Optimal Monetary Interventions in Credit Markets: Supplemental Material

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Here we give four extensions of our baseline model. The first section considers limited monitoring with a general feasibility constraint, that is, we allow ℓ to be any number in $(0, 2)$, and we also consider the one-round DM model with general limited monitoring. The second studies the model with $\sigma_1 < 1$. The third considers models with multiple DM rounds. The fourth considers alternative meeting patterns, where a buyer meet sellers from $\mathbb{S}_1$ and $\mathbb{S}_2$ with positive probabilities at both rounds of DM.

1 General limited monitoring

1.1 Models with one DM round

Here we consider limited monitoring with one DM round, and allow for general $\ell < 1$. There is a measure one of buyers and a measure one of sellers. We use u to denote the DM utility function for buyers and c to denote the cost function for sellers for the DM good. The probability of a successful matching in the DM is $\sigma \in (0, 1]$. Let $x^* > 0$ be the solution to $u'(x^*) = c'(x^*)$. All agents consume and produce with a linear preference in the CM. An allocation is given by

$$\mathcal{L} = [(x_1, x_2), (z_1, z_2)],$$

where x_1 denotes a buyer's DM consumption in monitored meetings and z_1 the corresponding sellers' CM consumption, x_2 denotes a buyer's DM consumption in non-monitored

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meetings and z_2 the corresponding sellers' CM consumption. The notion of implementation is the same as in the main text.

We have the following result.

Proposition 1.1. *The allocation $\mathcal{L}$ is implementable only if*

$$-\rho \max\{z_1, z_2\} + \sigma\ell[u(x_1) - z_1] + \sigma(1-\ell)[u(x_2) - z_2] \geq 0, \tag{1}$$

$$z_i \geq c(x_i), \;\; i = 1, 2, \tag{2}$$

$$x_i \leq x^*, \;\; i = 1, 2 \text{ and } x_1 = x^* \text{ if } z_1 < z_2. \tag{3}$$

Proof. (Sketch.) The constraint (2) is necessary to ensure seller participation. The constraint (3) is necessary to ensure that the buyer has no deviating offer to make both parties better off. Note that buyers in monitored meetings can pay at least as much (using both money and debt potentially) as in non-monitored meetings. Now we show the necessity of (1).

Suppose that the payment is given by $z_{1,c}$ (credit component) and $z_{1,m}$ (money component) in monitored meetings. Then the real balance buyers need to hold is $z_m \geq \max\{z_{1,m}, z_2\}$. Consider an agent who had a monitored DM meeting in the CM. His money holding is then $z_m - z_{1,m}$ and his debt is $z_{1,c}$. To follow the equilibrium behavior, the buyer needs to buy $z_{1,m}$ units of real balances and repay $z_{1,c}$ units of debts. He could, however, deviate to selling all of his money without repaying anything and staying in autarky thereafter. This deviation is not profitable only if

$$\begin{aligned} & -z_{1,m} - z_{1,c} + \frac{\delta}{1-\delta}\left\{\sigma\ell[u(x_1) - z_{1,c} - z_{1,m}] + \sigma(1-\ell)[u(x_2) - z_2]\right\} \\ \geq \;\; & z_m - z_{1,m} \geq \max\{z_{1,m}, z_2\} - z_{1,m}, \end{aligned}$$

which implies

$$-\rho \max\{z_1, z_2\} + \{\sigma\ell[u(x_1) - z_1] + \sigma(1-\ell)[u(x_2) - z_2]\} \geq 0.$$

Thus, (1) is necessary. □

As in the main text, to find the constrained efficient allocation, it is without loss of generality to assume that (2) binds and consider allocations with $z_i = c(x_i)$ for $i = 1, 2$. Moreover, it can be easily shown that the solution to

$$\max_{(x_1, x_2)} \sigma\ell[u(x_1) - c(x_1)] + \sigma(1-\ell)[u(x_2) - c(x_2)]$$

subject to (1), denoted by $(\hat{x}_1, \hat{x}_2)$, satisfies (3) and that $\hat{x}_1 = \hat{x}_2$. Thus, by Proposition 1.1, the constrained efficient allocation under $\ell \in [0, 1]$ cannot do better than $(\hat{x}_1, \hat{x}_2)$. However, since $\hat{x}_1 = \hat{x}_2 \equiv \hat{x}$ and since it satisfies

$$-\rho c(\hat{x}) + \sigma[u(\hat{x}) - c(\hat{x})] \geq 0,$$

Hu-Kennan-Wallace (2009) show that it is also implementable under a constant money supply without any monitoring. Thus, the use of debt is not essential for any $\ell \in [0, 1]$. We also remark that since condition (1) reflects an overall participation constraint without which buyers may simply prefer permanent autarky to the equilibrium arrangement, no policy that satisfies voluntary participation can improve the constrained efficient outcome.

1.2 Model with two DM rounds

Now we turn to our model with two DM rounds. We still assume $\sigma_1 = 1$, although the results are robust to this assumption in the same way as discussed in the next section. In the main text we only consider $\ell = 0, 1, 2$, but here we illustrate how results change for any $\ell \in (0, 2)$. We first remark that since the mechanism can always choose to monitor a smaller fraction of sellers than what is available given by ℓ (or, equivalently, to ignore records from some meetings), any implementable allocation without interventions under $\ell \in (0, 2)$ cannot have a higher welfare than (x_1^{C1}, x_2^{C1}) given in the main text, which is the solution to

$$\begin{aligned} \max_{(x_1,x_2)} \quad & u_1(x_1) - c_1(x_1) + \sigma_2[u_2(x_2) - c_2(x_2)] \\ \text{subject to} \quad & -\rho[c_1(x_1) + c_2(x_2)] + u_1(x_1) - c_1(x_1) + \sigma_2[u_2(x_2) - c_2(x_2)] \geq 0. \end{aligned} \tag{4}$$

Next, for the same reason, for any $\ell \geq 1$, EMP can be used to implement the allocation (x_1^{C2}, x_2^{C2}) given in the main text, which is the solution to

$$\begin{aligned} \max_{(x_1,x_2)} \quad & u_1(x_1) - c_1(x_1) + \sigma_2[u_2(x_2) - c_2(x_2)] \\ \text{subject to} \quad & \{-\rho c_1(x_1) + [u_1(x_1) - c_1(x_1)]\} + \frac{(\rho+1)\sigma_2}{\rho+\sigma_2}\{-\rho c_2(x_2) + \sigma_2[u_2(x_2) - c_2(x_2)]\} \geq 0, \\ & -\rho c_1(x_1) + u_1(x_1) - c_1(x_1) \geq 0. \end{aligned} \tag{5}$$

Finally, for all $\rho \leq \rho^M$ that is given in the main text, the first best allocation is still implementable in a pure currency economy without intervention.

The following theorem shows that results regarding $\ell = 1$ in the main text are robust in the sense that there is an interval $(\underline{\ell}, \overline{\ell})$ around $\ell = 1$ for which the EMP is optimal.

Theorem 1.1. *Suppose that* $\rho > \rho^M$*,* $\sigma_2 < 1$*, and that* $(\bar{x}_1, \bar{x}_2) \neq (x_1^{C1}, x_2^{C1})$*. Then, there exist an interval* $(\underline{\ell}, \overline{\ell})$ *around* $\ell = 1$ *such that for all* ℓ *in that interval, the constrained efficient allocation with EMP under* ℓ *cannot be implemented with a constant money supply.*

Proof. (Sketch.) We separate two cases.

(1) $-\rho c_1(x_1^{C1}) + u_1(x_1^{C1}) - c_1(x_1^{C1}) < 0$. For each $\ell \in (0, 1]$, define $(x_1(\ell), x_2(\ell))$ as the solution to

$$\max_{(x_1,x_2)} \quad \ell[u_1(x_1) - c_1(x_1)] + \sigma_2[u_2(x_2) - c_2(x_2)] \tag{6}$$

$$\text{subject to} \quad \frac{(1+\rho)\ell}{\rho+\ell}\{-\rho c_1(x_1) + \ell[u_1(x_1) - c_1(x_1)]\} + \{-\rho c_2(x_2) + \sigma_2[u_2(x_2) - c_2(x_2)]\} \geq 0.$$

Note that $(x_1(\ell), x_2(\ell))$ is (x_1^{C1}, x_2^{C1}) at $\ell = 1$ and it varies continuously with ℓ. As argued in the main text, when $\ell = 1$, (x_1^{C1}, x_2^{C1}) satisfies the pairwise core requirement. Now, using the same arguments and continuity, $(x_1(\ell), x_2(\ell))$ also satisfies the pairwise core requirement for ℓ sufficiently close to 1.

When the allocation $(x_1(\ell), x_2(\ell))$ satisfies the pairwise core, it is implementable under $\ell \in (0, 1]$ by the following EMP:

$$k = \frac{1}{\rho+\ell}\{-\rho c_1(x_1(\ell)) + \ell[u_1(x_1(\ell)) - c_1(x_1(\ell))]\}, \ \pi = \frac{\ell k}{c_2[x_2(\ell)]}, \tag{7}$$

and the following mechanism. We choose ℓ fraction of round-1 DM sellers to be monitored. We set $D = c_1[x_1(\ell)]$, and the real balance to be $c_2[x_2(\ell)]$. We choose the following o_1 and o_2. o_2 punishes buyers who do not bring enough real balances, but continuously. o_1 gives buyers with good records equilibrium surplus, zero surplus otherwise, if the meeting is monitored. If not monitored, the buyer receives expected round-2 DM surplus.

As mentioned earlier, $(x_1(\ell), x_2(\ell)) = (x_1^{C1}, x_2^{C1})$ when $\ell = 1$. Also, at $\ell = 1$, the welfare without intervention is strictly less than that of (x_1^{C1}, x_2^{C1}). Thus, for some $\underline{\ell} < 1$, if $\ell \in (\underline{\ell}, 1]$, $(x_1(\ell), x_2(\ell))$ satisfies the pairwise core and achieves a better welfare than that with no intervention.

Now we show that there exists $\overline{\ell} > 1$ such that for all $\ell < \overline{\ell} \leq 2$, (x_1^{C1}, x_2^{C1}) is not implementable without intervention. First note that for all $\ell < 2$, if some meetings in round-1 DM is not monitored, then cash is needed to trade in those meetings and hence it requires $-\rho c_1(x_1^{C1}) + u_1(x_1^{C1}) - c_1(x_1^{C1}) \geq 0$, which leads to a contradiction to our assumption. Thus, all round-1 DM has to be monitored if we want to implement it without intervention. Thus, for (x_1^{C1}, x_2^{C1}) to be implementable without intervention under ℓ, it must be the case that

$$-\rho[c_1(x_1^{C1}) + c_2(x_2^{C1})] + [u_1(x_1^{C1}) - c_1(x_1^{C1})] + (\ell - 1)\sigma_2[u_1(x_2^{C1}) - c_1(x_2^{C1})] \geq 0.$$

At $\ell = 1$ this does not hold. Then $\overline{\ell} > 1$ is the smallest number for this to hold.

