar argument to (78), the equilibrium profit for each bank is $\Pi(\bar{A}/m) - \gamma$. Thus, free entry requires $m = m^*$.

Now we turn to determination of equilibrium $\iota$. Since (15) does not hold, there is a unique $\iota \in (0, r)$ that solves (26), denoted by $\bar{\iota}$. Let $\bar{\eta} = \bar{\iota}/r \in (0,1)$. Moreover, for each $\eta \in [0,1)$, there is a unique $\iota \geq 0$ that solves

$$D(\iota) \leq \frac{1}{1-\eta}\rho\tau\bar{A}, \text{ with equality whenever } \iota > 0, \tag{81}$$

which we denote $\iota(\eta)$. Since $D(\iota)$ is a strictly decreasing function and since the right side of (81) is strictly increasing in $\eta$, $\iota(\eta)$ is a strictly decreasing function in $\eta$ until it hits zero and stays there. Moreover, by (26),

$$D(\bar{\iota}) = \frac{\rho\tau}{1 - \bar{\iota}/r}\bar{A} = \frac{\rho\tau}{1-\bar{\eta}}\bar{A},$$

and hence $\bar{\iota} = \iota(\bar{\eta})$. Thus, for any $\eta \leq \bar{\eta}$, $\iota(\eta) \geq \iota(\bar{\eta}) = \bar{\eta}r \geq \eta r$, that is, (20) is satisfied. Thus, for any $\eta \in (\underline{\eta}, \bar{\eta}]$, with $m = m^*$ and $\phi$ given by (16), $\iota(\eta) \in (0, r)$ satisfies (24) with equality.

Here we show that if $\eta > \bar{\eta} > 0$, then no active bank will issue deposit in equilibrium. Suppose, by contradiction, that active banks issue deposits. Now, since $\eta > \bar{\eta} > \underline{\eta}$, $\iota(\eta) > r$, and hence in equilibrium it must be the case that $\iota > r$. This then requires (20) to hold and hence in equilibrium it must be the case that $\iota > 0$. So the second line in the market clearing condition (24) with $A^r(\phi, \iota) = \bar{A}$ has to hold, which implies that in equilibrium $\iota = \iota(\eta) < \iota(\bar{\eta}) = \bar{\eta}r < \eta r$, a contradiction to (20). Note that in this argument we did not use the assumption that $\bar{h}$ does not bind or the fact that equilibrium $m = m^*$.

Now we show that if $\underline{\eta} \geq 0$ and if $\eta \leq \underline{\eta}$, then $\iota = r$ in equilibrium. In this case (20) holds trivially. Each active bank issues $D(r)/m^*$ units of deposits and holds $\bar{A}/m^*$ assets, and hold reserve in the amount of

$$\bar{z} = \frac{D(r) - \rho\tau\bar{A}}{m^*} \geq \eta\frac{D(r)}{m^*},$$

where the inequality follows from the fact that $\eta \leq \underline{\eta}$. Thus, the reserve requirement (19) is satisfied, and each active bank is willing to hold $\bar{z}$ units of reserves as $\iota = r$.

Finally, we show that (5) is sufficient for banks to furnish sufficient equity in equilibrium for any given $\eta \in [0, \bar{\eta}]$. In equilibrium, each active bank pays $\phi\bar{A}/m^* + z$ for assets, with $\phi$ given by (25) and

$$z \geq \frac{\eta}{1-\eta}\rho\tau\frac{\bar{A}}{m^*}, \tag{82}$$

and have $(\rho\tau\bar{A}/m^* + z)/R$ from deposits. Since (82) can hold with inequality only when $R = 1$ and in that case more reserve holdings do not affect feasibility, we may assume that (82) holds with equality. Thus, the required CM good is given by

$$
\begin{aligned}
& \left[\phi + \frac{\eta}{1-\eta}\rho\tau - \frac{(\rho\tau/(1-\eta))}{R}\right]\frac{\bar{A}}{m^*} \\
= & \left\{\frac{\tau + \rho\tau\frac{\iota - r\eta}{1-\eta}}{1+r} - \psi'\left(\frac{\bar{A}}{m^*}\right) + \frac{\eta\rho\tau}{1-\eta} - \frac{1+\iota}{1+r}\frac{\rho\tau}{1-\eta}\right\}\frac{\bar{A}}{m^*} \\
= & \left\{\frac{\tau}{1+r} - \psi'\left(\frac{\bar{A}}{m^*}\right) + \frac{\rho\tau}{1+r}\left[\frac{\iota - r\eta + (1+r)\eta - (1+\iota)}{1-\eta}\right]\right\}\frac{\bar{A}}{m^*} \\
= & (1-\rho)\phi^*\frac{\bar{A}}{m^*} - \rho\psi'(\bar{A}/m^*)\frac{\bar{A}}{m^*} < \bar{h},
\end{aligned}
$$

where we used $R = \frac{1+r}{1+\iota}$ and $\phi^* = \frac{\tau}{1+r} - \psi'(\bar{A}/m^*)$; the last inequality comes from (5).

**Proof of Theorem 3.1** Lemma 3.2 shows that in any equilibrium where active banks issue deposits, $m = m^*$ and hence the asset-management is at its efficient level. Hence, optimal reserve requirement would maximize term (a) in the social welfare given by (1). Now, Lemma 3.2 shows that for $\eta \leq \underline{\eta}$, equilibrium $\iota = r$, and for $\eta \in (\underline{\eta}, \bar{\eta}]$, equilibrium $\iota = \iota(\eta) < r$. Since in equilibrium $q = c^{-1}[D(\iota)] \in (0, q^*)$, welfare is strictly decreasing in $\iota$. Since the function $\iota(\eta)$ is strictly decreasing in $\eta$, it is optimal set it at its maximum, $\bar{\eta}$.

**Proof of Lemma 4.1** We first compute equilibrium profit, assuming that $\bar{h}$ does not bind. Since the bank profit (29) differs from (21) only in the additional term $\beta\iota\kappa$, the optimal asset holding is still determined by (79). Hence, if $a$ is the optimal holding, then the bank profit is given by

$$
\begin{aligned}
& \beta\left\{\frac{\iota - \eta r}{1-\eta}\rho\tau + [\tau - (1+r)\phi]\right\}a - \psi(a) - \gamma + \beta\iota\kappa \\
= & \psi'(a)a - \psi(a) - \gamma + \beta\iota\kappa = \Pi(a) - \gamma + \frac{\iota\kappa}{1+r},
\end{aligned} \tag{83}
$$

where the first line is from (29), and the first equality is from (79). Thus, if there are $m$ active banks in equilibrium and hence each holds $\bar{A}/m$ projects, the equilibrium profit is given by $\Pi(\bar{A}/m) - \gamma + \iota\kappa/(1+r)$.

The rest of the proof follows exactly the same logic as the proof of Lemma 3.2, but replace the right-side of (26) by $\frac{r}{r-\iota}\rho\tau\bar{A} + m\kappa$. This then determines $\bar{\eta}(\kappa, m)$. Note that, however, in this case $\bar{\eta}(\kappa, m)$ may be zero and this becomes the trivial case. Similarly, $\underline{\eta}(\kappa, m) > 0$ only if $D(r) > m\kappa$ and $\underline{\eta}(\kappa, m) = 1 - \frac{\rho\tau\bar{A}}{D(r)-m\kappa}$. The rest of the argument is exactly the same as that in the proof of Lemma 3.2.

**Proof of Lemma 4.2** Recall that from (33),

$$\mathcal{S}(\iota, m) = \max_{\eta,\kappa} \frac{\rho\tau}{1-\eta}\bar{A} + m\kappa \text{ subject to (20) and (31).}$$

The objective function is strictly increasing in $\eta$ and $\kappa$, while (20) and (31) give upper bounds for $\eta$ and $\kappa$, respectively. Thus, at the optimum, we have $\eta = \iota/r$ and $\kappa$ be determined by (31) at equality.

**Proof of Theorem 4.1** (a) Since (15) implies that the first-best allocation (for both $q$ and $m$) is implementable under free entry, there is no room for any intervention.

(b) Since (15) does not hold, the proof of Theorem 3.1 implies that under $(m, \eta, \kappa) = (m^*, \bar{\eta}, 0)$ the equilibrium allocation has $q < q^*$. We show that any optimal policy has $m < m^*$. First we consider the case where (32) is not binding. From Lemma 4.2, we have

$$\mathcal{S}(\iota, m) = \frac{r\rho\tau}{r-\iota}\bar{A} + m\frac{(1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right]}{r-\iota}.$$

Let $\iota(m)$ be the unique solution to the requirement that $D(\iota) \leq S(\iota, m)$ and with equality whenever $\iota > 0$. Since (15) does not hold and hence $\iota(m^*) > 0$, there is a range of $m$'s below $m^*$, $\iota(m)$ is determined by $D(\iota) = \mathcal{S}(\iota, m)$.

Let $F(\iota, m) \equiv \mathcal{S}(\iota, m) - D(\iota)$, and hence $\iota(m)$ is implicitly defined by $F(\iota, m) = 0$. We can compute the derivatives of $F$ as follows. For all $\iota < r$,

$$\begin{aligned}\frac{\partial}{\partial m}F(\iota, m^*) = \frac{\partial}{\partial m}\mathcal{S}(\iota, m^*) &= \frac{1+r}{r-\iota}\left[\Pi\left(\frac{\bar{A}}{m^*}\right) - \Pi'\left(\frac{\bar{A}}{m^*}\right)\left(\frac{\bar{A}}{m^*}\right) - \gamma\right] \\ &= -\frac{1+r}{r-\iota}\left[\psi''\left(\frac{\bar{A}}{m^*}\right)\frac{\bar{A}^2}{(m^*)^2}\right] < 0,\end{aligned}$$

where the third equality is obtained by using the fact that $\Pi\left(\frac{\bar{A}}{m^*}\right) - \gamma = 0$, and the fact that $\Pi'(a) = \psi''(a)a$. Similarly,

$$\frac{\partial}{\partial \iota}F(\iota, m^*) = \frac{\partial}{\partial \iota}\mathcal{S}(\iota, m^*) - D'(\iota) = \frac{r\rho\tau\bar{A} + m^*(1+r)\left[\Pi\left(\frac{\bar{A}}{m^*}\right) - \gamma\right]}{(r-\iota)^2} - D'(\iota) > 0,$$

since $D'(\iota) < 0$. By the implicit function theorem, in a neighbourhood of $m^*$, $\iota(m)$ is continuously differentiable with

$$\iota'(m^*) = -\frac{\frac{\partial}{\partial m}F(\iota, m^*)}{\frac{\partial}{\partial \iota}F(\iota, m^*)} > 0.$$

Assuming that (32) does not bind, the optimal banking regulation would then maximize welfare (1) subject to $F(m, \iota) = 0$ and $q = c^{-1}[D(\iota)]$, which can then be reduced to

$$\max_{m \in [m^*, \bar{m}]} G(m) \equiv \mathcal{W}[D(\iota(m)), m].$$

Now,

$$\begin{aligned}G'(m^*) &= \sigma\left\{\frac{u'[c^{-1}[D(\iota(m^*))]]}{c'[c^{-1}[D(\iota(m^*))]]} - 1\right\}D'(\iota(m^*))\iota'(m^*) - [\Pi(\bar{A}/m^*) - \gamma] \\ &= \sigma\left\{\frac{u'[c^{-1}[D(\iota(m^*))]]}{c'[c^{-1}[D(\iota(m^*))]]} - 1\right\}D'(\iota(m^*))\iota'(m^*) < 0,\end{aligned}$$

where the last inequality follows from the fact that $D(\iota(m^*)) < q^*$, $D'(\iota(m^*)) < 0$, and $\iota'(m^*) > 0$. Thus, the optimal $m$ must satisfy $m < m^*$.

