Endogenous Intermediation in Over-the-Counter Markets*

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Abstract

We provide a theory of trading through intermediaries in OTC markets. The role of intermediaries is to sustain unsecured trade. When agents trade without collateral, total surplus can increase. In our model, traders are connected through a network. Agents observe their neighbors' actions, and can trade with their counterparty in a given period through a path of intermediaries in the network. If trade is unsecured, agents can renege on their obligations. We show that trading through a network is essential to support unsecured trade, when agents infrequently meet the same counteparty in the market. However, intermediaries must receive fees to have the incentive to implement unsecured trades. While trade without collateral can be sustained in many networks, the efficiency gains are higher in a star network. The center agent in a star can receive higher fees as well. Moreover, concentrated intermediation is a stable structure, when agents incur linking costs.

Keywords: over-the-counter trading; strategic default; dynamic network formation.

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1 Introduction

Many financial transactions take place in the over-the-counter (OTC) markets in a decentralized fashion. One prominent feature of various OTC markets is a concentrated intermediation structure. For instance, Li and Schürhoff (2014) show that a relatively small group of dealers intermediate persistently a majority of the trades in the municipal bond market. Another documented feature is the prevalence of long-lived trading relations. Afonso, Kovner and Schoar (2014) find evidence that participants in the Fed Funds market frequently choose to interact with the same counterparty over time. These findings lead to questions about the role of intermediation and its connection to relationship trading in OTC markets.

This paper proposes a theory of endogenous intermediation in OTC markets. In a market where counterparties meet infrequently, trading through a network of intermediaries allows agents to access more favorable terms of trade than those they would obtain in direct, but one-shot, interactions. In particular, the role of intermediaries in our model is to sustain trade without collateral, by providing monitoring services. While there are positive gains from unsecured trade, intermediaries affect the division of the surplus. That is, intermediaries must be compensated to ensure they have the incentive to implement unsecured trades. The share of the surplus that accrues to intermediaries is endogenously determined by incentive compatibility, and depends on the network structure. We show that concentrated intermediation is both an efficient and stable structure when agents incur linking costs.

We consider a dynamic setting in which agents trade bilaterally. At each date half of the agents have liquidity surpluses and half have investment opportunities. An agent with a liquidity surplus is randomly paired with an agent with an investment opportunity at the beginning of each period. A liquidity agent is endowed with one unit of cash, which can be stored at no cost until the end of the period. An investment agent is endowed with a riskless asset and has access to an investment opportunity, both maturing at the end of the period. The investment opportunity represents a risky asset that yields a high return in the good states of the world and nothing in the bad states of the world. The return in the good states depends on the amount the agent invests in the project. To finance

the investment the agent with an investment opportunity needs to borrow cash from the agent with a liquidity surplus. The debt must be repaid at the end of the period.

In this environment, we consider two frictions. First, we assume that there is limited commitment, and that agents can renege on due payments at the end of the period. The idea is that agents in financial markets can strategically default and benefit from it at the expense of their counterparties. For instance, in the Fed funds market banks can delay the delivery of overnight loaned funds until the afternoon hours, while in the repo markets agents strategically postpone the delivery of the collateral. More generally, agents can use the funds borrowed to engage in excessive risk taking activities that would preclude them from repaying their debts.

Second, we consider that agents have limited access to information about other agents' past behavior. The interpretation is that, while over-the-counter markets are opaque and information about the terms of trade is not public, financial institutions may nevertheless have access to soft information about their long-term trading partners. In particular, we consider that traders are connected through an informational *network* that allows each agent to observe the repayments that his neighbors make.