(2) $-\rho c_1(x_1^{C1}) + u_1(x_1^{C1}) - c_1(x_1^{C1}) > 0$. Now, (x_1^{C2}, x_2^{C2}) is implementable under EMP and has higher welfare than no intervention at $\ell = 1$. For each $\ell \in (0, 1]$, define $(x_1(\ell), x_2(\ell))$ be the solution to

$$\begin{aligned} \max_{(x_1,x_2)} \quad & [u_1(x_1) - c_1(x_1)] + \ell\sigma_2[u_2(x_2) - c_2(x_2)] \qquad (8) \\ \text{s.t.} \quad & \{-\rho c_1(x_1) + [u_1(x_1) - c_1(x_1)]\} + \frac{(\rho+1)\ell\sigma_2}{\rho+\sigma_2}\{-\rho c_2(x_2) + \ell\sigma_2[u_2(x_2) - c_2(x_2)]\} \geq 0, \\ & -\rho c_1(x_1) + u_1(x_1) - c_1(x_1) \geq 0. \end{aligned}$$

Now, this allocation is implementable under $\ell \in (0, 1]$ by the following EMP:

$$k = \frac{1}{\rho + \sigma_2\ell}\{-\rho c_2(x_2(\ell)) + \ell[u_1(x_2(\ell)) - c_1(x_2(\ell))]\},\ \pi = \frac{\ell\sigma_2 k}{c_1[x_1(\ell)]}, \qquad (9)$$

and the following mechanism. We choose ℓ fraction of round-2 DM sellers to be monitored. We set $D = c_2[x_2(\ell)]$, and the real balance to be $c_1[x_1(\ell)]$. We choose the following o_1 and o_2. o_1 punishes buyers who do not bring enough real balances, but continuously. o_2 gives buyers with good records equilibrium surplus, zero surplus otherwise, if the meeting is monitored. If not monitored, the buyer receives no surplus. Note that the pairwise core requirement is not needed as only round-2 DM has credit trades and no money is used there in equilibrium.

Note that $(x_1(\ell), x_2(\ell))$ is (x_1^{C2}, x_2^{C2}) at $\ell = 1$ and it varies continuously with ℓ. Also, at $\ell = 1$, the welfare without intervention is strictly less than that of (x_1^{C2}, x_2^{C2}). Thus, for some $\underline{\ell} < 1$, if $\ell \in (\underline{\ell}, 1]$, $(x_1(\ell), x_2(\ell))$ achieves a better welfare than that with no intervention.

Now, note that in this case for all $\ell \geq 1$, (x_1^{C2}, x_2^{C2}) is implementable with EMP and it has better welfare than that of unlimited monitoring. □

2 Meeting probabilities with $\sigma_1 < 1$

When $\sigma_1 < 1$, it is not without loss of generality to consider symmetric allocations only. Indeed, a buyer's round-2 allocation may depend on his consumption and payments in his round-1 DM meeting. Thus, an allocation is given by

$$\mathcal{L} = [(x_1, x_2^0, x_2^1), (z_1, z_2^0, z_2^1)],$$

where x_1 denotes a buyer's round-1 DM consumption, conditional on meeting a seller; x_2^0 denotes a buyer's round-2 DM consumption, conditional on meeting a seller at round 2 but not meeting a seller at round 1; x_2^1 denotes a buyer's round-2 DM consumption, conditional on meeting a seller at round 2 and round 1; z_1 denotes the CM consumption of a round-1 seller, conditional on meeting a buyer; z_2^0 denotes the CM consumption of a round-2 seller, conditional on meeting a buyer who has not matched at round 1; z_2^1 denotes the CM consumption of a round-2 seller, conditional on meeting a buyer who has matched at round 1. For simplicity we only consider allocations that satisfy

$$u_1(x_1^*) \geq u_1(x_1) \geq z_1,\ u_2(x_2^*) \geq u_2(x_2^0) \geq z_2^0,\ u_2(x_2^0) \geq u_2(x_2^1) \geq z_2^1,\ z_2^1 \leq z_2^0 \leq z_1 + z_2^1.$$

Except for changes in σ_1, the environment stays the same as in the main text, and the notions of simple equilibria and implementation remains the same. Notice that the first-best allocation, (x_1^*, x_2^*), does not depend on σ_1.

First we give a lemma about implementation under unlimited monitoring, $\ell = 2$.

Lemma 2.1 (Implementability under unlimited monitoring). *Let $\ell = 2$ and assume that money has no value ($\phi = 0$). An allocation $\mathcal{L} = [(x_1, x_2^0, x_2^1), (z_1, z_2^0, z_2^1)]$ is implementable if and only if*

$$-\rho z + \sigma_1[u_1(x_1) - z_1] + \sigma_1\sigma_2[u_2(x_2^1) - z_2^1] + (1-\sigma_1)\sigma_2[u_2(x_2^0) - z_2^0] \geq 0, \quad (10)$$

$$z_1 \geq c_1(x_1),\ z_2^0 \geq c_2(x_2^0),\ z_2^1 \geq c_2(x_2^1), \quad (11)$$

$$[u_1(x_1) - z_1] + \sigma_2[u_2(x_2^1) - z_2^1] \geq \sigma_2[u_2(x_2^0) - z_2^0], \quad (12)$$

$$x_2 < x_2^* \text{ implies } z_2^0 = z, \quad (13)$$

$$u_1(x_1) - c_1(x_1) + \sigma_2[u_2(x_2^1) - z_2^1] \geq u_1(\hat{x}_1) - c_1(\hat{x}_1), \quad (14)$$

where $z = (z_1 + z_2^1)$ and $\hat{x}_1 = \min\{x_1^*, c^{-1}(z_1 + z_2^1)\}$.

Proof. (Sketch.) Because $z_2^0 \leq z_1 + z_2^1$, the biggest total repayment for a buyer is $z_1 + z_1^2$. Thus, (10) is necessary for the buyer to repay. The conditions in (11) are necessary for sellers to accept the trades. After meeting a seller at round-1 DM, the buyer may still skip the trade and hence (12) is necessary. For a buyer who meets for the first time at round-2 DM, (13) is necessary not to deviate to other offers that make both partners better off. The necessity of (14) is the same as before.

For sufficiency, we take $D = z_1 + z_1^2$. The function o_1 is given by $o_1(G, D) = (x_1, z_1)$ and $o_1(B, 0) = (0, 0)$. We also set $o_2(B, 0) = (0, 0)$. The function $o_2(G, \cdot)$ is constructed through ξ, given by $\xi(D) = u_2(x_2^0) - z_2^0$, $\xi(d) = u_2(x_2^1) - z_2^1$ if $D > d \geq D - z_1 = z_2^1$ and let $\xi(d) = 0$ if $d < z_2^1$. The value $\xi(d)$ will be the buyer's surplus in round-2 DM if his available debt limit is d when entering that round. Then, $o_2(G, d)$ solves

$$\max_{(x,y)\in\mathbb{R}_+\times[0,d]} -c_2(x) + y \qquad (15)$$
$$\text{s.t. } u_2(x) - y \geq \xi(d).$$

The condition (13) ensures that (x_2^0, z_2^0) is the unique solution to (15) for $d = D$. The construction of ξ ensures that the only potentially profitable deviation at round-1 DM for a buyer is either to skip or to make an offer involving $\hat{x}_1$. The former deviation is prevented by (12) while the latter by (14). □

Second, we consider a pure currency economy ($\ell = 0$) with a constant money supply.

Lemma 2.2 (Implementability under no monitoring)**.** *Let $\ell = 0$ and assume that the money supply is constant. An allocation $\mathcal{L} = [(x_1, x_2^0, x_2^1), (z_1, z_2^0, z_2^1)]$ is implementable only if (10)-(14), and*

$$u_2(x_2^0) - z_2^0 \geq u_2(x_2^1) - z_2^1, \qquad (16)$$
$$-\rho z_1 + \sigma_1[u_1(x) - z_1] + (1-\sigma_1)\sigma_2\{[u_2(x_2^0) - z_2^0] - [u_2(x_2^1) - z_2^1]\} \geq 0. \qquad (17)$$

Proof. (Sketch.) The necessity of (10)-(14) follows exactly the same reasoning as in Lemma 2.1. Because money holding is not observable and because $z_2^1 \leq z_2^0$, a buyer whose round-2 meeting is the first DM meeting can always pretend to have met a seller at round 1. Hence, (16) is necessary. Moreover, the buyer may also bring only z_2^1 units of money and skip round-1 trades. Thus, (17) is necessary. □

Since the first best allocation, (x_1^*, x_2^*), remains the same for any σ_1, and since the first best allocation is symmetric, using exactly the same arguments as in the main text, it is straightforward to show that the first-best is implementable under no monitoring if and only if it is implementable under limited monitoring ($\ell = 1$) with a constant money supply. Hence, Lemma 2.2 has the following corollary

Corollary 2.1. *Assume limited monitoring ($\ell = 1$). The first-best allocation, (x_1^*, x_2^*), is implementable in with a constant money supply if and only if*

$$\rho \leq \rho^M(\sigma_1) \equiv \min\left\{\frac{\sigma_1[u_1(x_1^*) - c_1(x_1^*)] + \sigma_2[u_2(x_2^*) - c_2(x_2^*)]}{c_1(x_1^*) + c_2(x_2^*)}, \frac{\sigma_1[u_1(x_1^*) - c_1(x_1^*)]}{c_1(x_1^*)}\right\}. \qquad (18)$$

Finally, we consider limited monitoring with expansionary monetary policies (EMP). The formulation of such policies is exactly the same as in the main text; in particular, the policy has two components, (k, π), where k is the amount of maximum debt purchase per trade, and π is the inflation rate. We also impose the feasibility constraint so that all debt purchases are financed by money creation.

For simplicity, we assume that if round i has monitored meetings, then, according to the proposal o_i, for any credit record r, the buyer's surplus is constant over all his money holdings. Thus, we cannot punish the buyer by insufficient money holdings in a credit meeting. We call such a proposal a *pure-credit proposal.* Obviously, this restriction only makes our result stronger.

Theorem 2.1 (Expansionary Monetary Policies). *Suppose that* $\ell = 1$.