Finally, up to now we have assumed that (32) does not bind. Suppose now that (32) is binding. In this case, we must have $m < \bar{m} < m^*$. Moreover, (31) must be binding as well, for otherwise we can increase $\kappa$ and increase $q$. This implies $\kappa > 0$. $\square$

**Proof of Propositions 4.1 (a) and Proposition 4.2 (a)** We begin with Proposition 4.1 (a). Let $m$ be fixed. Then, since (15) does not hold, under the optimal policy, equilibrium $\iota$ solves

$$D(\iota) = \frac{r\rho\tau}{r-\iota}\bar{A} + m\frac{(1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right]}{r-\iota}. \tag{84}$$

An increase in $\tau$ then leads to an increase in the right side of the above equation, which leads to a decrease in equilibrium $\iota$. Thus, by Lemma 4.2, optimal $\kappa$ and $\eta$ both decrease. Finally, since $\mathcal{L}$ is strictly increasing in $m\kappa$, this also implies that $\mathcal{L}$ decreases.

Now we turn to Proposition 4.2 (a). Let $D(\iota;\theta)$ denote the demand for deposit with $u(q) = \theta q^{\alpha}/\alpha$ and $c(q) = q$. Then

$$D(\iota;\theta) = (\theta\sigma)^{\frac{1}{1-\alpha}}\left(\frac{1}{\sigma+\iota}\right)^{\frac{1}{1-\alpha}}$$

and hence is strictly increasing in $\theta$. Thus, an increase in $\theta$ leads to an increase in equilibrium $\iota$ according to (84), and, by Lemma 4.2, optimal $\kappa$ and $\eta$ both increase. Again, since $\mathcal{L}$ is strictly increasing in $m\kappa$, this also implies that $\mathcal{L}$ increases. $\square$

**Proof of Propositions 4.1 (b) and 4.2 (b)** We parameterize consumer's DM utility function as $\theta u(q)$. The optimal banking regulation problem, or the *planner's problem*, is thus given by

$$\begin{aligned}
&\max_{m\geq 0,\iota\geq 0} \sigma[\theta u(q) - c(q)] - \left[m\gamma + m\psi\left(\frac{\bar{A}}{m}\right)\right], \\
\text{s.t.}\quad & D(\iota;\theta) \leq \frac{r}{r-\iota}\rho\tau\bar{A} + m\frac{(1+r)[\psi'(\bar{A}/m)\bar{A}/m - \psi(\bar{A}/m) - \gamma]}{r-\iota}, \\
& c(q) = D(\iota;\theta).
\end{aligned}$$

Let

$$F(\iota, m;\theta,\tau) = -D(\iota;\theta) + \frac{r}{r-\iota}\rho\tau\bar{A} + m\frac{(1+r)[\psi'(\bar{A}/m)\bar{A}/m - \psi(\bar{A}/m) - \gamma]}{r-\iota}.$$

Then,

$$
\begin{aligned}
\frac{\partial}{\partial m}F &= \frac{(1+r)}{r-\iota}\left\{\left[\psi'\left(\frac{\bar{A}}{m}\right)\frac{\bar{A}}{m}-\psi\left(\frac{\bar{A}}{m}\right)-\gamma\right]-\psi''\left(\frac{\bar{A}}{m}\right)\left(\frac{\bar{A}}{m}\right)^2\right\},\\
\frac{\partial}{\partial \iota}F &= -D'(\iota;\theta)+\frac{r\rho\tau\bar{A}+m(1+r)\left[\psi'\left(\frac{\bar{A}}{m}\right)\frac{\bar{A}}{m}-\psi\left(\frac{\bar{A}}{m}\right)-\gamma\right]}{(r-\iota)^2}.
\end{aligned}
$$

Let $\iota(m;\theta,\tau)$ be the smallest $\iota \geq 0$ such that $F(\iota,m;\theta,\tau)\geq 0$, and, since $F$ is strictly increasing in $\iota$, whenever $\iota(m;\theta,\tau)>0$, $F[\iota(m;\theta,\tau),m;\theta,\tau]=0$ (as in the market clearing condition (35)). Moreover, by the implicit function theorem, $\iota(m;\theta,\tau)$ is differentiable in $m$, $\theta$, and $\tau$ when $\iota(m;\theta,\tau)>0$. Since (15) does not hold, there is a range of $m$ below $m^*$ such that $\iota(m;\theta,\tau)>0$. It is easy to see that it is never optimal to have $\iota=0$ at the optimal and hence we can take the FOC for the planner's problem. Now we eliminate the constraints in the planner's problem by replacing $q$ with $q=c^{-1}[D(\iota(m;\theta,\tau);\theta)]$, and the FOC for the social planer's problem is then given by

$$
\sigma\left[\frac{\theta u'(q)}{c'(q)}-1\right]D'(\iota;\theta)\frac{\partial}{\partial m}\iota(m;\theta,\tau)+\left[\Pi\left(\frac{\bar{A}}{m}\right)-\gamma\right]=0.
$$

We can further simplify this by noting that $\frac{\partial}{\partial m}\iota(m;\theta,\tau)=-\frac{\partial}{\partial m}F/\frac{\partial}{\partial \iota}F$, and the FOC becomes

$$
\frac{(1+r)\iota(m;\theta,\tau)\left\{\left[\Pi\left(\frac{\bar{A}}{m}\right)-\gamma\right]-\psi''\left(\frac{\bar{A}}{m}\right)\left(\frac{\bar{A}}{m}\right)^2\right\}}{(r-\iota)+\frac{D[\iota(m;\theta,\tau);\theta]}{-D'[\iota(m;\theta,\tau);\theta]}}+\left[\Pi\left(\frac{\bar{A}}{m}\right)-\gamma\right]=0. \tag{85}
$$

To obtain (85), we used the fact that, by (12),

$$
\iota=\sigma\left[\frac{\theta u'(q)}{c'(q)}-1\right],
$$

and plugged in the expressions for $\frac{\partial}{\partial m}F$ and $\frac{\partial}{\partial \iota}F$ that were derived earlier, and used the observation that with $\iota=\iota(m;\theta,\tau)$,

$$
\frac{\partial}{\partial \iota}F=-D'(\iota;\theta)+\frac{D(\iota;\theta)}{r-\iota}.
$$

Given that $\psi(a) = \lambda a^x / x$,

$$\Pi\left(\frac{\bar{A}}{m}\right) = \lambda\frac{x-1}{x}\left(\frac{\bar{A}}{m}\right)^x, \Pi\left(\frac{\bar{A}}{m}\right) - \psi''\left(\frac{\bar{A}}{m}\right)\left(\frac{\bar{A}}{m}\right)^2 = -\lambda\frac{(x-1)^2}{x}\left(\frac{\bar{A}}{m}\right)^x.$$

After rearranging, the FOC (85) becomes

$$\frac{(1+r)\iota(m;\theta,\tau)}{r - \iota(m;\theta,\tau) - \frac{D[\iota(m;\theta,\tau);\theta]}{D'[\iota(m;\theta,\tau);\theta]}} = \frac{\lambda\frac{x-1}{x}\left(\frac{\bar{A}}{m}\right)^x - \gamma}{\lambda\frac{(x-1)^2}{x}\left(\frac{\bar{A}}{m}\right)^x + \gamma} = \frac{\lambda\frac{x-1}{x}\bar{A}^x - \gamma m^x}{\lambda\frac{(x-1)^2}{x}\bar{A}^x + \gamma m^x}. \tag{86}$$

Now, define

$$H_1(\iota;\theta) \equiv \frac{(1+r)\iota}{r - \iota - \frac{D(\iota;\theta)}{D'(\iota;\theta)}} \text{ and } H_2(m) \equiv \frac{\lambda\frac{x-1}{x}\bar{A}^x - \gamma m^x}{\lambda\frac{(x-1)^2}{x}\bar{A}^x + \gamma m^x}.$$

The FOC (86) can then be rewritten as

$$H_1[\iota(m;\theta,\tau);\theta] = H_2(m). \tag{87}$$

Since (37) implies that $H_1$ is strictly increasing in $\iota$ and since $\iota(m;\theta,\tau)$ is strictly increasing in $m$, the left side of (87) is strictly increasing in $m$. $H_2$ is strictly decreasing in $m$. Note that since (15) does not hold, $H_1[\iota(m^*;\theta,\tau),\theta] > 0$ and $H_2(m^*) = 0$. Thus, there is a unique $m < m^*$ that solves the FOC, a solution we denote by $\bar{m}(\theta,\tau)$. Define

$$\bar{\iota}(\theta,\tau) \equiv \iota[\bar{m}(\theta,\tau)'\theta,\tau],$$

the equilibrium $\iota$ under the optimal $\bar{m}(\theta,\tau)$.