(i) A symmetric allocation, $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$, *is implementable (using pure-credit proposals) with EMP and* $C = \{1\}$ *if and only if (11), (14), and*

$$\frac{(\rho+1)\sigma_1}{\rho+\sigma_1}\{-\rho z_1 + \sigma_1[u_1(x_1) - z_1]\} + \{-\rho z_2 + \sigma_2[u_2(x_2) - z_2]\} \geq 0, \tag{19}$$

$$-\rho z_2 + \sigma_2[u_2(x_2) - z_2] \geq 0. \tag{20}$$

(ii) An allocation, $\mathcal{L} = [(x_1, x_2^0, x_2^1), (z_1, z_2^0, z_2^1)]$, *is implementable (using pure-credit proposals) with EMP and* $C = \{2\}$ *if and only if (11), (13), and*

$$\{-\rho z_1 + \sigma_1[u_1(x) - z_1]\} + \frac{(\rho+1)\sigma_2}{\rho+\sigma_2}\{-\rho z_2^1 + \sigma_2[u_2(x_2^1) - z_2^1]\} \geq 0, \tag{21}$$

$$-\rho z_1 + \sigma_1[u_1(x_1) - z_1] \geq 0, \tag{22}$$

$$u_2(x_2^0) - z_2^0 = u_2(x_2^1) - z_2^1. \tag{23}$$

Proof. (Sketch.) (i) The necessity of (11) and (14) is clear. To void buyers from skipping holding cash, (20) is necessary because of pure-credit-proposal. Now we show the necessity of (19). Consider an arbitrary EMP (k, π). For a buyer in the CM to be willing to follow the equilibrium behavior, it must be the case that

$$-(z_1 - k) - (1+\pi)z_2 + \frac{\delta}{1-\delta}\{\sigma_1[u_1(x_1) - (z_1 - k)] + \sigma_2[u_2(x_2) - z_2] - \pi z_2\} \geq 0,$$

that is,

$$-\rho z_1 - \rho z_2 + \sigma_1[u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_2] + (1+\rho)(k - \pi z_2) \geq 0.$$

Feasibility of the EMP implies that $\sigma_1 k = \pi z_2$, the above inequality is equivalent to

$$-\rho z_1 - \rho z_2 + \sigma_1[u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_2] + (1+\rho)(1-\sigma_1)k \geq 0. \tag{24}$$

Now, because of pure-credit proposals, for the buyer to be willing to hold z_2 units of real balances under inflation rate π it must be the case that

$$-[\rho + (1+\rho)\pi]z_2 + \sigma_2[u_2(x_2) - z_2] \geq 0,$$

which gives an upper bound on π and hence, by feasibility of EMP, an upper bound on k. Plugging this upper bound in (24) we obtain (19).

For sufficiency, suppose first that $-\rho z_1 + \sigma_1[u_1(x_1) - z_1] \geq 0$. Since we are considering a symmetric allocation, as shown in the main text, we can implement $\mathcal{L}$ without the EMP. Note that we have (20). Suppose now, instead, that $-\rho z_1 + \sigma_1[u_1(x_1) - z_1] < 0$, and, by (19), $-\rho z_2 + \sigma_2[u_2(x_2) - z_2] > 0$. The EMP is defined as follows. Let

$$k = \frac{1}{\sigma_1 + \rho}\{\rho z_1 - \sigma_1[u_1(x_1) - z_1]\} = z_1 - \frac{\sigma_1}{\sigma_1 + \rho}u_1(x_1) \in (0, z_1), \tag{25}$$

and let $\pi = \sigma_1 k / z_2$. Consider the following proposal: $\phi_t M_t = z_2$ for each t, $C = \{1\}$, $D = z_1 - k$.

The construction of o_2 follows similar steps in the main text. Let $\epsilon \in (0, z_2)$ be so small that

$$\epsilon < \frac{1}{2}\min\left\{\frac{c_1'(x_1)\sigma_2[u_2(x_2) - z_2]}{u_1'(x_1) - c_1'(x_1)}, \frac{\sigma_2[u_2(x_2) - z_2]}{\rho + (1+\rho)\pi}\right\}. \tag{26}$$

Note that condition (26) is exactly the same as in the main text. Similarly, we set

$$\xi(m) = \begin{cases} u_2(x_2) - z_2 & \text{if } m \geq z_2; \\ 0 & \text{if } m \leq z_2 - \epsilon; \\ \left[1 - \frac{z_2 - m}{\epsilon}\right][u_2(x_2) - z_2] & \text{if } m \in (z_2 - \epsilon, z_2), \end{cases}$$

and let $o_2(m)$ solve

$$\begin{aligned} &\max_{(x,y)\in\mathbb{R}_+\times[0,m]} -c_2(x) + y \\ &\text{s.t. } u_2(x) - y \geq \xi(m). \end{aligned} \tag{27}$$

Now we define $o_1(m, r, d)$. Note that d can only take two values: $d = D$ when $r = G$ and $d = 0$ when $r = B$. Let $\eta(m, G) = u_1(x_1) - (z_1 - k) + \sigma_2\xi(m)$ and let $\eta(m, B) = \sigma_2\xi(m)$. η is continuous as ξ is. When $r = G$, for each $m \in \mathbb{R}_+$, let $o_1(m, G, D)$ be a solution to

$$\begin{aligned} &\max_{(x,y_c,y_m)\in\mathbb{R}_+\times[0,D+k]\times[0,m]} -c_1(x) + y_c + y_m \\ &\text{s.t. } u_1(x) - \max(y_c - k, 0) - y_m + \sigma_2\xi(m - y_m) \geq \eta(m, G). \end{aligned} \tag{28}$$

When $r = B$, for each $m \in \mathbb{R}_+$, let $o_1(m, B, 0)$ be a solution to

$$\begin{aligned} &\max_{(x,y_m)\in\mathbb{R}_+\times[0,m]} -c_1(x) + y_m \\ &\text{s.t. } u_1(x) - y_m + \sigma_2\xi(m - y_m) \geq \eta(m, B). \end{aligned} \tag{29}$$

Following exactly the same argument as in the main text, when $m = z_2$, $(x_1, z_1, 0)$ is a solution to (28).

Now we show that the following strategies form a simple equilibrium. All agents respond with *yes* to the proposed trades and buyers offer the proposed trades, on both equilibrium and off-equilibrium paths. Buyers under state G always repay their debts up to D, and buyers under state B never repay anything. All buyers leave the CM with z_2 units of real balances.

By construction, the outcome functions $o_1(m, r, d)$ and $o_2(m)$ ensure that buyers and sellers always prefer to say *yes*, buyers are willing to offer the proposed trade, and buyers always announce their money holdings truthfully. Implicitly in these arguments is the standard result that the CM continuation value with m units of real balances and y_c units of debt is linear:

$$W_G(m, y_c) = m - \min\{D, \max\{y_c - k, 0\}\} + W_G(0, 0).$$

Moreover, the DM value functions are given by

$$V_G(m) = \eta(m, G) + m + W_G(0, 0),\ V_B(m) = \eta(m, B) + m + W_B(0),$$

and hence

$$W_G(0, z_1) = -[(z_1 - k) + (1 + \pi)z_2] + \frac{\delta}{1 - \delta}\{\sigma_1[u_1(x_1) - (z_1 - k)] + \sigma_2[u_2(x_2) - z_2] - \pi z_2\},$$

and

$$W_B(0) = -(1 + \pi)z_2 + \frac{\delta}{1 - \delta}\{\sigma_2[u_2(x_2) - z_2] - \pi z_2\}.$$

Since, by our construction of o_1, the amount of money carried into round-1 DM does not affect the buyer's surplus, to show that buyers are willing to leave the CM with z_2 units of real balances, it is sufficient to show that it is better than 0. The other cases are taken care of by the same arguments as in the main text. This will be the case if and only if

$$-[\rho + (1 + \rho)\pi]z_2 + \sigma_2[u_2(x_2) - z_2] \geq 0.$$

Using $\pi z_2 = \sigma_1 k = \sigma_1[z_1 - \frac{\sigma_1 u_1(x_1)}{\sigma_1 + \rho}]$ by (25), we can rewrite this inequality as

$$-\rho z_2 + \sigma_2[u_2(x_2) - z_2] \geq (1 + \rho)\pi z_2 = (1 + \rho)\sigma_1 k,$$

which is equivalent to (19). Note that this also proves the necessity of (19).

Finally, we show that a buyer under state G has incentive to repay $D = z_1 - k$ whenever his debt is at least z_1 (where the EMP pays k for him). Because the buyer's payoff are affected by his record only through η and the buyer holds z_2 units of real balances when leaving the CM regardless of his records, he has incentive to repay D if and only if

$$-(z_1 - k) + \frac{\delta}{1-\delta}\eta(z_2, G) \geq \frac{\delta}{1-\delta}\eta(z_2, B),$$

which is equivalent to

$$-\rho(z_1 - k) + \sigma_1[u_1(x_1) - (z_1 - k)] \geq 0.$$

This holds by (25). Note that (25) is necessary as we cannot improve upon the EMP constructed.

(ii) The necessity of (11) and (13) is clear. To void buyers from skipping round-1 DM trade and holding cash, (22) is necessary. (23) is necessary because of pure-credit-proposal: buyers with different money holdings obtain the same surplus at round-2 DM, which is monitored. The necessity of (21) can be proved with the same outline as that in (i).

Now we turn to sufficiency. If $-rz_2^1 + \sigma_2[u_2(x_2^1) - z_2^1] \geq 0$, then $\mathcal{L}$ with a constant money supply. The proposals (o_1, o_2) to make the mechanism satisfy the pairwise core requirement will use the same arguments as the other case. So suppose that $-\rho z_2^1 + \sigma_2[u_2(x_2^1) - z_2^1] < 0$, and, by (21), $-\rho z_1 + \sigma_1[u_1(x_1) - z_1] > 0$.

The EMP is defined as follows. Let

$$k = \frac{1}{\sigma_2 + \rho}\{\rho z_2^1 - \sigma_2[u_2(x_2^1) - z_2^1]\} = z_2^1 - \frac{\sigma_2}{\sigma_2 + \rho}u_2(x_2^1) \in (0, z_2^1), \tag{30}$$

and let $\pi = \sigma_2 k / z_1$. Consider the following proposal: $\phi_t M_t = z_1$ for each t, $C = \{2\}$, $D = z_2^1 + k$, and o_1, o_2 are given as follows. For any $m \geq 0$, $o_2(m, G, D)$ solves

$$\begin{aligned} &\max_{(x, y_c, y_m)} -c_2(x) + y_m + y_c \\ \text{s.t.} \quad & u_2(x) - y_m - \max\{y_c - k, 0\} \geq u_2(x_2^1) - (z_2^1 - k), \\ & y_c \leq D, y_m \leq m. \end{aligned} \tag{31}$$

For any $m \geq 0$, $o_2(m, B, 0)$ solves

$$\begin{aligned} &\max_{(x, y_m)} -c_2(x) + y_m \\ \text{s.t.} \quad & u_2(x) - y_m \geq 0, \\ & y_m \leq m. \end{aligned} \tag{32}$$

The solutions to (31) and (32) exist and are unique. Moreover, when $m = 0$, $o_2(0, G, D) = (x_2^1, z_2^1, 0)$. For $m > 0$, $o_2(m, G, D) = (x(m), z_2^1, y_m)$ satisfies

$$y_m = \min\{u_2(x_2^*) - u_2(x_2^1) + k, m\}, \; u_2(x(m)) = u_2(x_2^1) - k + y_m.$$

So $z_2^0 = z_2^1 + y_m$ and $x_2^0 = x^*$. It is easy to verify that, by (13) and (23), $o_2(z_1, G, D) = (x_2^0, z_2^1, z_2^0 - z_2^1)$.

Now we define o_2. Suppose that $m \geq z_1$. Then, $o_1(m)$ solves

$$\max_{(x,y),y\leq m} -c_1(x) + y \quad \text{s.t. } u_1(x) - y \geq u_1(x_1) - z_1. \tag{33}$$

Suppose that $m < z_2$. Then, $o_2(m)$ solves

$$\max_{(x,y),y\leq m} -c_1(x) + y \quad \text{s.t. } u_1(x) - y \geq 0. \tag{34}$$

Now we specify the equilibrium strategies. All agents always respond with *yes* to the proposed trades, on both equilibrium and off-equilibrium paths. The buyers always hold and acquire z_1 units of real balances, irrespective of their credit records; they repay their debts when their records are G and their debts are below D, and they never repay anything otherwise.

Given the proposals, the continuation valued are given as follows. Note that we have (23) that links (x_2^1, z_2^1) and (x_2^0, z_2^0).

$$\begin{aligned}
V_G(m) &= \mathbf{1}_{m\geq z_1}\sigma_1[u_1(x_1) - z_1] + m + \sigma_2[u_2(x_2^1) - (z_2^1 - k)] + W_G(0,0),\\
V_B(m) &= \mathbf{1}_{m\geq z_1}\sigma_1[u_1(x_1) - z_1] + m + W_B(0),\\
W_G(0, z_2^1) &= -[(1+\pi)z_1 + (z_2^1 - k)] + \frac{\delta}{1-\delta}\left\{\sigma_1[u_1(x_1) - z_1] + \sigma_2[u_2(x_2^1) - (z_2^1 - k)] - \pi z_1\right\},\\
W_B(0) &= -(1+\pi)z_1 + \frac{\delta}{1-\delta}\left\{\sigma_1[u_1(x_1) - z_1] - \pi z_1\right\}.
\end{aligned}$$

Using similar arguments as in the main text, it is easy to verify that buyers are willing to hold z_1 units of real balances, irrespective of their credit records, if

$$-[\rho + (1+\rho)\pi]z_1 + \sigma_1[u_1(x_1) - z_1] \geq 0,$$

which holds by (30) and (21). Similarly, buyers are willing to repay $z_2^1 - k$ and keep a record G in the CM if

$$-(z_2^1 - k) + \delta V_G(z_1) \geq V_B(z_1),$$

that is, if

$$-(z_2^1 - k) + \frac{\delta}{1-\delta}\sigma_2[u_2(x_2^1) - (z_1^2 - k)] \geq 0,$$

which holds by (30). □

Optimality of EMP

Here we show that, for any $\sigma_1 \in (0,1)$, generically there exist $\bar{\rho}(\sigma_1) > \rho^M(\sigma_1)$ such that for all $\rho \in (\rho_M(\sigma_1), \bar{\rho}(\sigma_1)]$, an active EMP is necessary to implement the constrained efficient allocation and hence a positive inflation rate is optimal.

The mechanism designer's problem is to choose an optimal mechanism, together with an optimal expansionary monetary policy, that maximizes the social welfare, subject to the implementability constraints. For a given allocation, $\mathcal{L} = [(x_1, x_2^0, x_2^1), (z_1, z_2^0, z_2^1)]$, its welfare is given by

$$\mathcal{W}(\mathcal{L}) = \frac{1}{\rho}\left\{\sigma_1[u_1(x_1) - c_1(x_1)] + \sigma_2\sigma_1[u_2(x_2^1) - c_2(x_2^1)] + \sigma_2(1-\sigma_1)[u_2(x_2^0) - c_2(x_2^0)]\right\}. \tag{35}$$

We say that an allocation is *constrained efficient* if it maximizes the social welfare given by (35) among all allocations that are implementable with EMP.

As shown earlier, if $\rho \leq \rho^M(\sigma_1)$, then the first best allocation is implementable in a pure currency economy without interventions. From now on we fix a $\sigma_1 \in (0,1)$ and hence ignore the dependence on it. We also assume that, when $\rho = \rho^M$, either

$$-\rho c_2(x_2^*) + \sigma_2[u_2(x_2^*) - c_2(x_2^*)] > 0, \text{ or} \tag{36}$$

$$-\rho c_1(x_1^*) + \sigma_1[u_1(x_1^*) - c_1(x_1^*)] > 0. \tag{37}$$

By definition of ρ^M, we cannot have both (36) and (37) to hold, but we assume that one of them holds.

To show that an active EMP is generically essential, we being with the case where the first-best is implementable. We have the following corollary.

Corollary 2.2. *Assume limited monitoring ($\ell = 1$).*

(i) Suppose that (36) holds at $\rho = \rho^M$. Then, the first-best allocation, (x_1^*, x_2^*), is implementable with EMP if and only if $\rho \leq \rho^*$, where $\rho^* > \rho^M$ is the unique solution to

$$\frac{(\rho+1)\sigma_1}{\rho+\sigma_1}\{-\rho c_1(x_1^*) + \sigma_1[u_1(x_1^*) - c_1(x_1^*)]\} + \{-\rho c_2(x_2^*) + \sigma_2[u_2(x_2^*) - c_2(x_2^*)]\} = 0. \tag{38}$$

(ii) Suppose that (37) holds at $\rho = \rho^M$. Then, the first-best allocation, (x_1^*, x_2^*), is implementable with EMP if and only if $\rho \leq \rho^*$, where $\rho^* > \rho^M$ is the unique solution to

$$\{-\rho c_1(x_1^*) + \sigma_1[u_1(x_1^*) - c_1(x_1^*)]\} + \frac{(\rho+1)\sigma_2}{\rho+\sigma_2}\{-\rho c_2(x_2^*) + \sigma_2[u_2(x_2^*) - c_2(x_2^*)]\} = 0. \tag{39}$$

Corollary 2.2 follows directly from Theorem 2.1. Indeed, when (36) holds at $\rho = \rho^M$, the left-side of (38) is strictly decreasing in ρ and hence ρ^* is uniquely determined. Moreover, for all $\rho \geq \rho^*$, (36) holds with strictly inequality and hence Theorem 2.1 (i) applies. Similarly, when (37) holds at $\rho = \rho^M$, the left-side of (39) is strictly decreasing in ρ and hence ρ^* is uniquely determined. Moreover, for all $\rho \geq \rho^*$, (37) holds with strictly inequality and hence Theorem 2.1 (ii) applies. Finally, because, by Corollary 2.1, the first-best allocation is implementable with money alone only if $\rho \leq \rho^M$, for all $\rho \in (\rho^M, \rho^*]$, EMP is optimal with a positive inflation rate. Note that we can use exactly the same arguments as in the main text to show that for all such ρ's, without interventions the first-best is not implementable even if debt is used.

Now we show that even for lower discount factors, EMP can still be optimal with a positive inflation rate, at least for a range. First we define two allocations, corresponding to the conditions (19) and (21). Let (x_1^{C1}, x_2^{C1}) be the solution to

$$\max_{(x_1,x_2)} \sigma_1[u_1(x_1) - c_1(x_1)] + \sigma_2[u_2(x_2) - c_2(x_2)] \tag{40}$$
$$\text{s.t.}$$
$$\frac{(\rho+1)\sigma_1}{\rho+\sigma_1}\{-\rho c_1(x_1) + \sigma_1[u_1(x_1) - c_1(x_1)]\} + \{-\rho c_2(x_2) + \sigma_2[u_2(x_2) - c_2(x_2)]\} \geq 0.$$

Let (x_1^{C2}, x_2^{C2}) be the solution to

$$\max_{(x_1,x_2)} \sigma_1[u_1(x_1) - c_1(x_1)] + \sigma_1\sigma_2[u_2(x_2) - c_2(x_2)] + (1-\sigma_1)\sigma_2[u_2(x_2^*) - c_2(x_2^*)] \tag{41}$$
$$\text{s.t.}$$
$$\{-\rho c_1(x_1) + \sigma_1[u_1(x_1) - c_1(x_1)]\} + \frac{(\rho+1)\sigma_2}{\rho+\sigma_2}\{-\rho c_2(x_2) + \sigma_2[u_2(x_2) - c_2(x_2)]\} \geq 0.$$

Theorem 2.2. *Assume limited monitoring ($\ell = 1$).*

(i) Suppose that (36) holds at $\rho = \rho_M$. Then, there exists $\bar{\rho} > \rho^*$ such that for all $\rho \in (\rho_M, \bar{\rho}]$, (x_1^{C1}, x_2^{C1}) is implementable with EMP (as the symmetric allocation) and it has a strictly higher welfare than any allocation implementable with a constant money supply.

(ii) Suppose that (37) holds at $\rho = \rho_M$. Then, there exists $\bar{\rho} > \rho_{EM}$ such that for all $\rho \in (\rho_M, \bar{\rho}]$, (x_1^{C2}, x_2^{C2}) is implementable with EMP (with $x_2^1 = x_2^{C2}$ and with $x_2^0 \in [x_2^{C2}, x_2^*]$ and z_2^0 satisfying (23)) and it has a strictly higher welfare than any allocation implementable with a constant money supply.

Proof. First we note that because of concavity and by the Theorem of Maximum, the allocations (x_1^{Ci}, x_2^{Ci}) are continuous in ρ. For each ρ, define $\mathcal{W}^i(\rho)$ as the welfare for (x_1^{Ci}, x_2^{Ci}) and let $\mathcal{W}^M(\rho)$ be the welfare of the best allocation that satisfies the necessary conditions in Lemma 2.2 (or, if the optimum does not exist, the supremum welfare). Note that $\mathcal{W}^M(\rho)$ is an upper bound on the social welfare subject to implementation with money alone. Note that all these functions are weakly decreasing and the first two are continuous.

(i) By Corollary 2.1 and Corollary 2.2 (i), $\mathcal{W}^M(\rho^*) < \mathcal{W}^1(\rho^*)$. Thus, there exists $\hat{\rho} > \rho^*$ such that for all $\rho \in (\rho_M, \hat{\rho}]$, $\mathcal{W}^M(\rho) < \mathcal{W}^1(\rho)$.

Now we show that (x_1^{C1}, x_2^{C1}) is implementable for a neighborhood around ρ^*. Note that, since (36) holds at $\rho = \rho^M$ and since $(x_1^{C1}, x_2^{C1}) = (x_1^*, x_2^*)$ at $\rho = \rho^*$, (36) holds at $\rho = \rho^*$ as well. Thus, by continuity,

$$-\rho c_2(x_2^{C1}) + \sigma_2[u_2(x_2^{C1}) - c_2(x_2^{C1})] > 0$$

around a neighborhood around ρ^*. Thus, (20) holds for the allocation associated with (x_1^{C1}, x_2^{C1}) under such ρ's. Finally, as (14) holds for $(x_1^{C1}, x_2^{C1}) = (x_1^*, x_2^*)$ at $\rho = \rho^*$, by continuity it holds for a neighborhood as well. Thus, there exists some $\bar{\rho} \in (\rho^*, \hat{\rho}]$ such that for all $\rho \leq \bar{\rho}$, (x_1^{C1}, x_2^{C1}) is implementable with EMP under $C = \{1\}$ by Theorem 2.1 (i).