Now, suppose that $\tau' > \tau$. Since $\iota(m;\theta,\tau)$ is strictly decreasing in $\tau$ and hence, for any given $m$, the left side of (87) is strictly decreasing in $\tau$. Thus, $\bar{m}(\theta,\tau') > \bar{m}(\theta,\tau)$. This in turn implies that

$$H_1[\bar{\iota}(\theta,\tau'),\theta] = H_2[\bar{m}(\theta,\tau')] < H_2[\bar{m}(\theta,\tau)] = H_1[\bar{\iota}(\theta,\tau),\theta].$$

Since $H_1$ is strictly increasing in $\iota$, this implies that $\bar{\iota}(\theta,\tau') < \bar{\iota}(\theta,\tau)$. Thus, as $\tau$ increases, optimal $m$ increases, and the equilibrium $\iota$ decreases. This then, by Lemma 4.2, implies that optimal $\kappa$ and $\eta$ both decrease. Finally, note that optimal

$$m\kappa = m\frac{\Pi\left(\frac{\bar{A}}{m}\right)-\gamma}{r-\iota} = m\frac{\lambda^{\frac{x-1}{x}}\left(\frac{\bar{A}}{m}\right)^{x}-\gamma}{r-\iota} = \frac{\lambda^{\frac{x-1}{x}}\left(\frac{(\bar{A})^{x}}{m^{x-1}}\right)-m\gamma}{r-\iota}, \tag{88}$$

which is strictly decreasing in $m$ (because $x>1$) but strictly increasing in $\iota$. Thus, optimal $m\kappa$ decreases as well and hence $\mathcal{L}$ decreases.

Now we turn to comparative statics for $\theta$, and assume $u(q) = q^{\alpha}/\alpha$ and $c(q) = q$. Then

$$\begin{aligned} D(\iota;\theta) &= (\theta\sigma)^{\frac{1}{1-\alpha}}\left(\frac{1}{\sigma+\iota}\right)^{\frac{1}{1-\alpha}}, D'(\iota;\theta) = -(\theta\sigma)^{\frac{1}{1-\alpha}}\frac{1}{1-\alpha}\left(\frac{1}{\sigma+\iota}\right)^{\frac{2-\alpha}{1-\alpha}}, \\ \frac{D(\iota;\theta)}{D'(\iota;\theta)} &= -(1-\alpha)(\sigma+\iota). \end{aligned}$$

We can then rewrite $H_1$ as

$$H_1(\iota;\theta) = \frac{1+r}{\frac{r+(1-\alpha)\sigma}{\iota}-\alpha},$$

That is, $H_1$ only depends on $\iota$ but not $\theta$. Now, when $\theta$ increases, $\iota(m;\theta,\tau)$ increases as well and hence the left-side of (87) increases. As a result, $\bar{m}(\theta,\tau)$ decreases. Thus, $H_2[\bar{m}(\theta,\tau)]$ increases. Hence, $H_1[\bar{\iota}(\theta,\tau)]$ must increase as well, and hence $\bar{\iota}(\theta,\tau)$ increases. This then implies that optimal $\kappa$ and $\eta$ both increase. Finally, it follows from (88) that optimal $m\kappa$ increases and hence $\mathcal{L}$ increases. $\square$

**Proof of Lemma 4.3** It is easy to verify that for any given $\mathbf{m}$, $[A_1(\mathbf{m}), ...., A_N(\mathbf{m})]$ given by (40) uniquely solves

$$\min_{(A_1,\ldots,A_N)} \sum_{n=1}^{N} [m_n\lambda_n\psi(A_n) + m_n\gamma]$$

s.t. $\sum_{n=1}^{N} m_n A_n = \bar{A}$. Moreover, these solutions can be characterized as follows: for any $\mathbf{m}$, define $C(\mathbf{m})$ as the solution to

$$\sum_{n=1}^{N} m_n(\psi')^{-1}\left(\frac{C}{\lambda_n}\right) = \bar{A}. \tag{89}$$

$C(\mathbf{m})$ is well-defined by strict convexity of $\psi$. Then,

$$A_n(\mathbf{m}) = (\psi')^{-1}\left(\frac{C(\mathbf{m})}{\lambda_n}\right).$$

Now, we can compute the derivatives:

$$\begin{aligned} \frac{\partial}{\partial m_n} C &= -\frac{A_n(\mathbf{m})}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}}, && (90) \\ \frac{\partial}{\partial m_n} A_{n'} &= -\frac{A_n(\mathbf{m})}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \frac{1}{\lambda_{n'}\psi''[A_{n'}(\mathbf{m})]}. && (91) \end{aligned}$$

Now, define

$$\Psi(\mathbf{m}) \equiv \sum_{n=1}^N \left[m_n \lambda_n \psi(A_n(\mathbf{m})) + m_n \gamma\right], \quad (92)$$

and we can rewrite the original problem, (38), as

$$\min_{\mathbf{m}} \Psi(\mathbf{m}) \text{ s.t. } m_n \le \mu_n,\ n = 1, ..., N.$$

By (91), we have

$$\begin{aligned} \frac{\partial}{\partial m_n}\Psi(\mathbf{m}) &= \lambda_n \psi(A_n(\mathbf{m})) + \gamma - \sum_{k=1}^N m_k \lambda_k \psi'(A_k(\mathbf{m})) \frac{A_n(\mathbf{m})}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \frac{1}{\lambda_k \psi''[A_k(\mathbf{m})]} \\ &= \lambda_n \psi(A_n(\mathbf{m})) + \gamma - \lambda_n \psi'(A_n(\mathbf{m})) A_n(\mathbf{m}) \frac{\sum_{k=1}^N m_k \frac{1}{\lambda_k \psi''[A_k(\mathbf{m})]}}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \\ &= -\lambda_n [\psi'(A_n(\mathbf{m})) A_n(\mathbf{m}) - \psi(A_n(\mathbf{m}))] + \gamma \\ &= -\lambda_n \Pi(A_n(\mathbf{m})) + \gamma, && (93) \end{aligned}$$

where the second equality follows from (40). Since for any $\mathbf{m}$, $\lambda_n \Pi(A_n(\mathbf{m}))$ is strictly decreasing in $n$ among which $m_n > 0$. This implies the optimal solution has the form given by (41)-(44). Note that (39) guarantees that $\bar{n} \le N$. □

**Proof of Theorem 4.2** (a) Since (58) holds, the proposed policy implements the first best allocation and hence is optimal.

(b) Suppose that (58) does not hold.

(b.1) The proof follows similar logic to the proof of Theorem 4.1, but we need some preliminary lemmas that are listed in Appendix B4. In particular, we refer to Lemma 6.3 for optimal policy for given $\mathbf{m}$ and $\iota$, and the corresponding definition of $\mathcal{S}(\iota, \mathbf{m})$ in (118) and $\iota(\mathbf{m})$ in (119). Note that $\iota(\mathbf{m}^*) > 0$ because (58) does not hold. Now,

$$
\begin{aligned}
\frac{\partial}{\partial m_{\bar{n}}}\mathcal{S}(\iota, \mathbf{m}^*) &= \frac{1+r}{r-\iota}\lambda_{\bar{n}}\psi'(A_{\bar{n}}(\mathbf{m}^*))A_{\bar{n}}(\mathbf{m}^*) \\
&+ \frac{(1+r)\left[-\frac{\partial \Psi(\mathbf{m}^*)}{\partial m_{\bar{n}}} + \sum_{n=1}^{N} m_n\lambda_n[\psi''(A_n(\mathbf{m}^*))A_n(\mathbf{m}^*) + \psi'(A_n(\mathbf{m}^*))]\frac{\partial}{\partial m_{\bar{n}}}A_n(\mathbf{m}^*)\right]}{r-\iota} \\
&= \frac{1+r}{r-\iota}\lambda_{\bar{n}}\psi'(A_{\bar{n}}(\mathbf{m}^*))A_{\bar{n}}(\mathbf{m}^*) \\
&+ \frac{(1+r)\left[\sum_{n=1}^{N} m_n\lambda_n[\psi''(A_n(\mathbf{m}^*))A_n(\mathbf{m}^*) + \psi'(A_n(\mathbf{m}^*))]\frac{\partial}{\partial m_{\bar{n}}}A_n(\mathbf{m}^*)\right]}{r-\iota},
\end{aligned}
$$

where the second equality follows from (93) and form definition of $\bar{n}$, which implies that

$$
\frac{\partial}{\partial m_{\bar{n}}}\Psi(\mathbf{m}^*) = -\left[\lambda_{\bar{n}}\Pi(A_{\bar{n}}(\mathbf{m}^*)) - \gamma\right] = 0.
$$

Now, by (91),

$$
\begin{aligned}
&\sum_{n=1}^{N} m_n\lambda_n\psi''(A_n(\mathbf{m}^*))A_n(\mathbf{m}^*)\frac{\partial}{\partial m_{\bar{n}}}A_n(\mathbf{m}^*) \\
&= -\sum_{n=1}^{N} m_n\lambda_n\psi''(A_n(\mathbf{m}^*))A_n(\mathbf{m}^*)\frac{A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N}\frac{m_j}{\lambda_j\psi''[A_j(\mathbf{m})]}}\frac{1}{\lambda_n\psi''[A_n(\mathbf{m}^*)]} \\
&= -\left(\sum_{n=1}^{N} m_n A_n(\mathbf{m}^*)\right)\frac{A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N}\frac{m_j}{\lambda_j\psi''[A_j(\mathbf{m})]}} = -\frac{\bar{A}A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N}\frac{m_j}{\lambda_j\psi''[A_j(\mathbf{m})]}},
\end{aligned}
$$

and

$$
\begin{aligned}
& \sum_{n=1}^{N} m_n \lambda_n \psi'(A_n(\mathbf{m}^*)) \frac{\partial}{\partial m_{\bar{n}}} A_n(\mathbf{m}^*) \\
= \;& -\sum_{n=1}^{N} m_n \lambda_n \psi'(A_n(\mathbf{m}^*)) \frac{A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N} \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \frac{1}{\lambda_n \psi''[A_n(\mathbf{m}^*)]} \\
= \;& -\lambda_{\bar{n}} \psi'(A_{\bar{n}}(\mathbf{m}^*)) A_{\bar{n}}(\mathbf{m}^*) \frac{\sum_{n=1}^{N} \frac{m_n}{\lambda_n \psi''[A_n(\mathbf{m}^*)]}}{\sum_{j=1}^{N} \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \\
= \;& -\lambda_{\bar{n}} \psi'(A_{\bar{n}}(\mathbf{m}^*)) A_{\bar{n}}(\mathbf{m}^*),
\end{aligned}
$$

where the second last equality follows from the fact that $\lambda_{\bar{n}} \psi'(A_{\bar{n}}(\mathbf{m}^*)) = \lambda_n \psi'(A_n(\mathbf{m}^*))$ for all $n$ with $m_n^* > 0$. Combining the terms, we obtain

$$
\begin{aligned}
\frac{\partial}{\partial m_{\bar{n}}} \mathcal{S}(\iota, \mathbf{m}^*) \;=\;& \frac{1+r}{r-\iota} \lambda_{\bar{n}} \psi'(A_{\bar{n}}(\mathbf{m}^*)) A_{\bar{n}}(\mathbf{m}^*) \\
&- \frac{1+r}{r-\iota} \left[ \lambda_{\bar{n}} \psi'(A_{\bar{n}}(\mathbf{m}^*)) A_{\bar{n}}(\mathbf{m}^*) + \frac{\bar{A} A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N} \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \right] \\
=\;& -\frac{(1+r)}{r-\iota} \left[ \frac{\bar{A} A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N} \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \right] < 0.
\end{aligned}
$$

This then implies that, at the margin, a small decrease in $m_{\bar{n}}$ can increase the total liquidity given by $\mathcal{S}(\iota, \mathbf{m}^*)$, and since (58) does not hold, it is optimal to decrease $m_{\bar{n}}$. The formal argument follows exactly the same reasoning as that of Theorem 4.1.