(ii) By Corollary 2.1 and Corollary 2.2 (ii), $\mathcal{W}^M(\rho^*) < \mathcal{W}^2(\rho^*)$. Thus, there exists $\hat{\rho} > \rho^*$ such that for all $\rho \in (\rho^M, \hat{\rho}]$, $\mathcal{W}^M(\rho) < \mathcal{W}^2(\rho)$.

Now we show that (x_1^{C2}, x_2^{C2}) is implementable for a neighborhood around ρ^*, and here we take $x_2^0 = x_2^*$ with $z_2^0 = u_2(x_2^*) - [u_2(x_2^{C2}) - c_2(x_2^{C2})]$. Note that $z_2^0 = c_2(x_2^*) < c_1(x_1^{C2}) + c_2(x_2^{C2})$ at $\rho = \rho^*$ and hence it holds in a neighborhood of ρ^*. Moreover, note that (37) holds for (x_1^{C2}, x_2^{C2}) with strict inequality at $\rho = \rho^*$ and hence it holds for a neighborhood around ρ^*. Thus, (22) holds for the allocation associated with (x_1^{C2}, x_2^{C2}) under such ρ's. Finally, (13) and (23) hold by construction. Thus, there exists some $\bar{\rho} \in (\rho^M, \hat{\rho}]$ such that for all $\rho \leq \bar{\rho}$, (x_1^{C1}, x_2^{C1}) is implementable with EMP under $C = \{2\}$ by Theorem 2.1 (ii). □

Theorem 2.2 shows that the EMP is generically optimal for a wide range of discount rates, not just the ones that allow the first-best allocation. However, it may not apply to really low discount rates. Nevertheless, for such discount rates, as we have proved in the main text, the EMP policy is optimal with a positive inflation rate and hence the welfare under EMP is strictly higher than constant money supply when $\sigma_1 = 1$, at least generically. By a similar continuity argument used in this section we can show the essentiality of EMP for any discount rate, but for sufficiently high σ_1's.

3 Multiple DM rounds

Here we consider our model with I rounds of DM. The environment is exactly the same except for this change, and $\sigma_i \in [0,1]$ is the probability of a successful meeting for round i. For each round $i = 1, ..., I$, the utility function for buyer is $u_i(x_i)$ while the cost function for the seller is $c_i(x_i)$. The first best allocation is given by $(x_1^*, ..., x_I^*)$, where $u_i'(x_i^*) = c_i'(x_i^*)$ for $i = 1, ..., I$, with the associated welfare denoted by $\mathcal{W}^*$. We also use u_i^* to denote $u_i(x_i^*)$ and c_i^* to denote $c_i(x_i^*)$. We still use ℓ to denote the number of rounds for which monitoring is available.

Here we focus on the case where the first best allocation is implementable under $\ell = I - 1$. Because we are interested in the case where there is matching uncertainty, considering the case where the first best is not implementable would require us to consider asymmetric allocations. However, our results here will survive for discount factors that are close to the threshold for which the first best is just implementable, using arguments from the previous section. For $\ell < I - 1$ we can obtain qualitatively similar results, but we need to consider many cases and the details become rather cumbersome.

First consider a pure currency economy. We have the following lemma regarding implementation of the first best.

Lemma 3.1 (Implementation under no monitoring)**.** *Consider a pure currency economy with $\ell = 0$. Then, the first best allocation is implementable with a constant money supply if and only if*

$$\sum_{i=1}^{\nu} [-\rho c_i^* + \sigma_i(u_i^* - c_i^*)] \geq 0 \text{ for all } \nu = 1, 2, ..., I. \tag{42}$$

Proof. (Sketch.) We only prove necessity. Sufficiency can be proved using arguments similar to those in the main text. Note that for the first-best allocation, the pairwise core requirement is never binding.

Let z be the equilibrium real balances buyers have to hold when leaving the CM. To respect seller participation constraint, it must be the case that

$$z \geq \sum_{i=1}^{I} c_i^*.$$

Let $z_i \geq c_i^*$ be the CM consumption for round-i sellers. For any $\nu \leq I - 1$, the buyer can choose to hold only $z - \sum_{i=1}^{\nu} z_i$ units of real balances when leaving the CM and to participate the last $I - \nu$ DM round trades. To ensure that this deviation is not profitable, it must be the case that

$$-z+\delta\left\{\sum_{i=1}^{I}\sigma_i[u_i^*-z_i]+z\right\}\geq-\left(z-\sum_{i=1}^{\nu}z_i\right)+\delta\left\{\sum_{i=\nu+1}^{I}\sigma_i[u_i^*-z_i]+\left(z-\sum_{i=1}^{\nu}z_i\right)\right\},$$

that is,

$$-\rho\sum_{i=1}^{\nu}z_i+\sum_{i=1}^{\nu}\sigma_i[u_i^*-z_i]\geq 0.$$

The inequality (42) immediately follows by noting that $z_i \geq c_i^*$ for all i. □

With $I \geq 3$, a full characterization of implementation of the first best allocation under limited monitoring ($1 \leq \ell \leq I-1$) with EMP is difficult. However, the unlimited monitoring case is easy and it gives an upper bound on those other cases. We omit its proof as it uses exactly the same arguments as those in the main text.

Lemma 3.2 (Implementation under monitoring)**.** *Consider an economy with $\ell \leq I$. Then, the first best allocation is implementable only if*

$$\sum_{i=1}^{I}[-\rho c_i^*+\sigma_i(u_i^*-c_i^*)]\geq 0. \tag{43}$$

Now we move to implementation with EMP. Consider a proposal with $C \subset \{1, ..., I\}$ and $|C| \leq \ell = I-1$, that is, round-i DM has monitored meetings for each $i \in C$. The EMP sets a maximum amount k_i of private debt that the mechanism will redeem from round i trades for each $i \in C$. That is, for any recorded promise at period t and each round $i \in C$, $(b, s_i, z_{i,c}^b)$, the mechanism will print new money and use it to purchase $\min\{k_i, z_{i,c}^b\}$ of the debt from the seller s_i. Feasibility then implies that the inflation rate π must satisfy

$$\sum_{i\in C}\int_{b\in\mathbb{B}}\min\{z_{i,c}^b,k_i\}db=\pi\phi_t M_{t-1}, \tag{44}$$

where M_{t-1} is the amount of money in the economy in period t, right before the monetary intervention.

Here we show that, generically, there is a range of ρ's for which the EMP is necessary to implement the first best allocation. By continuity, we can also show that for a range of lower ρ's EMP is also essential. Let ρ_M be the highest ρ that satisfies (42) and let ρ_C be the highest ρ that satisfies (43). To define the genericity condition, and let

$$\rho_i=\frac{\sigma_i(u_i^*-c_i^*)}{c_i^*}, \tag{45}$$

and let $i^* = \arg\max_{i=1,...,I}\rho_i$. Then, $\rho_M \leq \rho_C \leq \rho_{i^*}$, and generically, $\rho_C < \rho_{i^*}$. Here is the theorem.

Theorem 3.1. *Suppose that $\rho_C < \rho_{i^*}$ and that $\sigma_i < 1$ for all $i = 1, ..., I$. Consider the economy with I rounds of DM and with $\ell = I - 1$. Then, there exist $\bar{\rho} > \rho_C$ and $\hat{\rho}$ such that $\rho_M \leq \hat{\rho} < \bar{\rho}$ and that the first best allocation is implementable with EMP for all $\rho \leq \bar{\rho}$ and is not implementable without EMP for all $\rho > \hat{\rho}$.*

Proof. Let $C = \{1, ..., I\} - \{i^*\}$. Let $\tilde{\rho}$ be the highest ρ such that

$$\sum_{i \neq i^*} [-\rho c_i^* + \sigma_i(u_i^* - c_i^*)] \geq 0. \tag{46}$$

Since $\rho_C < \rho_{i^*}$, $\tilde{\rho} < \rho_C < \rho_{i^*}$.

First we show that for all $\rho \leq \tilde{\rho}$, the first best allocation is implementable with a constant money supply. After this we construct $\bar{\rho}$.

(a) Suppose that $\rho \leq \tilde{\rho} < \rho_{i^*}$. Here we only need a constant money supply. Let $C = \{1, ..., I\} - \{i_{i^*}\}$, let $D = \sum_{i \neq i^*} c_i^*$, and let the proposed real balance be $c_{i^*}^*$. For each $i \in C$, let

$$o_i(m, G, d) = (x_i^*, c_i^*, 0) \text{ if } d \geq c_i^* \text{ and } o_i(m, G, d) = (c_i^{-1}(d + m), d + m) \text{ otherwise;}$$

let

$$o_i(m, B, 0) = \arg \max_{(x,y), y \leq m} \{-c_i(x) + y : u_i(x) - y = 0\}.$$

Stage i^* has monetary trades and

$$o_{i^*}(m) = (x_{i^*}^*, c_{i^*}^*) \text{ if } m \geq c_{i^*}^* \text{ and } o_{i^*}(m) = \arg \max_{(x,y), y \leq m} \{-c_{i^*}(x) + y : u_{i^*}(x) - y = 0\}.$$

It is straightforward to check that the sellers are willing to respond with yes for all proposed trades. Because in all monitored meetings, buyers receive full surpluses, they have no incentive to give offers other than the proposed ones. Because in round i^*, only money is acceptable to sellers, buyers cannot propose other trades that dominate the proposed ones. In the CM, buyers with record G are willing to repay their debt up to D if and only if (46) holds, i.e., if and only if $\rho \leq \tilde{\rho}$. Similarly, as carrying real balances exceeding $c_{i^*}^*$ does not increase a buyer's trade surplus in the DM's, and carrying less than that results in zero surplus in round i^* trade and no change in surplus in other rounds, a buyer's optimal real balance holding leaving CM is $c_{i^*}^*$ if and only if

$$-\rho c_{i^*}^* + \sigma_{i^*}[u_{i^*}^* - c_{i^*}^*] \geq 0,$$

i.e., $\rho \leq \rho_{i^*}$.

(b) Suppose that $\tilde{\rho} < \rho$. To define $\bar{\rho}$, first we need some notation. For all $\rho \in [\tilde{\rho}, \bar{\rho}]$, define $\lambda(\rho)$ as the unique $\lambda \in (0,1)$ such that

$$\sum_{i \neq i^*} [-\rho c_i^* + \sigma_i (u_i^* - c_i^*)] + \lambda \left[\sum_{i \neq i^*} (\rho + \sigma_i) c_i^* \right] = 0. \tag{47}$$

Note that $\lambda(\rho)$ is continuous in ρ and $\lambda(\tilde{\rho}) = 0$. Later, this function $\lambda(\rho)$ will be used to construct the EMP as follows. Let $k_i = \lambda(\rho) c_i^*$. For inflation, let

$$\pi(\rho) = \frac{\sum_{i \neq i^*} \sigma_i k_i}{c_{i^*}^*} = \frac{\sum_{i \neq i^*} \sigma_i \lambda(\rho) c_i^*}{c_{i^*}^*}.$$

Now we construct $\bar{\rho}$. First we show that, at $\rho = \rho_C$,

$$-\rho c_{i^*} + \sigma_{i^*} [u_{i^*}^* - c_{i^*}^*] > (1 + \rho) \pi(\rho) c_{i^*}^*. \tag{48}$$

To see this, since (43) holds at $\rho = \rho_C$,

$$\begin{aligned} -\rho c_{i^*} + \sigma_{i^*} [u_{i^*}^* - c_{i^*}^*] &\geq -\sum_{i \neq i^*} [-\rho c_i^* + \sigma_i (u_i^* - c_i^*)] \\ &= \lambda \left[\sum_{i \neq i^*} (\rho + \sigma_i) c_i^* \right] \\ &> \lambda \left[\sum_{i \neq i^*} (\rho + 1) \sigma_i c_i^* \right] \\ &= (1 + \rho) \pi(\rho) c_{i^*}^*. \end{aligned}$$

Thus, for some $\bar{\rho} > \rho_C$, (48) holds (with weak inequality) for all $\rho \in [\rho_C, \bar{\rho}]$.

Now we show that for all $\rho \in (\tilde{\rho}, \bar{\rho}]$, EMP can implement the first best allocation. First we define the proposal and the EMP. Let $C = \{1, ..., I\} - \{i_{i^*}\}$, let $D = \sum_{i \neq i^*} [1 - \lambda(\rho)] c_i^*$, and let the proposed real balance be $c_{i^*}^*$. Set $k_i = \lambda(\rho) c_i^*$ for each $i \in C$, and set inflation rate to be $\pi(\rho)$.

For each $i \in C$, let

$o_i(m, G, d) = (x_i^*, c_i^*, 0)$ if $d \geq c_i^* - k_i$ and $o_i(m, G, d) = (c_i^{-1}(d + m + k_i), d + m + k_i)$ otherwise;

let

$$o_i(m, B, 0) = \arg \max_{(x,y), y \leq m} \{-c_i(x) + y : u_i(x) - y = 0\}.$$

Stage i^* has monetary trades and

$$o_{i^*}(m) = (x_{i^*}^*, c_{i^*}^*) \text{ if } m \geq c_{i^*}^* \text{ and } o_{i^*}(m) = \arg \max_{(x,y), y \leq m} \{-c_{i^*}(x) + y : u_{i^*}(x) - y = 0\}.$$

It is straightforward to check that the sellers are willing to respond with yes for all proposed trades. Because in all monitored meetings, buyers receive full surpluses, they have no incentive to give offers other than the proposed ones. Because in round i^*, only money is acceptable to sellers, buyers cannot propose other trades that dominate the proposed ones. In the CM, buyers with record G are willing to repay their debt up to D if and only if

$$-\rho \sum_{i \neq i^*} (c_i^* - k_i) + \sum_{i \neq i^*} \sigma_i [u_i^* - c_i^* + k_i] \geq 0,$$

which holds with equality by the definition of $\lambda(\rho)$. Similarly, as carrying real balances exceeding $c_{i^*}^*$ does not increase a buyer's trade surplus in the DM's, and carrying less than that results in zero surplus in round i^* trade and no change in surplus in other rounds, a buyer's optimal real balance holding leaving CM is $c_{i^*}^*$ if and only if (48) holds with weak inequality, which is the case for all $\rho \leq \bar{\rho}$. □

Theorem 3.1 shows that, for any finite number of DM rounds, there is always a range of discount rates for which EMP is necessary to achieve the first best allocation, i.e., for $\rho \in (\hat{\rho}, \bar{\rho}]$. Although this theorem is only for first best allocations, we strongly conjecture that the essentiality of EMP holds for a range of lower discount rates as well, at least generically. These results demonstrate that results in our two-round DM model are robust to many DM rounds.

However, unlike the model with two rounds of DM, it can be the case that the first best is achievable with limited monitoring without EMP but not achievable in a pure currency economy, i.e., for $\rho \in (\rho_M, \hat{\rho}]$. In general $\hat{\rho}$ may not be strictly larger than ρ_M, but here we provide some sufficient conditions for the case $I = 3$ when this actually happens.

Proposition 3.1. *Consider an economy with three DM rounds, i.e., $I = 3$, and with $\ell = 2$. Suppose that at $\rho = \rho_M$,*

$$-\rho c_1^* + \sigma_1 [u_1^* - c_1^*] = 0; \tag{49}$$

$$-\rho (c_1^* + c_2^*) + \sigma_1 [u_1^* - c_1^*] + \sigma_2 [u_2^* - c_2^*] > 0; \tag{50}$$

$$-\rho c_3 + \sigma_3 [u_3^* - c_3^*] > 0. \tag{51}$$

Then, there exists $\hat{\rho} > \rho_M$ such that for all $\rho \in (\rho_M, \hat{\rho}]$, the first best is implementable with $C = \{1, 2\}$ and a constant money supply but not implementable under no monitoring and a constant money supply.

Proof. (Sketch.) Let $\hat{\rho}$ be the largest ρ for which both (50)-(51) hold with weak inequalities. Then, $\hat{\rho} > \rho_M$. Consider any $\rho \in (\rho_M, \hat{\rho}]$. Then the first best is not implementable under no monitoring as (49) fails. However, it can be implemented under limited monitoring with $C = \{1, 2\}$. Take $D = c_1^* + c_2^*$ and take real balances to be $z = c_3^*$. Then, (50) ensures that buyers are willing to repay their debts, and, using similar arguments to those in HKW, (51) ensures that buyers are willing to hold z units of real balances. $\square$

4 Alternative meeting patterns

Here we consider a different version of our model as follows. It still has an infinite horizon and agents are still divided into buyers and sellers. The set of buyers is denoted $\mathbb{B}$ and the set of sellers is partitioned into two subsets, $\mathbb{S}_1$ and $\mathbb{S}_2$ both with measure one. The two sets $\mathbb{S}_1$ and $\mathbb{S}_2$ may represent two sectors of the economy.

Each period is divided into three stages with two DM rounds and one CM round. However, the meeting pattern differs from our previous model. In round-1 DM, a buyer may meet a seller from $\mathbb{S}_1$ or $\mathbb{S}_2$ or none. The probability of a successful meeting is $2\sigma_1 \leq 1$ and the probability of meeting a seller from $\mathbb{S}_j$ is σ_1 for both $j = 1, 2$. For simplicity we assume that only buyers with a successful meeting at round-1 has a chance to meet a seller from a different sector at round-2, which happens with probability $\sigma_2/\sigma_1 \leq 1$.[1] We also assume that a seller may meet at most one buyer at each period. Note that the ex ante probability of a buyer to meet a sector-j seller is $\sigma = \sigma_1 + \sigma_2$ for both $j = 1, 2$.

The utility and cost functions for meeting sellers from $\mathbb{S}_i$ remain the same. The record-keeping technology works in the same way as before. In particular, when the technology is only available to sector j, then any meeting with a seller from $\mathbb{S}_j$ has access to the technology, regardless of the timing of the meeting.

For implementation, a proposal is still given by

$$\mathcal{P} = \left[C, D, \left(o_1^j, o_2^j\right)_{j=1,2}, (\phi, \mu)\right], \tag{52}$$

but the proposed trade $\left(o_i^j\right)_{i,j=1,2}$ depends both on the stage i and the sector j.

Because of the symmetry assumed in the meeting patterns, we focus only on symmetric allocations, denoted by

$$\mathcal{L} = [(x_1, x_2), (z_1, z_2)],$$

[1]This assumption allows us to focus on symmetric allocations across different meeting patterns for different buyers, and it plays a very similar role as $\sigma_1 = 1$ in previous sections. Results in this section are robust to this assumption in the same way as results in previous sections are robust to the assumption $\sigma_1 = 1$.

where x_j is the sector-j DM consumption of a buyer with a successful meeting with a sector j seller in either DM round, and z_j is the CM consumption of a sector-j seller who had a successful meeting with a buyer in either DM round. Note that the first-best allocation is symmetric.

Lemma 4.1 below characterizes the set of implementable allocations for the two extreme cases: $\ell = 2$ and $\ell = 0$, without introducing any monetary policy.

Lemma 4.1 (Implementability without policy). *(i) Assume unlimited monitoring ($\ell = 2$) and no money. An allocation $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$ is implementable if and only if*

$$-\rho(z_1 + z_2) + \sigma \{[u_1(x_1) - z_1] + [u_2(x_2) - z_2]\} \geq 0, \tag{53}$$

$$z_1 \geq c_1(x_1), \;\; z_2 \geq c_2(x_2), \tag{54}$$

$$u_j(x_j) - c_j(x_j) + (\sigma_2/\sigma_1)[u_{-j}(x_{-j}) - z_{-j}] \geq u_j(\tilde{x}_j) - c_j(\tilde{x}_j), \tag{55}$$

for both $j = 1, 2$, where $\tilde{x}_j = \min\{x_j^*, c_j^{-1}(z_1 + z_2)\}$.

(ii) Assume no monitoring ($\ell = 0$) and a constant money supply. An allocation $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$ with $z_j \geq z_{-j}$ is implementable only if (53), (54), (55), and

$$-\rho z_j + \sigma[u_j(x_j) - z_j)] + \sigma_1[u_{-j}(x_{-j}) - z_{-j}] \;\; \geq \;\; 0, \tag{56}$$

$$-\rho z_{-j} + \sigma_1[u_1(x_1) - z_1)] + \sigma_1[u_2(x_2) - z_2)] \;\; \geq \;\; 0. \tag{57}$$

Proof. We only prove necessity here; sufficiency can be proved with similar arguments as in the main text.

(i) Condition (53) is necessary to ensure that the buyers are willing to participate the whole scheme, and (54) ensures that for sellers. Condition (55) is necessary to guarantee no better offers to make at round-1 DM, and is a pairwise core requirement.

(ii) The two additional conditions come from the following two deviations. We may assume without loss of generality that the equilibrium amount of real balance is $z = z_1 + z_2$. When leaving CM, the buyer may hold only z_1 units of real balance, and, by doing so, he can still participate round-2 DM trades for both sectors since $z_1 \geq z_2$. Thus, unless (57) holds, this would be profitable deviation. Similarly, when leaving CM, the buyer may hold only z_2 units of real balance, and, by doing so, he can still participate round-2 DM trades for sector-2. Thus, unless (56) holds, this would be profitable deviation. □

To characterize the set of implementable allocations under limited monitoring ($\ell = 1$) without interventions is rather difficult, but we remark that any allocation that is implementable under limited monitoring and with a constant money supply is also implementable under unlimited monitoring ($\ell = 2$).