(b.2) First note that for any $\iota > 0$, $\kappa_n = \bar{\kappa}_n[\mathbf{m}, \iota(\mathbf{m})] = \frac{(1+r)[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma]}{r-\iota}$ is optimal under $\mathbf{m}$. We first show that $\lambda_n \Pi[A_n(\mathbf{m})]$ decreases in $n$, and hence so does $\kappa_n(\mathbf{m})$. Suppose that $n < n'$. Then, equilibrium requires $\lambda_n \psi'(A_n) = \lambda_{n'} \psi'(A_{n'})$, and hence,

$$
\begin{aligned}
& \lambda_n \left[ \psi'(A_n) A_n - \psi(A_n) \right] > \lambda_{n'} \left[ \psi'(A_{n'}) A_{n'} - \psi(A_{n'}) \right] \\
\Leftrightarrow \;& A_n - \frac{\psi(A_n)}{\psi'(A_n)} > A_{n'} - \frac{\psi(A_{n'})}{\psi'(A_{n'})}.
\end{aligned}
$$

Now,

$$\frac{\mathrm{d}}{\mathrm{d}a}\left[a - \frac{\psi(a)}{\psi'(a)}\right] = 1 - \frac{[\psi'(a)]^2 - \psi(a)\psi''(a)}{[\psi'(a)]^2} = \frac{\psi(a)\psi''(a)}{[\psi'(a)]^2} > 0,$$

and the result follows from the fact that $A_n > A_{n'}$.

Here we show that leverage decreases with $n$, assuming that $\psi(a) = \lambda a^x/x$ for some $x > 1$. Rearranging terms, we have

$$\mathcal{L}_n = \rho + \frac{1+r}{r-\iota}\frac{[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma]}{\tau A_n(\mathbf{m})}.$$

We want to show $\frac{\lambda_n \Pi(A_n(\mathbf{m}))-\gamma}{A_n(\mathbf{m})}$ is strictly decreasing in $n$. Given that $\psi(a) = \lambda a^x/x$, we have $\psi'(a) = \lambda a^{x-1}$, and $\Pi(a) = \psi'(a)a - \psi(a) = \frac{x-1}{x}\lambda a^x$. Thus,

$$\frac{\lambda_n \Pi[A_n(\mathbf{m})]}{A_n(\mathbf{m})} = \lambda_n \frac{x-1}{x}\lambda[A_n(\mathbf{m})]^{x-1}.$$

Note that $\lambda_n \lambda [A_n(\mathbf{m})]^{x-1} = \lambda_n \psi'(A_n(\mathbf{m}))$, and that $\lambda_n \psi'[A_n(\mathbf{m})]$ is constant across $n$ for all active banks. This in turn implies that $\frac{\lambda_n \Pi(A_n(\mathbf{m}))}{A_n(\mathbf{m})}$ is constant across $n$ as well. Because $\frac{\gamma}{A_n(\mathbf{m})}$ increases in $n$, we obtain the result that $\frac{\lambda_n \Pi(A_n(\mathbf{m}))-\gamma}{A_n(\mathbf{m})}$ is strictly decreasing in $n$.

Finally, we show that the same result holds if we define leverage including reserves, that is,

$$\mathcal{L}'_n = \frac{\rho\tau A_n(\mathbf{m}) + \bar{\kappa}_n(\mathbf{m})}{\tau A_n(\mathbf{m})} = \frac{\rho}{1-\eta+\eta\rho} + \frac{1-\eta}{\tau(1-\eta+\eta\rho)}\frac{1+r}{r-\iota}\frac{\lambda_n \Pi(A_n(\mathbf{m}))-\gamma}{A_n(\mathbf{m})}.$$

Note that it is also sufficient to show that $\frac{\lambda_n \Pi(A_n(\mathbf{m}))-\gamma}{A_n(\mathbf{m})}$ is strictly decreasing in $n$.

**Proof of Lemma 5.1** We show that banks have no incentive to gamble if and only if $\omega \le \omega_1$. Note that, by a similar argument as before, the profit at the optimal asset holding $a$ (for either prudent behavior or gambling behavior) is given by $\Pi(a) - \gamma$ and since $\Pi$ is strictly increasing, it suffices to show that $A^s \le \bar{A}/m$. This can be done by comparing the first-order conditions for the profits under prudent behavior and under gambling, and this would be the case if and only if

$$
\begin{aligned}
& \frac{\iota - r\eta}{1-\eta}[\rho\tau_\ell + \omega(\tau - \tau_\ell)] + \tau - (1+r)\phi \\
\geq \quad & \frac{\iota - r\eta}{1-\eta}[\rho\tau_\ell + \omega(\tau - \tau_\ell)] + \tau - (1+r)\phi \\
& + \nu\tau_h + (1-\nu)\tau_\ell - \tau + (1-\nu)\omega(\tau - \tau_\ell) + (1+r)e \\
\iff \quad & \omega \leq \omega_1.
\end{aligned}
$$

**Proof of Theorem 5.1** From Lemma 5.1, if $\omega_1 \geq \rho$, then banks will not gamble under $\omega = \rho$. Thus, in that case, moral hazard is not an issue in the sense that one can ignore it when setting the pledgeability constraints.

So now suppose that $\omega_1 < \rho$, and we show that for any given $m$, it is optimal to set $\omega = \omega_1$. Note that since $m$ determines asset-management efficiency, the regulator's problem is reduced to maximize DM trade (up to $q^*$), or, equivalently, to have the lowest equilibrium $\iota$ among all $(\eta, \kappa, \omega)$ that are incentive compatible. To do this, instead of working with constraint (72), we consider a different constraint that we later show is more relaxed: we assume that the gambling bank also chooses $\bar{A}/m$ and derive the incentive compatibility based on that choice (instead of the optimal one). Now, the profit to a gambling bank is given by

$$
\begin{aligned}
& \pi^s(a, z, d; \phi, R) \qquad (94) \\
= \quad & \frac{d}{R} - z - \phi a - [\psi(a) - ea + \gamma] + \beta\{[\nu\tau_h + (1-\nu)\tau_\ell]a + z - \nu d - (1-\nu)d_\ell\} \\
= \quad & \beta \left\{ \begin{array}{c} \frac{\iota - r\eta}{1-\eta}[\rho\tau_\ell + \omega(\tau - \tau_\ell)]a + \iota\kappa + [\tau - (1+r)\phi]a - (1+r)[\psi(a) + \gamma] \\ +(1-\nu)\kappa + [\nu\tau_h + (1-\nu)\tau_\ell - \tau + \omega(1-\nu)(\tau - \tau_\ell)]a + (1+r)ea \end{array} \right\}.
\end{aligned}
$$

Hence, the gain from gambling under the assumed asset choice is the difference between two expressions (69) and (94) with $a = \bar{A}/m$, which is given by

$$
\Phi \equiv \beta\{[\omega(1-\nu)(\tau - \tau_\ell) + \nu(\tau_h - \tau) - (\tau - \nu\tau - (1-\nu)\tau_\ell)]\bar{A}/m + (1+r)e\bar{A}/m + (1-\nu)\kappa\}. \quad (95)
$$

We can write the incentive compatibility constraint as

$$-\Phi + (1-\nu)\frac{\beta}{1-\beta}\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota \cdot \kappa}{1+r}\right] \geq 0,$$

which can be simplified to

$$\begin{aligned} & -r\left[(\omega-1)(1-\nu)(\tau-\tau_\ell) + (1+r)e + \nu(\tau_h - \tau)\right]\frac{\bar{A}}{m} \\ + \quad & (1-\nu)\left\{-(r-\iota)\kappa + (1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right]\right\} \geq 0. \end{aligned} \tag{96}$$

Now we show that (96) relaxes (72), which can be rewritten as

$$-r(1+r)\left[\Pi(A^s) - (1-\nu)\Pi\left(\frac{\bar{A}}{m}\right)\right] + (1-\nu)\left\{-(r-\iota)\kappa + (1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right]\right\} \geq 0.$$

Moreover, $\Pi(A^s) - \Pi\left(\frac{\bar{A}}{m}\right)$ and $[(\omega-1)(1-\nu)(\tau-\tau_\ell) + (1+r)e + \nu(\tau_h - \tau)]$ have the same sign for all $\omega$ (it is positive for $\omega > \omega_1$, negative for $\omega < \omega_1$, zero for $\omega = \omega_1$), and

$$\Pi(A^s) - \Pi\left(\frac{\bar{A}}{m}\right) > \left[(\omega-1)(1-\nu)(\tau-\tau_\ell) + (1+r)e + \nu(\tau_h - \tau)\right]\left(\frac{\bar{A}}{m}\right)$$

for all $\omega > \omega_1$. When both terms are negative, the corresponding constraints are weaker than (31). It then follows that (96), combined with (31), is indeed weaker than (72) combined with (31). When $\omega = \omega_1$, they are equivalent.

Now, define

$$\mathcal{S}(\iota) = \max_{\eta,\omega,\kappa} \frac{\rho\tau_\ell + \omega(\tau-\tau_\ell)}{1-\eta}\bar{A} + m\kappa$$

subject to (31) and (96). Note that the minimum equilibrium $\iota$ subject to (31) and (96) is determined by $D(\iota) \leq S(\iota)$ (at equality whenever $\iota > 0$). Note that $S(\iota)$ is strictly increasing in $\iota$: as $\iota$ increases both constraints (96) and (31) are more relaxed, and the objective function is strictly increasing in $\iota$.