Now we turn to implementation under limited monitoring ($\ell = 1$) and with expansionary monetary policy. Consider a mechanism with $C = \{j, \}$, i.e, sector j has monitored meetings. In such meetings, buyers may issue private debts to sector-j sellers. The EMP sets a maximum amount k of debts (in terms of the CM good) that the policy will redeem using newly created money. More precisely, for any recorded promise at period t, (b, s_j, z_c), the policy will redeem $\min\{k, z_c\}$. Let π be the net money growth rate. Thus, for each t, $M_{t+1} = (1+\pi)M_t$ and we focus only on proposals with constant real balances, that is, $\phi_{t+1}M_{t+1} = \phi_t M_t$. Then, if, for each buyer b, z_c^b is the amount of debt that b has with a sector-j seller (which, obviously, would be zero if the buyer did not meet any sector-j seller), feasibility requires a corresponding inflation rate π such that

$$\int_{b\in\mathbb{B}} \min\{z_c^b, k\}db = \pi\phi_t M_{t-1}. \tag{58}$$

The following theorem characterizes implementable allocations with EMP. As in Section 2, we only consider pure-credit proposals.

Theorem 4.1 (Expansionary Monetary Policies). *Assume limited monitoring ($\ell = 1$). An allocation, $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$, is implementable (using pure-credit proposals) with EMP if and only if (54) and (55) hold, and, for some $j = 1, 2$,*

$$\frac{(\rho+1)\sigma}{\rho+\sigma}\{-\rho z_{-j} + \sigma[u_{-j}(x_{-j}) - z_{-j}]\} + \{-\rho z_j + \sigma[u_j(x_j) - z_j]\} \geq 0, \tag{59}$$

$$-\rho z_j + \sigma[u_j(x_j) - z_j] \geq 0. \tag{60}$$

Proof. (Sketch.) We prove sufficiency. Suppose that (59) and (60) hold for $j = 2$. The only relevant case is where $-\rho z_1 + \sigma[u_1(x_1) - z_1] < 0$, and $-\rho z_2 + \sigma[u_2(x_2) - z_2] > 0$.

The EMP is defined as follows. Let

$$k = \frac{1}{\sigma+\rho}\{\rho z_1 - \sigma[u_1(x_1) - z_1]\} = z_1 - \frac{\sigma}{\sigma+\rho}u_1(x_1) \in (0, z_1), \tag{61}$$

and let $\pi = \sigma k/z_2$. Consider the following proposal: $\phi_t M_t = z_2$ for each t, $C = \{1\}$, $D = z_1 - k$, and o_i^j, $i, j = 1, 2$, are given as follows. Note that because when buyers meet a sector 1 seller (a credit meeting), his available debt limit is either D or 0, on any equilibrium and off-equilibrium paths. Thus, we formulate the proposals o_i^1 without d as an argument (implicitly, we assume $d = D$ if $r = G$ and $d = 0$ otherwise).

(1) First we define the proposals for round-2 DM's, o_2^1 and o_2^2.

(1.a) Here we define o_2^1, the proposal for the round-2 credit meeting. Let

$$\xi_G^1 = u_1(x_1) - z_1 + k \text{ and } \xi_B^1 = 0.$$

Suppose that $r = G$. Then, $o_2^1(m, G)$ solves

$$\max_{(x,y_c,y_m)} -c_1(x) + y_m + y_c \tag{62}$$
$$\text{s.t.} \quad u_1(x) - y_c - y_m + k \geq \xi_G^1,$$
$$k \leq y_c \leq D + k = z_1, y_m \leq m.$$

Suppose that $r = B$. Then, $o_2^1(m, B)$ solves

$$\max_{(x,y_m)} -c_1(x) + y_m \tag{63}$$
$$\text{s.t.} \quad u_1(x) - y_m \geq \xi_B^1,$$
$$y_m \leq m.$$

(1.b) Here we define o_2^2, a non-credit meeting. Let $\epsilon \in (0, z_2)$ be so small that

$$\epsilon < \frac{1}{2} \min \left\{ \frac{c_1'(x_1)(\sigma_2/\sigma_1)[u_2(x_2) - z_2]}{u_1'(x_1) - c_1'(x_1)}, \frac{\sigma_2[u_2(x_2) - z_2]}{\rho + (1+\rho)\pi} \right\}. \tag{64}$$

Let

$$\xi^2(m) = \begin{cases} u_2(x_2) - z_2 & \text{if } m \geq z_2; \\ 0 & \text{if } m \leq z_2 - \epsilon; \\ \left[1 - \frac{z_2 - m}{\epsilon}\right][u_2(x_2) - z_2] & \text{if } m \in (z_2 - \epsilon, z_2). \end{cases}$$

Then, $o_2^2(m)$ solves

$$\max_{(x,y), y \leq m} -c_2(x) + y \tag{65}$$
$$\text{s.t.} \ u_2(x) - y \geq \xi^2(m).$$

The solutions to (62) and (63) exist, and for all the solutions the constraints on buyer's reservation utilities are binding. Moreover, $o_2^1(0, G) = (x_1, z_1, 0)$. Similarly, the solutions to (65) exist, and for all the solutions the constraints on buyer's reservation utilities are binding with $o_2^2(z_2) = (x_2, z_2)$.

(2) Here we define the proposals for round-1 DM's, o_1^1 and o_1^2.

(2.a) Here we define o_1^1, the proposal for the round-1 credit meeting. Let

$$\eta^1(m, G) = [u_1(x_1) - z_1] + (\sigma_2/\sigma_1)\xi^2(m),$$
$$\eta^1(m, B) = (\sigma_2/\sigma_1)\xi^2(m).$$

Suppose that $r = G$. Then, $o_1^1(m, G)$ solves

$$\max_{(x,y_c,y_m)} -c_1(x) + y_m + y_c \tag{66}$$
$$\text{s.t.} \quad u_1(x) - \max(y_c - k, 0) - y_m + (\sigma_2/\sigma_1)\xi^2(m - y_m) \geq \eta^1(m, G) \tag{67}$$
$$y_c \leq z_1, y_m \leq m.$$

Suppose that $r = B$. Then, $o_1^1(m, B)$ solves

$$\max_{(x,y_m)} -c_1(x) + y_m \tag{68}$$

$$\text{s.t.} \quad u_1(x) + (\sigma_2/\sigma_1)\xi^2(m - y_m) \geq \eta^1(m, B), \tag{69}$$

$$y_m \leq m.$$

Because ξ^2 is continuous, solutions to the above two maximization problems exist. Moreover, at the optimum, the constraints are binding.

Now we show that when $m = z_2$, $(x_1, z_1, 0)$ is a solution to (66). Suppose, by contradiction, (x, y_c, y_m) gives seller a higher surplus without violating the constraint. We may assume that $y_c \geq k$, for otherwise we can increase y_c to give the seller even a higher surplus without changing the buyer's. Hence,

$$\begin{aligned} & u_1(x) - (y_c - k) - y_m + (\sigma_2/\sigma_1)\xi^2(z_2 - y_m) \\ \geq \;& u_1(x_1) - (z_1 - k) + (\sigma_2/\sigma_1)\xi^2(z_2), \; y_m + y_c - c_1(x) > z_1 - c_1(x_1), \end{aligned}$$

and hence

$$u_1(x) - c_1(x) + (\sigma_2/\sigma_1)\xi^2(z_2 - y_m) > u_1(x_1) - c_1(x_1) + (\sigma_2/\sigma_1)[u_2(x_2) - z_2]. \tag{70}$$

Consider two cases.

(a) $y_m \geq \epsilon$. Then, $\xi^2(z_2 - y_m) = 0$ and we can obtain a contradiction to (55).

(b) $y_m \in (0, \epsilon)$. Then,

$$\xi^2(z_2 - y_m) = \left(1 - \frac{y_m}{\epsilon}\right)[u_2(x_2) - z_2].$$

However, because $y_m - c_1(x) > (z_1 - y_c) - c_1(x_1) \geq -c_1(x_1)$,

$$c_1'(x_1)[x - x_1] \leq c_1(x) - c_1(x_1) < y_m.$$

From the above two conditions and the definition of ϵ, (64),

$$\begin{aligned} & [u_1(x) - c_1(x)] - [u_1(x_1) - c_1(x_1)] \leq [u_1'(x_1) - c_1'(x_1)](x - x_1) < \frac{[u_1'(x_1) - c_1'(x_1)]y_m}{c_1'(x_1)} \\ < \;& \frac{y_m}{\epsilon}(\sigma_2/\sigma_1)[u_2(x_2) - z_2] = (\sigma_2/\sigma_1)[\xi^2(z_2) - \xi^2(z_2 - y_m)], \end{aligned}$$

a contradiction to (70). This also implies that the buyer has no profitable deviating offers at round-1 DM.

(2.b) Here we define o_1^2, the proposal for the round-1 non-credit meeting. Suppose that $m \geq z_1$. Let

$$\eta^2(m) = u_2(x_2) - z_2 \text{ if } m \geq z_2 \text{ and } \eta^2(m) = 0 \text{ otherwise.}$$

Then, $o_1^2(m)$ solves

$$\max_{(x,y_m),y_m\leq m} -c_2(x)+y \tag{71}$$
$$\text{s.t.} \quad u_2(x)-y_m \geq \eta^2(m).$$

The solutions to (66)-(67) and (68)-(69) exist, and for all the solutions the constraints on buyer's reservation utilities are binding. When $(m,r)=(z_2,G)$, $o_1(m,r)=(x_1,z_1,0)$. Similarly, the solutions to (71) exist, and for all the solutions the constraints on buyer's reservation utilities are binding with $o_2^2(z_2)=(x_2,z_2)$.

Now we specify the equilibrium strategies. All agents always respond with *yes* to the proposed trades and the buyer always offers the proposed trade at the renegotiation stage, on both equilibrium and off-equilibrium paths. The buyers always hold and acquire z_2 units of real balances; they repay their debts up to D when their records are G, and they never repay anything with record B.

By construction, the outcome functions, $\{o_i^j\}_{i,j=1,2}$, ensure that buyers and sellers always prefer to say *yes*, buyers are willing to offer the proposed trade, and buyers always announce their money holdings truthfully. Moreover, because $\eta^1(m,G)-\eta^2(m,B)=u_1(x_1)-(z_1-k)$ is independent of m and η^2 does not depend on r while $\xi_G^1-\xi_B^1$ doe snot depend on m, we can write the continuation values as follows. Let $V_B^c=0$,

$$V_G^c=\frac{\sigma}{1-\delta}[u_1(x_1)-(z_1-k)],$$

$$V(m)=\sigma_1\eta^2(m)+\sigma_2\xi^2(m)+W(0),$$

and

$$W(0)=-(1+\pi)z_2+\frac{\delta}{1-\delta}\left\{\sigma u_2(x_2)+(1-\sigma)z_2-(1+\pi)z_2\right\}.$$

Then, the continuation value for a buyer entering DM with credit record r and real balances m is $V_r^c+V(m)$. This implies that the choice of real balances in the CM and the repayment decision for the debts are independent from each other.