Now we show that at the optimum, we have $\eta = \iota/r$, $\kappa$ determined by (31) at equality, and

$\omega = \omega_1$. For any fixed $\iota$, the maximization problem in $S(\iota)$ is a linear programming problem in $(\kappa, \omega)$ and can be reduced to (by using $\eta = \iota/r$)

$$\begin{aligned} & \max_{\kappa,\omega} \frac{\omega r(\tau - \tau_\ell)}{r - \iota} \bar{A} + m\kappa, \\ \text{s.t.} \quad & -r\omega(\tau - \tau_\ell)\frac{\bar{A}}{m} - (r - \iota)\kappa + C \geq 0, \\ & -(r - \iota)\kappa + D \geq 0. \end{aligned}$$

where by (59),

$$\begin{aligned} C &= r\left[(\tau - \tau_\ell)\frac{\bar{A}}{m} - \frac{(1+r)e + \nu(\tau_h - \tau)}{1 - \nu}\frac{\bar{A}}{m}\right] + (1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right] \\ &> (1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right] = D. \end{aligned}$$

The optimal choice is given by

$$\kappa = \frac{D}{r - \iota}, \ \omega = \frac{C - D}{r(\tau - \tau_\ell)\frac{\bar{A}}{m}} = \omega_1.$$

□

**Proof of Theorem 5.2** Since $\omega_1 < \rho$ and given the results in Theorem 5.1, the optimal banking regulation problem, or the *planner's problem*, is

$$\begin{aligned} & \max_{m \geq 0, \iota \geq 0} \sigma[u(q) - c(q)] - \left[m\gamma + m\psi\left(\frac{\bar{A}}{m}\right)\right], \\ \text{s.t.} \quad & D(\iota) \leq \frac{r}{r - \iota}[\rho\tau_\ell + \omega_1(\tau - \tau_\ell)]\bar{A} + m\frac{(1+r)[\psi'(\bar{A}/m)\bar{A}/m - \psi(\bar{A}/m) - \gamma]}{r - \iota}, \\ & c(q) = D(\iota). \end{aligned}$$

Note that by (66), $\omega_1$ is strictly decreasing in $\tau_h$ and in $e$. Now, let

$$F(\iota, m; \omega_1) = -D(\iota) + \frac{r}{r - \iota}[\rho\tau_\ell + \omega_1(\tau - \tau_\ell)]\bar{A} + m\frac{(1+r)[\psi'(\bar{A}/m)\bar{A}/m - \psi(\bar{A}/m) - \gamma]}{r - \iota}.$$

Then,

$$\begin{aligned}\frac{\partial}{\partial m}F &= \frac{(1+r)}{r-\iota}\left\{\left[\psi'\left(\frac{\bar{A}}{m}\right)\frac{\bar{A}}{m}-\psi\left(\frac{\bar{A}}{m}\right)-\gamma\right]-\psi''\left(\frac{\bar{A}}{m}\right)\left(\frac{\bar{A}}{m}\right)^2\right\},\\ \frac{\partial}{\partial \iota}F &= -D'(\iota)+\frac{r[\rho\tau_\ell+\omega_1(\tau-\tau_\ell)]\bar{A}+m(1+r)\left[\psi'\left(\frac{\bar{A}}{m}\right)\frac{\bar{A}}{m}-\psi\left(\frac{\bar{A}}{m}\right)-\gamma\right]}{(r-\iota)^2}.\end{aligned}$$

Let $\iota(m;\omega_1)$ be the implicit function defined by $F(\iota,m;\omega_1)=0$, $m\le m^*$, which exists since the first-best is not implementable, and it is differentiable by Implicit Function Theorem. Then,

$$\begin{aligned}\iota'(m;\omega_1) &= -\frac{(1+r)(r-\iota)\left\{\left[\psi'\left(\frac{\bar{A}}{m}\right)\frac{\bar{A}}{m}-\psi\left(\frac{\bar{A}}{m}\right)-\gamma\right]-\psi''\left(\frac{\bar{A}}{m}\right)\left(\frac{\bar{A}}{m}\right)^2\right\}}{-D'(\iota)(r-\iota)^2+\left\{r[\rho\tau_\ell+\omega_1(\tau-\tau_\ell)]\bar{A}+m(1+r)\left[\psi'\left(\frac{\bar{A}}{m}\right)\frac{\bar{A}}{m}-\psi\left(\frac{\bar{A}}{m}\right)-\gamma\right]\right\}}\\ &= -\frac{(1+r)\left\{\left[\psi'\left(\frac{\bar{A}}{m}\right)\frac{\bar{A}}{m}-\psi\left(\frac{\bar{A}}{m}\right)-\gamma\right]-\psi''\left(\frac{\bar{A}}{m}\right)\left(\frac{\bar{A}}{m}\right)^2\right\}}{-D'(\iota)(r-\iota)+D(\iota)},\end{aligned}$$

where the second equality follows from the fact that $\iota=\iota(m;\omega_1)$ and $F[\iota(m;\omega_1),m;\omega_1]=0$. Moreover, for any $m$ and $\omega_1$ such that $\iota(m;\omega_1)>0$, $\iota(m;\omega_1)$ strictly increases with $\omega_1$.

We can then eliminate the constraints on the social planner's problem by plugging in $q=c^{-1}[D(\iota(m;\omega_1))]$. Thus, the FOC for the social planer's problem, with $\psi(a)=\lambda a^x/x$, is given by

$$\sigma\left\{\frac{u'(q)}{c'(q)}-1\right\}\frac{-(1+r)\left[\lambda\frac{(x-1)^2}{x}\left(\frac{\bar{A}}{m}\right)^x+\gamma\right]}{(r-\iota)+\frac{D(\iota)}{-D'(\iota)}}+\left[\lambda\frac{x-1}{x}\left(\frac{\bar{A}}{m}\right)^x-\gamma\right]=0,$$

in which $q=c^{-1}[D(\iota)]$ and $\iota$ is an implicit function of $m$ and $\omega_1$ with $\iota'(m;\omega_1)>0$. Since, by (12),

$$\iota=\sigma\left[\frac{u'(q)}{c'(q)}-1\right],$$

this condition can be simplified as

$$\frac{(1+r)\iota(m;\omega_1)}{[r-\iota(m;\omega_1)]+\frac{D[\iota(m;\omega_1)]}{-D'[\iota(m;\omega_1)]}}=\frac{\left[\lambda\frac{x-1}{x}\left(\frac{\bar{A}}{m}\right)^x-\gamma\right]}{\left[\lambda\frac{(x-1)^2}{x}\left(\frac{\bar{A}}{m}\right)^x+\gamma\right]}. \tag{97}$$

Now, by (37), the left-side of (97) is strictly increasing in $m$, while the right-side is strictly decreasing. Since the first-best is not implementable, there exists a unique $\tilde{m}$ that solves (97) which is the optimal $m$. As $\tau_h$ or $e$ increases and hence $\omega_1$ decreases, $\iota(m;\omega_1)$ increases for any $m$, and this decreases the optimal $\tilde{m}$. $\square$

**Proof of Lemma 5.2** For any given $\iota$ and $m$, market clearing is given by

$$D(\iota) \leq \frac{1}{1-\eta}\rho\tau\bar{A} + m\kappa,$$

with equality whenever $\iota > 0$. The parameters $\eta$ and $\kappa$ are subject to (75) and (31). Since the right-side of the above inequality is strictly increasing in $\eta$ and in $\kappa$, and to set both at equality under (75) and (31) gives the upper bound for both parameters, it is optimal to do so. $\square$

**Proof of Theorem 5.3** (a) This follows exactly the same reasoning as that for Theorem 4.1 (a).

(b) We follow the same logic as that for Theorem 4.1 (b). Since (15) does not hold, Lemma 3.1 (b) implies that under $(m,\kappa) = (m^*, 0)$ the equilibrium allocation has $q < q^*$. We show that any optimal policy has $m < m^*$. From Lemma 5.2, we have

$$\mathcal{S}(\iota, m) = \frac{\iota_m \rho\tau}{\iota_m - \iota}\bar{A} + m\frac{(1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right]}{r-\iota}.$$

Let $\iota(m)$ be the unique solution to the requirement that $D(\iota) \leq S(\iota, m)$ and with equality whenever $\iota > 0$. Since (15) does not hold and hence $\iota(m^*) > 0$, there is a range of $m$'s below $m^*$, $\iota(m)$ is determined by $D(\iota) = \mathcal{S}(\iota, m)$.

Let $F(\iota, m) \equiv \mathcal{S}(\iota, m) - D(\iota)$, and hence $\iota(m)$ is implicitly defined by $F(\iota, m) = 0$. We can compute the derivatives of $F$ as follows. For all $\iota < r$,

$$\begin{aligned}
\frac{\partial}{\partial m}F(\iota, m^*) = \frac{\partial}{\partial m}\mathcal{S}(\iota, m^*) &= \frac{1+r}{r-\iota}\left[\Pi\left(\frac{\bar{A}}{m^*}\right) - \Pi'\left(\frac{\bar{A}}{m^*}\right)\left(\frac{\bar{A}}{m^*}\right) - \gamma\right] \\
&= -\frac{1+r}{r-\iota}\left[\psi''\left(\frac{\bar{A}}{m^*}\right)\frac{\bar{A}^2}{(m^*)^2}\right] < 0.
\end{aligned}$$

Similarly,

$$\frac{\partial}{\partial \iota}F(\iota, m^*) = \frac{\partial}{\partial \iota}\mathcal{S}(\iota, m^*) - D'(\iota) = \frac{\iota_m \rho \tau \bar{A}}{(\iota_m - \iota)^2} + \frac{m^*(1+r)\left[\Pi\left(\frac{\bar{A}}{m^*}\right) - \gamma\right]}{(r-\iota)^2} - D'(\iota) > 0.$$

By the implicit function theorem, in a neighborhood of $m^*$, $\iota(m)$ is continuously differentiable with

$$\iota'(m^*) = -\frac{\frac{\partial}{\partial m}F(\iota, m^*)}{\frac{\partial}{\partial \iota}F(\iota, m^*)} > 0.$$

Now, to maximize welfare (1) subject to $F(m, \iota) = 0$ and $q = c^{-1}[D(\iota)]$ can then be reduced to

$$\max_{m \in [m^*, \bar{m}]} G(m) \equiv \mathcal{W}[D(\iota(m)), m].$$

Now,

$$\begin{aligned}
G'(m^*) &= \sigma\left\{\frac{u'[c^{-1}[D(\iota(m^*))]]}{c'[c^{-1}[D(\iota(m^*))]]} - 1\right\} D'(\iota(m^*))\iota'(m^*) - [\Pi(\bar{A}/m^*) - \gamma] \\
&= \sigma\left\{\frac{u'[c^{-1}[D(\iota(m^*))]]}{c'[c^{-1}[D(\iota(m^*))]]} - 1\right\} D'(\iota(m^*))\iota'(m^*) < 0,
\end{aligned}$$

where the last inequality follows from the fact that $D(\iota(m^*)) < q^*$, $D'(\iota(m^*)) < 0$, and $\iota'(m^*) > 0$. Thus, the optimal $m$ must satisfy $m < m^*$.

(c) Let $m \leq m^*$ be given so that the first-best is not implementable. The right-side of (77) is strictly decreasing in $\iota_m$ and hence in $\zeta$. Thus, an increase in $\zeta$ leads to a higher equilibrium $\iota$ according to (77), which then increases optimal $\kappa$ according to Lemma 5.2. □

# Appendix B: Supplemental Material (For Online Publication)

## B1. Linearity of $W(d)$

We use $V_t(d)$ to denote a household's continuation value upon entering period-$t$ DM with deposit $d$, and $W_t(d)$ to denote a household's continuation value upon entering period-$t$ CM with deposit $d$, facing a sequence of bank returns $\{R_t\}_{t=0}^{\infty}$. Then, a household with deposits $d$ entering period-$t$ CM faces the following problem:

$$\begin{aligned} W_t(d) &= \max_{x_t,h_t,d_t} \{x_t - h_t + \beta V_{t+1}(d_t)\} \\ \text{s.t. } x_t &= h_t + d - \frac{d_t}{R_t},\ x_t \geq 0, h_t \geq 0,\ d_t \geq 0, \end{aligned} \tag{98}$$

where $d_t$ is the deposits taken out of the period-$t$ CM market. Note that $d_t$ is the *promised value* in terms the period-$(t+1)$ CM goods. Substituting for $h_t$ from the budget constraint to rewrite (98) as

$$W_t(d) = d + \max_{d_t} \left\{ -\frac{d_t}{R_t} + \beta V_{t+1}(d_t) \right\} = d + W_t(0),$$

where

$$W_t(0) \equiv \max_{d_t} \left\{ -\frac{d_t}{R_t} + \beta V_{t+1}(d_t) \right\}.$$

Therefore, the demand for deposits does not depend on $d_t$, the initial deposits that the household brings to the period-$t$ CM. Moreover, $W_t(d)$ is linear in $d$.

## B2. Optimal pledgeability constraint

Here we show that, as long as (32) does not bind, the constraints (27) and (28) are indeed optimal. A sufficient condition for this to hold is the following. Let $\bar{r}$ be the unique solution to

$$\rho\tau\bar{A} + \bar{m}\frac{(1+r)\left[\Pi\left(\frac{\bar{A}}{\bar{m}}\right) - \gamma\right]}{r} = c(q^*). \tag{99}$$

Then, for any $r \leq \bar{r}$, by setting $m = \bar{m}$ the DM production $q^*$ is implementable, but it will imply inefficiency in asset management and hence cannot be optimal. This implies that any optimal $m$ must satisfy $m > \bar{m}$, and hence the constraint (32) is not binding.

Now we show that the constraints (27) and (28) are optimal among all other constraints of the form:

$$d \leq \tilde{\rho}\tau a + z + \kappa, \tag{100}$$

$$z \geq \max\{0, \eta(d - \kappa')\}. \tag{101}$$

Here $\tilde{\rho}$ is the fraction of assets that a bank pledges for its deposits. Thus, upon bankruptcy, the depositors can claim $\tilde{\rho}$ fraction of its assets for repayment. However, as assumed in the main text, the court can only seize up to $\rho$ fraction. As a result, when $\tilde{\rho} > \rho$, the difference is unsecured and requires self-enforcing.

Now, under the general constraints (100) and (101), the bank's equilibrium profits and market clearing conditions remain the same as in Section 4, except for replacing $\rho$ by $\tilde{\rho}$. In particular, under market clearing, a bank's equilibrium profit is still given by

$$\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{1}{1+r}\left[\frac{\iota - \eta r}{1-\eta}(\kappa - \eta\kappa') + r\eta\kappa'\right].$$

Thus, incentive compatibility requires

$$-\kappa - \tilde{\rho}\tau\frac{\bar{A}}{m} - z + \sum_{t=0}^{\infty}\beta^t\left\{\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{1}{1+r}\left[\frac{\iota - \eta r}{1-\eta}(\kappa - \eta\kappa') + r\eta\kappa'\right]\right\} \geq -\min\{\rho, \tilde{\rho}\}\tau\frac{\bar{A}}{m} - z.$$

The right-side of the inequality is the real value seized by the court upon default. Since the court can only seize up to $\rho$ fraction of bank assets, when $\tilde{\rho} > \rho$, only $\rho$ fraction can be used for repayment. We can then rewrite this incentive constraint as

$$-r \max\{\tilde{\rho} - \rho, 0\}\tau \frac{\bar{A}}{m} - \frac{r - \iota}{1 - \eta}(\kappa - \eta\kappa') + (1 + r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right] \geq 0. \tag{102}$$

The incentive compatibility constraint for $\eta$ is still given by (20).

Moreover, for a given $m$, under the general constraints (100) and (101) the total amount of liquidity provided by the banking sector is given by

$$\frac{\tilde{\rho}\tau}{1 - \eta}\bar{A} + m\frac{1}{1 - \eta}(\kappa - \eta\kappa'). \tag{103}$$

Now, we have four policy parameters: $(\kappa, \kappa', \eta, \tilde{\rho})$. As before, under the assumption that $m \geq \bar{m}$ does not bind, optimal policy is the one that maximizes (103) subject to (102) and (20).

The following lemma then generalizes Lemma 4.2.

**Lemma 6.1.** *Let $m \in [\bar{m}, m^*]$ and $\iota < r$ be given. The optimal $(\kappa, \kappa', \eta, \tilde{\rho})$ that solves*

$$\mathcal{S}(m, \iota) = \max_{\kappa, \kappa', \eta, \tilde{\rho}} \frac{\tilde{\rho}\tau}{1 - \eta}\bar{A} + m\frac{1}{1 - \eta}(\kappa - \eta\kappa'), \tag{104}$$

*subject to (20) and (102) is given by $\eta = \iota/r$, $\kappa$ from (34), $\kappa' = \kappa$, and $\tilde{\rho} = \rho$.*

*Proof.* We consider two cases. First, consider $\tilde{\rho} \leq \rho$. In this case, $\tilde{\rho}$ does not enter the constraint. Recall that

$$\mathcal{S}(m, \iota) = \max_{\tilde{\rho}, \eta, \kappa, \kappa'} \frac{\tilde{\rho}\tau}{1 - \eta}\bar{A} + m\frac{1}{1 - \eta}(\kappa - \eta\kappa'),$$

subject to (20) and (102). It is straightforward to see that since $\kappa$ and $\kappa'$ enter both the objective function and the constraint (102) only through $\kappa - \eta\kappa'$, it is optimal to set $\kappa = \kappa'$ and we may rewrite (31) as

$$\kappa \leq \frac{1 + r}{r - \iota}\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right] \tag{105}$$

and we can rewrite

$$\mathcal{S}(\iota, m) = \max_{\tilde{\rho},\eta,\kappa} \frac{\tilde{\rho}\tau}{1-\eta}\bar{A} + m\kappa.$$

Thus, at the optimum, we have $\eta = \iota/r$ and $\kappa$ be determined by (105) at equality. Thus, it is optimal to choose $\tilde{\rho}$ as large as possible, and hence $\tilde{\rho} = \rho$ is optimal.

Second, consider the case where $\tilde{\rho} \geq \rho$. For the same reason as before, it is optimal to choose $\kappa = \kappa'$ and $\eta = \iota/r$. We may then rewrite the maximization problem as

$$\max_{\tilde{\rho},\kappa} \frac{r\tilde{\rho}\tau}{r-\iota}\bar{A} + m\kappa \tag{106}$$

$$\text{subject to} \quad -r(\tilde{\rho}-\rho)\tau\frac{\bar{A}}{m} - (r-\iota)\kappa + (1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right] \geq 0, \tag{107}$$

$$\tilde{\rho} \geq \rho. \tag{108}$$

Note that we need the last constraint to rewrite (102) as (107). The problem is a linear programming problem and it is straightforward to verify that $\tilde{\rho} = \rho$ is optimal. $\square$

The proof of Lemma 6.1 shows that any $\tilde{\rho} \geq \rho$ is as good as $\tilde{\rho} = \rho$, but that only takes the incentive compatibility constraint (102) into account. When $\tilde{\rho} > \rho$, a bank who plans to default in the following CM would also choose a different asset holding than the equilibrium level, and that would give rise to an additional incentive compatibility constraint. Thus, without further regulations, setting $\tilde{\rho} = \rho$ is strictly optimal.

## B3. Derivation of threshold $\bar{m}$

Here we first derive an interval for $m$ in which chartered banks can furnish all required capital, and then derive another interval in which unchartered banks have no incentive to become active and hold assets. The condition (32) is obviously then the intersection of the two.

We begin with the feasibility issue. For a given policy parameter $(\eta, m, \kappa)$ and $\iota$, in equilibrium each chartered bank holds assets worth of $\phi(m)\bar{A}/m + \frac{\eta}{1-\eta}\rho\tau\bar{A}/m$ (the value of projects and reserves), and promises deposits $d = \frac{\eta}{1-\eta}\rho\tau\bar{A}/m + \kappa$; hence, feasibility requires

$$\phi(m)\bar{A}/m + \frac{\eta}{1-\eta}\rho\tau\bar{A}/m - \frac{\frac{1}{1-\eta}\rho\tau\bar{A}/m + \kappa}{R} \leq \bar{h}. \tag{109}$$

Rearranging terms using $\phi(m)$ given by (25), and replacing $\kappa$ with its optimal level given by (31) at equality, this can be simplified to

$$[\beta(1-\rho)\tau - \psi'(\bar{A}/m)]\bar{A}/m - \frac{(1+r)[\Pi(\bar{A}/m) - \gamma]}{R(r-\iota)} \leq \bar{h}. \tag{110}$$

By (5), it follows that when $m = m^*$ (110) holds with strict inequality and hence there is a threshold $\bar{m}_1(\iota) < m^*$ such that (110) holds for all $m \geq \bar{m}_1(\iota)$.

Now we show that feasibility is not an issue for $r$ not too large; that is $\bar{m}_1(\iota) = 0$. Let

$$K(a) = \beta(1-\rho)\tau a - \psi'(a)a - (1+\iota)\frac{\Pi(a) - \gamma}{r-\iota}.$$

As mentioned, by (5), $K(A^*) < \bar{h}$ with $A^* \equiv \bar{A}/m^*$. Now,

$$K'(a) = \beta(1-\rho)\tau - \frac{1+r}{r-\iota}\psi''(a)a - \psi'(a).$$

Since $\Pi(a)$ is convex, $\psi''(a)a$ is increasing in $a$. Now, let $\bar{r}$ be the largest $r$ such that

$$\beta(1-\rho)\tau - \frac{1+r}{r}\psi''(\bar{A}/m^*)\bar{A}/m^* - \psi'(\bar{A}/m^*) \leq 0.$$

Note that $\bar{r} > 0$ exits. Then, for any $r \leq \bar{r}$, $K'(A) \leq 0$ for all $A \geq A^*$, and hence $K(A) < \bar{h}$ for all $A \geq A^*$, that is, (110) holds for all $m \in [0, m^*]$.