Here we show that buyers are willing to leave the CM with z_2 units of real balances. Now, a buyer who leaves with m units of real balances has the expected payoff (regardless of the amount of repayment to his debts)

$$\begin{aligned} & -(1+\pi)m+\delta\left[\sigma_1\eta^2(m)+\sigma_2\xi^2(m)+m+W(0)\right] \\ = \; & \delta\left\{-[\rho+(1+\rho)\pi]m+\sigma_1\eta^2(m)+\sigma_2\xi^2(m)+W(0)\right\}. \end{aligned}$$

Because both $\xi^2(m)$ and $\eta^2(m)$ are constant for all $m\geq z_2$ but the cost of holding money increases with m, any $m>z_2$ is strictly dominated by $m=z_2$.

Here we show that for any $\epsilon' \in (0, \epsilon]$, $z_2 - \epsilon'$ is strictly dominated by z_2. This will be the case if

$$-[\rho+(1+\rho)\pi](z_2-\epsilon')+\sigma_1\eta^2(z_2-\epsilon')+\sigma_2\xi^2(z_2-\epsilon') < -[\rho+(1+\rho)\pi]z_2+\sigma_1\eta^2(z_2)+\sigma_2\xi^2(z_2),$$

which is equivalent to

$$[\rho + (1 + \rho)\pi]\epsilon' < \sigma_2[\xi^2(z_2) - \xi^2(z_2 - \epsilon')] + \sigma_1\eta^2(z_2) = \sigma_2\frac{\epsilon'}{\epsilon}[u_2(x_2) - z_2] + \sigma_1\eta^2(z_2),$$

which holds by (64). Moreover, for any $m \leq z_2 - \epsilon$, it is strictly dominated by zero as $\xi^2(m)$ and $\eta^2(m)$ are constant below $z_2 - \epsilon$. Thus, to show that holding z_2 is optimal, it is sufficient to show that it is better than 0, and this will be the case if and only if

$$-[\rho + (1 + \rho)\pi]z_2 + \sigma[u_2(x_2) - z_2] \geq 0.$$

Using $\pi z_2 = \sigma k = \sigma\{z_1 - \frac{\sigma}{\sigma+\rho}u_1(x_1)\}$, we can rewrite this inequality as

$$\{-\rho z_2 + \sigma[u_2(x_2) - z_2]\} + \frac{\sigma(1 + \rho)}{\sigma + \rho}\{-\rho z_1 + [u_1(z_1) - z_1]\} \geq 0,$$

which corresponds to (59) for $j = 2$.

Finally, we show that a buyer with record G and with debt below $D = z_1 - k$ is willing to repay his debt. This is true if and only if

$$-(z_1 - k) + \delta V_G^c \geq \delta V_B^c.$$

Now, because

$$V_G^c - V_B^c = \frac{1}{1 - \delta}\sigma[u_1(x_1) - z_1 + k],$$

by substituting k by (61), the condition becomes

$$-\rho(z_1 - k) + \sigma[u_1(x_1) - z_1 + k] = -\rho z_1 + \sigma[u_1(x_1) - z_1] + (\rho + \sigma)k \geq 0,$$

but by (61),

$$-\rho z_1 + \sigma[u_1(x_1) - z_1] + (\rho + \sigma)k = 0.$$

□

Optimality of EMP

To discuss the optimality of the EMP policy, we shall assume that $\sigma_1 = \sigma_2 = \sigma/2 < 1/2$. This gives us the sharpest result for optimality of EMP, and the other cases can be handled using a similar methodology to the one used in Section 2.

For a given allocation, $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$, its welfare is given by

$$\begin{aligned}\mathcal{W}(\mathcal{L}) &= \sum_{t=0}^{\infty} \delta^t \{\sigma[u_1(x_1) - c_1(x_1)] + \sigma[u_2(x_2) - c_2(x_2)]\} \\ &= \frac{\sigma(1+\rho)}{\rho} \{[u_1(x_1) - c_1(x_1)] + [u_2(x_2) - c_2(x_2)]\}.\end{aligned} \tag{72}$$

We say that an allocation is *constrained efficient* if it maximizes the social welfare given by (72) among all implementable allocations under $\ell = 1$ with EMP. Although our focus is on constrained-efficient allocations under $\ell = 1$, results about constrained-efficient allocations under $\ell = 0$ and 2 will be useful to understand the constrained-efficient allocations under $\ell = 1$. The first best allocation is given by (x_1^*, x_2^*) that satisfies $u_j'(x_j^*) = c_j'(x_j^*)$ for $j = 1, 2,$. Without loss of generality and appealing to genericity, we assume that $c_1(x_1^*) > c_2(x_2^*)$.

We remark here that to maximize the social welfare, it is without loss of generality to have the constraint (54) binding, i.e., to consider only allocations of the form $\mathcal{L} = [(x_1, x_2), (c_1(x_1), c_2(x_2))]$, and, hence, we also say that a pair, (x_1, x_2), is a constrained-efficient allocation if $[(x_1, x_2), (c_1(x_1), c_2(x_2))]$ is a constrained-efficient allocation. Note that this applies to all cases: $\ell = 0, 1, 2$.

In particular, if the first-best allocation is implementable under $\ell = 0$ and a constant money supply, then credit is not essential in the sense that it is not needed to implement the constrained-efficient allocation. For the first-best allocation, conditions in Lemma 4.1 (ii) are also sufficient, and hence, to determine whether a first-best allocation, (x_1^*, x_2^*), is implementable under $\ell = 0$ amounts to check whether the conditions (53) and (57) hold (note that as $\sigma_1 = \sigma_2$, (56) is implied by (53)) under that allocation, and we have the following corollary. Note also that (55) is trivially satisfied for any first-best allocation.

Corollary 4.1. *The first-best allocation, (x_1^*, x_2^*), is implementable under $\ell = 0$ and with a constant money supply if and only if*

$$\begin{aligned}&\rho \leq \rho^M \\ &\equiv \min\left\{\frac{\sigma[u_1(x_1^*) - c_1(x_1^*)] + \sigma[u_2(x_2^*) - c_2(x_2^*)]}{c_1(x_1^*) + c_2(x_2^*)}, \frac{\sigma[u_1(x_1^*) - c_1(x_1^*)] + \sigma/2[u_2(x_2^*) - c_2(x_2^*)]}{c_1(x_1^*)}\right\}.\end{aligned} \tag{73}$$

The first term inside the min operator corresponds to (53) with $((x_1, x_2), (z_1, z_2)) = ((x_1^*, x_2^*), (c_1(x_1^*), c_2(x_2^*)))$, and the second term corresponds to (57). By Corollary 4.1, when $\rho \leq \rho^M$, credit is not essential, and we are only interested in the case where $\rho > \rho^M$. To study the constrained-efficient allocations for $\rho > \rho^M$, it is useful to first consider the constrained-efficient outcomes under $\ell = 2$.

Lemma 4.2. *The allocation (x_1^C, x_2^C) is the constrained-efficient allocation under $\ell = 2$ if and only if it maximizes the social welfare, (72), subject to (53) with $(z_1, z_2) = (c_1(x_1), c_2(x_2))$.*

Proof. The proof follows the same argument as the analogous lemma in the main text. In particular, because $\sigma_1 = \sigma_2$, the pairwise core requirement (55) is identical for $j = 1, 2$ and is not binding at (x_1^C, x_2^C). □

Lemma 4.2 shows that in order to find the constrained-efficient allocation, only the participation constraint (53) is relevant, but the pairwise core requirement (55) is not binding.

As shown below, (x_1^C, x_2^C) is always implementable under $\ell = 1$ with EMP. Thus, whenever such allocation is not implementable under $\ell = 0$, coexistence of money and credit is necessary to implement (x_1^C, x_2^C), and, by Lemma 4.1, EMP is also necessary. Hence, both money and credit are coessential and EMP is essential. Moreover, we will show that even when (x_1^C, x_2^C) *is* implementable under $\ell = 0$, as long as it is not the first-best, EMP can actually implement an even better allocation. The essentiality of credit and hence of EMP, however, would fail when $u_1(x) = u_2(x)$, $c_1(x) = c_2(x)$ for all x, and $\sigma = 1$. However, it will fail only in such knife-edge cases but credit is essential generically. To define the genericity we need some more notation. For any $\rho > 0$, define $(\bar{x}_1, \bar{x}_2)$ as the unique positive solution to

$$u_1(\bar{x}_1) - (1 + \rho)c_1(\bar{x}_1) = 0 = \sigma_2 u_2(\bar{x}_2) - (\sigma_2 + \rho)c_2(\bar{x}_2). \tag{74}$$

Generically, $(\bar{x}_1, \bar{x}_2) \neq (x_1^C, x_2^C)$, as the latter has to satisfy the FOC's implied by the maximization problem as well. We have the following theorem.

Theorem 4.2. *Let $\ell = 1$ and $\sigma_1 = \sigma_2 = \sigma/2 < 1/2$. Suppose that $\rho > \rho^M$ and that $(\bar{x}_1, \bar{x}_2) \neq (x_1^C, x_2^C)$.*

(1) The constrained efficient allocation can be implemented with EMP but not in a pure currency economy with a constant money supply.

(2) There exists $\rho^* \geq \rho^M$ such that the first-best is implementable at ρ^* with EMP and if $\rho > \rho^*$, a constant money supply cannot implement the constrained efficient allocation.

Proof. We separate two cases.

(a) Suppose that at $\rho = \rho^M$, (53) holds with equality. Then, for all $\rho > \rho^M$, $(x_1^C, x_2^C) \neq (x_1^*, x_2^*)$ and (53) is binding at (x_1^C, x_2^C). Using earlier arguments we can show that (x_1^C, x_2^C) satisfies (55) with strict inequality for both $j = 1, 2$.

Since $(\bar{x}_1, \bar{x}_2) \neq (x_1^C, x_2^C)$, for some $j = 1, 2$, both (59) and (60) hold with strict inequality for (x_1^C, x_2^C). Without loss of generality let such j be 1. Then we may increase (x_1^C, x_2^C) slightly but the constraint (59) is still satisfied as $\sigma < 1$, and such allocation is implementable with EMP under $C = \{2\}$. Finally, since any allocation implementable with a constant money supply under $\ell = 1$ has to satisfy (53), an active EMP is necessary to implement the constrained efficient allocation.

(b) Suppose that at $\rho = \rho^M$, (53) holds with strict inequality. Let ρ^* be the largest ρ such that (53) holds for the first best allocation; then, $\rho^* > \rho^M$ and for all $\rho \leq \bar{\rho}$, (x_1^C, x_2^C) is the first best allocation. Since $(\bar{x}_1, \bar{x}_2) \neq (x_1^C, x_2^C)$ at $\bar{\rho}$, for some $j = 1, 2$, both (59) and (60) hold with strict inequality for (x_1^C, x_2^C). Hence, we can still implement the first best for a range of higher ρ's with EMP, say, below $\bar{\rho} > \rho^*$. Moreover, as (53) is binding at (x_1^C, x_2^C) for any $\rho \geq \rho^*$, and since $(\bar{x}_1, \bar{x}_2) \neq (x_1^C, x_2^C)$, we can use similar arguments to (a) to find an even better allocation with EMP. $\square$