Now we turn to the incentive issue for unchartered banks for a given $(\eta, \iota)$. Let $\phi(m)$ be given by (25), which also determines the asset price for any given $\kappa$ under the charter system. Note that for any given $(\eta, \iota)$, $\phi(m)$ is strictly increasing in $m$. Now, For any given $\eta$, $\iota$, $\phi$, and

$m < m^*$, an unchartered bank chooses asset holding by solving

$$\pi^{nc}[\phi(m)] = \max_{0 \leq a \leq \bar{h}/\phi} -\phi a + \beta \tau a - \psi(a), \tag{111}$$

but will hold a positive amount only if $\pi^{nc}[\phi(m)] > \gamma$. When $m = m^*$, $\phi(m^*) \geq \phi^*$, and hence (5) ensures that, either the constraint is binding and $a = \bar{h}/\phi^* < \bar{A}/m^*$, or, $\phi(m^*) > \phi^*$ and the optimal holding is strictly less than $\bar{A}/m^*$. Since when $\phi = \phi^*$ by holding $\bar{A}/m^*$ assets a unchartered bank makes zero profit, it then follows that $\pi^{nc}[\phi(m^*)] < \gamma$. By the Theorem of Maximum $\pi^{nc}$ is continuous in $\phi$, and now we show that it is strictly decreasing in $\phi$. Let $\phi_1 < \phi_2$, and let $a_2$ solves (111) for $\phi_2$. Then $a_2$ is also feasible for the problem under $\phi_1$, but gives a strictly higher value. Finally, at $\phi = 0$ we have $\pi^{nc}(\phi) > \gamma$ by (3). Thus, there exists a unique $\bar{\phi} < \phi^*$ for which $\pi^{nc}(\bar{\phi}) = \gamma$. Let $\bar{m}_2(\eta, \iota)$ be the unique $m$ such that

$$\bar{\phi} = \phi(m).$$

Now, if $m < \bar{m}_2(\eta, \iota)$, $\phi < \bar{\phi}$ and hence $\pi^{nc}(\phi) > \gamma$. This implies that if only $m$ banks hold assets, unchartered banks can deviate to hold assets and make profits. We set $\bar{m}(\eta, \iota) = \min\{\bar{m}_1(\iota), \bar{m}_2(\eta, \iota)\}$, and for all $m \in [\bar{m}(\eta, \iota), m^*]$, both feasibility of capital for chartered banks and incentive for unchartered banks not to hold assets are respected.

Finally, we show that equilibria where unchartered banks hold assets cannot be optimal. Suppose that $m^u$ measure of unchartered banks are active and hold assets, and $m$ chartered banks are active. Since unchartered banks have free entry to hold assets, it must be the case that $\phi = \bar{\phi}$ and unchartered banks make zero profits. We show that the regulator can increase welfare by increasing $m$ to $m' = m + \epsilon < m + m^u$. Since the asset price must be the same under $m$ and $m'$ because unchartered banks are active in both, the optimal asset holding for each chartered bank is the same, as well as their profits. However, as $m < m'$, under $m'$ chartered banks hold more pledgeable assets than under $m$ and this will lead to higher welfare.

## B4. Heterogenous banks

We first characterize equilibrium allocation for a given set of policy parameters, $(\eta, \mathbf{m}, \{\kappa_n\})$, and then show how the regulator chooses policy parameters to maximize the social welfare. As before, we only focus on $\eta$'s for which $\iota \in [\eta r, r)$.

**Lemma 6.2.** *Let $\mathbf{m} \leq \mathbf{m}^*$ with $m_1 > 0$, $\eta$, and $\{\kappa_n\}$ be given. There is a unique allocation $(\phi, \iota, q, d)$ that satisfies the market-clearing conditions, and can be characterized as follows:*

$$A_n = A_n(\mathbf{m}),$$

$$\phi = \frac{\left[1 + \frac{\rho(\iota - r\eta)}{1-\eta}\right]\tau - (1+r)\lambda_1\psi'(A_1)}{1+r}, \tag{112}$$

$$D(\iota) \leq \frac{\rho\tau}{1-\eta}\bar{A} + \sum_{n=1}^{N} m_n\kappa_n, \text{ with equality if } \iota > 0. \tag{113}$$

*Moreover, the profit for bank of type $n$ is given by*

$$\lambda_n\Pi(A_n(\mathbf{m})) - \gamma + \frac{\iota\kappa_n}{1+r}. \tag{114}$$

*Proof.* From the FOC, (51), and the market clearing condition, (53), it follows that in equilibrium the asset holdings solve (40), that is, $A_n = A_n(\mathbf{m})$. Then, (112) follows from (51). □

The assumption that $m_1 > 0$ is with no loss of generality; if, instead, $m_1 = 0$ but $m_n > 0$ for some other $n$, then we can simply replace 1 by $n$ in (112) and (113). Note also that since we are only concerned with market clearing and not entry, banks may make negative profits (because of the fixed cost $\gamma$). However, a full equilibrium analysis also requires incentive compatibility for repayment of $\kappa$, which would require nonnegative profits. As before, banks that fail to repay depositors will be closed and hence lose their future profits. Thus, given a policy, $\mathbf{m}$ and $\{(\kappa_n, \kappa_n')\}$, a bank of type-$n$ is willing to repay deposits if and only if

$$-\kappa_n + \sum_{t=0}^{\infty} \beta^t \left\{\lambda_n\Pi(A_n(\mathbf{m})) - \gamma + \frac{1}{1+r}\left[\frac{\iota - \eta r}{1-\eta}(\kappa_n - \eta\kappa_n') + r\eta\kappa_n'\right]\right\} \geq 0.$$

This constraint can be simplified as

$$-\frac{r-\iota}{1-\eta}(\kappa_n - \eta\kappa_n') + (1+r)[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma] \geq 0. \tag{115}$$

The following lemma characterize optimal $\{\kappa_n\}$ for a given $\mathbf{m}$.

**Lemma 6.3.** *Let* $\mathbf{m}$ *be given such that*

$$\lambda_n \Pi(A_n(\mathbf{m})) \geq \gamma \text{ for all } n \text{ with } m_n > 0.$$

*For any given* $\iota$*, it is optimal to set* $\eta = \iota/r$*, and to set* $\kappa_n' = \kappa_n$*.*

*(a) Let* $\hat{\kappa}_n(\mathbf{m}) = \frac{1+r}{r}[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma]$ *for each* $n = 1, ..., N$*. If*

$$c(q^*) \leq \rho \frac{\tau}{1+r}\bar{A} + \sum_{n=1}^{N} m_n \hat{\kappa}_n(\mathbf{m}), \tag{116}$$

*then* $\iota = 0$ *and* $q = q^*$ *in equilibrium under* $\eta = 0$*.*

*(b) Suppose that (116) does not hold. Then, there exists an optimal* $\{\kappa_n\}$ *under* $\mathbf{m}$*, denoted by* $\{\bar{\kappa}_n(\mathbf{m})\}$*, such that the constraint (115) is binding for all* $n$ *with* $m_n > 0$*.*

*Proof.* Let $\bar{\kappa} = \sum_{n=1}^{N} m_n(\kappa_n - \eta\kappa_n')$. Define

$$\mathcal{S}(\iota, \mathbf{m}) = \max_{\eta, \kappa_n, \kappa_n'} \frac{\rho\tau}{1-\eta}\bar{A} + \frac{\bar{\kappa}}{1-\eta},$$

subject to

$$-\frac{r-\iota}{1-\eta}\bar{\kappa} + (1+r)\sum_{n=1}^{N} m_n \left[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma\right] \geq 0. \tag{117}$$

This implies that

$$\begin{aligned} \mathcal{S}(\iota, \mathbf{m}) &= \frac{\rho\tau}{1-\eta}\bar{A} + \frac{(1+r)\sum_{n=1}^{N} m_n \left[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma\right]}{r-\iota} \\ &= \frac{\rho\tau}{1-\eta}\bar{A} + \frac{(1+r)[-\Psi(\mathbf{m}) + \sum_{n=1}^{N} m_n \lambda_n \psi'(A_n(\mathbf{m}))A_n(\mathbf{m})]}{r-\iota}, \end{aligned} \tag{118}$$

where $\Psi$ is given by (92). Note that, for any fixed $\mathbf{m}$, $S(\iota, \mathbf{m})$ is strictly increasing in $\iota$.

For each $n$, let

$$\bar{\kappa}_n(\iota, \mathbf{m}) = \frac{(1-\eta)(1+r)\left[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma\right]}{r - \iota}.$$

Fixed some $\mathbf{m}$, we consider two cases.

(i) If $D(0) \leq S(0, \mathbf{m})$, then $q^*$ is implementable under $\eta = 0$ with $\kappa_n = \bar{\kappa}_n(0, \mathbf{m})$, which is optimal under $\mathbf{m}$.

(ii) Otherwise, let $\iota(\mathbf{m}) > 0$ be the unique solution to

$$D(\iota) = \mathcal{S}(\iota, \mathbf{m}). \tag{119}$$

Then, $\kappa_n = \bar{\kappa}_n[\iota(\mathbf{m}), \mathbf{m}]$ is optimal under $\mathbf{m}$. $\square$

## B5. Is gambling always bad?

In the main text we have focused on equilibrium with regulations that induce banks to be prudent, and have shown that even under the charter system the best equilibrium with prudent banks requires $\omega = \min\{\rho, \omega_1\}$, the same capital requirement as under market discipline. However, this can be very costly in terms of liquidity provision; in particular, when $\tau_h$ is high so that $\omega_1$ is close to zero, it seems inefficient to insist on prudent behavior. To investigate the optimality of prudent behavior, it is necessary to understand whether an equilibrium with banks gambling exists in the first place. For the analysis below, we begin with a given $m$. Moreover, while a constraint similar to (32) is needed, for simplicity we focus on the case where it does not bind.

In contrast to the main text, when the regulator expects banks to gamble, the pledgeability constraint has to take this into account. In particular, since returns are now stochastic, the repayment should also depend on the bank returns. Specifically, when a bank with asset holding $a$ has return $\tau_h$, it can repay up to $d_h = \rho\tau a + z + \kappa$ with $\kappa$ the unsecured deposits; note that the court cannot seize the difference $\tau_h - \tau$. When the return is $\tau_\ell$, the bank can only repay $d_\ell = \rho\tau_\ell a + z + \kappa$. Note that this differs from (71) by $\kappa$, as gambling is expected and the regulator still requires banks repay $\kappa$ to stay in the business. Thus, the pledgeability constraint is bounded by the expected amount the bank can repay, and hence is given by

$$d \leq \rho[\nu\tau + (1-\nu)\tau_\ell]a + z + \kappa, \tag{120}$$

and, as in the main text, we set the reserve requirement as (28).

Since all banks are expected to gamble, there is no extra capital requirement than what the court could enforce. Thus, the profit for a gambling bank is given by

$$\begin{aligned} & \pi^s(a, d; \phi, R) \\ = \; & \frac{d}{R} - z - \phi a - [\psi(a) - ea + \gamma] + \beta\{(\nu\tau_h + (1-\nu)\tau_\ell)a + z - \nu d_h - (1-\nu)d_\ell\} \\ = \; & \beta \left\{ \begin{array}{c} \frac{\iota - r\eta}{1-\eta}\rho[\nu\tau + (1-\nu)\tau_\ell]a + [\nu\tau_h + (1-\nu)\tau_\ell - (1+r)\phi]a + \iota\kappa - (1+r)ea \\ -(1+r)[\psi(a) + \gamma] \end{array} \right\}. \end{aligned} \tag{121}$$

The demand for deposits is not affected by the stochastic returns since both buyers and sellers are only concerned with the expected return in the CM. The market clearing condition for deposits is then given by

$$D(\iota) \leq \rho\frac{[\nu\tau + (1-\nu)\tau_\ell]}{1-\eta}\bar{A} + m\kappa, \text{ with equality whenever } \iota > 0. \tag{122}$$

Finally, banks need incentives to gamble. To do so, we need to consider the profit for a bank not to gamble under pledgeability constraint (120), which is given by

$$\begin{aligned} & \pi^p(a, d; \phi, R) = \frac{d}{R} - z - \phi a - [\psi(a) + \gamma] + \beta\{\tau a + z - d_h\} \\ = \; & \beta \left\{ \begin{array}{c} \frac{\iota - r\eta}{1-\eta}\rho[\nu\tau + (1-\nu)\tau_\ell]a + [\nu\tau_h + (1-\nu)\tau_\ell - (1+r)\phi]a + \iota\kappa - (1+r)ea \\ +\underbrace{[\tau - \nu\tau_h - (1-\nu)\tau_\ell]a}_{(a)} - \underbrace{\rho(1-\nu)(\tau - \tau_\ell)a}_{(b)} - \underbrace{(1+r)[\psi(a) + ea + \gamma]}_{(c)} \end{array} \right\}. \end{aligned} \tag{123}$$

Now, compared to (121), (123) implies that a prudent bank benefits from a higher average return, reflected in the term (a), but loses on two grounds: since a bank with return $\tau$ repays more and a prudent bank always has return $\tau$, this hurts the profit of a prudent bank, as expressed in term (b); the prudent bank has a higher variable cost, as in term (c). Taking $\phi$ as given, the FOC then gives an optimal asset holding for a prudent bank, $A^p$. Incentive compatibility then requires

$$\Pi(A^p) \leq \Pi^s(\bar{A}/m).$$

We have the following theorem.

**Theorem 6.1.** *Let $m \leq m^*$ be given. Then, an equilibrium with all banks gambling exists and is unique if and only if $\omega_1 \leq \rho$, where $\omega_1$ is given by (66). Moreover, an optimal gambling equilibrium yields higher welfare than the optimal prudent equilibrium if and only if first-best is not implementable under the optimal prudent equilibrium and*

$$\omega_1 < \rho\nu. \tag{124}$$

*Proof.* First we show that banks have no incentive to be prudent if and only if $\rho \leq \omega_1$. Note that we only need to show that $A^p \leq \bar{A}/m$. This can be done by comparing the first-order conditions for the profits given by (121) and (123), and this would be the case if and only if

$$\begin{aligned} & \frac{\iota - r\eta}{1-\eta}\rho[\nu\tau + (1-\nu)\tau_\ell] + [\nu\tau_h + (1-\nu)\tau_\ell - (1+r)\phi] + (1+r)e \\ \geq \;& \frac{\iota - r\eta}{1-\eta}\rho[\nu\tau + (1-\nu)\tau_\ell] + [\nu\tau_h + (1-\nu)\tau_\ell - (1+r)\phi] \\ + \;& [\tau - \nu\tau_h - (1-\nu)\tau_\ell)]a - \rho(1-\nu)(\tau - \tau_\ell), \end{aligned}$$

which holds if and only if $\omega_1 \leq \rho$.

Now for any fixed $m$, it is easy to show that the optimal gambling equilibrium is characterized by the following

$$D(\iota) \leq \mathcal{S}^g(\iota),$$

where

$$\mathcal{S}^g(\iota) = \max_{\kappa} \frac{\rho r[\nu\tau + (1-\nu)\tau_\ell]}{r-\iota}\bar{A} + m\kappa,$$

subject to (31). Note that under this equilibrium banks profits are still given by $\Pi(\bar{A}/m) + \iota\kappa/(1+r) - \gamma$.

However, as shown in Theorem 5.1, the optimal prudent equilibrium is characterized by

$$D(\iota) \leq \mathcal{S}^p(\iota),$$

where

$$\mathcal{S}^p(\iota) = \max_{\kappa} \frac{r[\rho\tau_\ell + \omega(\tau - \tau_\ell)]}{r-\iota}\bar{A} + m\kappa,$$

subject to (31).

Since the $\kappa$ term will be replaced by the same expression, $S^g(\iota) > S^p(\iota)$ if and only if

$$\rho r[\nu\tau + (1-\nu)\tau_\ell] > r[\rho\tau_\ell + \omega(\tau - \tau_\ell)],$$

which is equivalent to (124). □

## B6. Money creation or withdrawal cannot improve welfare

Here we consider inflationary/deflationary policy where the central bank charges a tax on banks to finance deflation. We assume that the transfer is given at the beginning of CM, before the settlement of the deposit obligation. We use $T$ to denote the value of the transfer (when positive) or tax (when negative) in terms of CM good, and use $\zeta$ for the net money creation rate (negative value means shrinking). Now, if $\bar{z}$ is the equilibrium reserves holding for each bank at the end of CM, then

$$T = \frac{\zeta}{1+\zeta}\bar{z}. \tag{125}$$

We also consider general pledgeability constraint:

$$d \leq \rho\tau a + z + T + \kappa, \tag{126}$$

$$z + T \geq \eta(d - \kappa'), \tag{127}$$

where $z$ is the value of reserves in the coming CM, the same units as $d$. Note that since the transfer or the tax is given before settlement, banks can use them to repay deposits and hence should be added to the right-side in the pledgeability constraint, (126). Similarly, $T$ should be added to the left-side of (127) as the sum will be the available amount of reserves for repayment next period. However, note that the fact that $T$ appears in (127) is with no loss of generality since we allow $\kappa'$ to be arbitrary. Assuming everything binds, we have

$$\begin{aligned} d &= \frac{1}{1-\eta}[\rho\tau a - \eta\kappa' + \kappa], \\ z + T &= \frac{\eta}{1-\eta}[\rho\tau a + \kappa - \kappa']. \end{aligned}$$

The profit is then given by

$$\begin{aligned} & \pi(a, d, z; \phi, R) = -\phi a - (1+\zeta)z + d/R - \psi(a) - \gamma + \beta\{\tau a + z + T - d\} \\ = \; & \beta\{[\tau - (1+r)\phi]a - (1+r)[\psi(a) + \gamma] + \iota d - \iota_m z + T\} \\ = \; & \beta\{[\tau - (1+r)\phi]a - (1+r)[\psi(a) + \gamma] + \iota d - \iota_m(z+T) + (1+\iota_m)T\} \\ = \; & \beta\left\{\frac{\iota - \eta\iota_m}{1-\eta}\rho\tau a + [\tau - (1+r)\phi]a - (1+r)[\psi(a) + \gamma]\right\} \\ + \; & \beta\left\{\frac{\iota - \eta\iota_m}{1-\eta}(\kappa - \eta\kappa') + \iota_m\eta\kappa' + (1+\iota_m)T\right\}. \end{aligned}$$

Hence, equilibrium profit will be

$$\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \beta(1+\iota_m)T\right] + \frac{1}{1+r}\left[\frac{\iota - \eta\iota_m}{1-\eta}(\kappa - \eta\kappa') + \iota_m\eta\kappa'\right],$$

where

$$\beta(1+\iota_m)T = \zeta\left[\frac{\eta}{1-\eta}[\rho\tau\bar{A}/m + \kappa - \kappa']\right] \tag{128}$$

is the transfer to each active bank. Hence, the incentive compatibility constraint requires

$$-r\kappa + \frac{\iota - \eta\iota_m}{1-\eta}(\kappa - \eta\kappa') + \iota_m\eta\kappa' + (1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \beta(1+\iota_m)T\right] \geq 0,$$

which can be simplified as

$$-\frac{r-\iota}{1-\eta}(\kappa - \eta\kappa') - \frac{\eta(r-\iota_m)}{1-\eta}\rho\frac{\bar{A}}{m} + (1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right] \geq 0. \tag{129}$$

For a given $m$, the optimal regulation solves

$$\max_{\eta,\kappa} \frac{1}{1-\eta}\rho\tau\bar{A} + m\kappa, \tag{130}$$

subject to $\eta \leq \iota/\iota_m$ and (129). This may then be rewritten as

$$\begin{aligned} & \max_{\eta} \frac{1}{1-\eta}\rho\tau\bar{A}/m - \frac{\eta(r-\iota_m)}{(1-\eta)(r-\iota)}\rho\tau\bar{A}/m + \frac{(1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right]}{r-\iota} \\ = \quad & \max_{\eta}\left[\frac{r}{r-\iota} - \frac{\iota - \eta\iota_m}{1-\eta}\right]\rho\tau\bar{A}/m + \frac{(1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right]}{r-\iota}, \end{aligned}$$

subject to $\eta \leq \min\{1, \iota/\iota_m\}$. Since the total liquidity available is increasing in $\eta$, it is optimal to set $\eta = \min\{1, \iota/\iota_m\}$. Then, the total liquidity either coincide with the right-side of (35) or is smaller (when $\eta \leq 1$ is binding). However, note that when $\iota_m < r$, i.e., when there is deflation, the constraint $\eta \leq 1$ can be binding. Thus, the optimal monetary policy is to have a constant money (reserve) supply.