To counteract the problem of limited commitment, agents can trade against collateral or rely on self-enforcing contracts. Trading against collateral involves an opportunity cost for the agent with an investment opportunity. More generally, there is a welfare loss when trade is secured, as the value of collateral for the borrower is higher than the value of collateral for the lender. For this reason, unsecured trading is desirable. When trade takes place without collateral, repayments may be enforced if agents can be threatened with exclusion from the unsecured market in case they default on their obligations. The information observed through the network allows agents to implement trade without collateral. For this, however, transactions must take place through intermediaries in the network.[1]

We obtain three sets of results. The first set of results highlights the role of intermediaries in implementing better terms of trade. We start by showing that unsecured trade is not sustainable for large economies without a network. At the same time, we show that a

[1] A credit bureau that collects and makes credit records public can make intermediaries redundant. However, there are significant difficulties associated with creating such institution. Typically, financial market participants are reluctant to disclose to regulators not only information about themselves, but also information about their counterparties. Indeed, financial institutions see putting a counterparty into default as a very serious step.

star network (i.e. a network in which one agent intermediates all transactions) can sustain trade without collateral, independently of how large the number of market participants is. In fact, unsecured trade can be sustained in many networks, as long as intermediation chains are not too long. However, trading through a network involves a trade-off. On the one hand, a higher level of investment in the risky project generates a higher expected surplus. On the other hand, it is more difficult to implement. We show that the expected surplus that can be generated when trading takes place without collateral in a star network is higher than in many other networks, as the number of market participants grows large.

We also find that intermediaries must be compensated to ensure they have the incentive to implement unsecured trades. In particular, since intermediaries transfer funds between liquidity and investment agents, they must receive appropriate fees to overcome the temptation to retain the funds for themselves. The fees in our model are endogenously determined by incentive compatibility. The incentive compatibility constraint for agents who use the intermediation service sets an upper bound for the fees intermediaries receives, while the incentive compatibility constraint for intermediaries sets a lower bound. When intermediation is concentrated, this implies that a few dealers receive most of the fees. Moreover, by comparing different network structure we highlight the relative advantage that network positions offer some agents over others. We find that the center agent in a star network can receive a higher fee than any intermediary in other classes of networks we study.

The second set of results focuses on welfare improvements that trading through a network can bring in the presence of linking costs. Maximizing expected welfare involves a trade-off. On the one hand, a higher level of investment increases welfare. On the other hand, in a network that implements a high level of investment there may be higher linking costs as well. We show that the star network is a constrained efficient network when it can sustain a level of investment sufficiently close to the first-best. However, it is more beneficial for agents to trade directly against collateral if linking costs are too high, or if the implementable investment level is too low.

The third set of results concerns network formation and stability, when agents incur linking costs. In particular, we investigate whether agents have an incentive to participate in a network and identify structures that are stable when traders are allowed to change

their links. For this, we first propose a set-up for a dynamic network formation game, and introduce an appropriate stability concept. We show that a star network is stable. However, we find that networks in which there is more than one intermediary, such as an interlinked star, can also be stable. Although in stylized, these results capture the observed features of OTC markets we have described above.

Related Literature

This paper relates to several strands of literature. The more relevant studies are those on intermediation in OTC markets, trading in networks and contract enforcement.

A series of papers, starting with Duffie, Garleanu and Pedersen (2005), has studied trading in over-the-counter markets. While initially these studies have been concerned with explaining asset prices through trading frictions, several recent additions to the literature are interested in the role of intermediaries in OTC markets. Hugonnier, Lester and Weill (2014), Neklyudov(2014) and Chang and Zhang (2015) propose models in which intermediaries facilitate trade between counterparties that otherwise would need to wait a long time to trade. In our model, agents also trade through intermediaries to overcome frictions that arise from search. However, our focus is on informational frictions, as is in Glode and Opp (2015) and Fainmesser (2014). While in the first paper the role of intermediaries is to reduce adverse selection and restore efficient trading, in the second one intermediaries can informally enforce the repayment of loans by borrowers, as in our model. In both studies, however, which agents are intermediaries remains exogenous. In contrast, in the model we provide, certain agents endogenously assume the role of intermediaries to facilitate repeated interactions between traders in the market. Gofman (2014) and Manea (2014) analyze how the presence of intermediaries affects the efficient allocation of assets when agents bargain through intermediaries in a fixed network. We show that intermediation can alleviate inefficiencies in over-the-counter markets. In addition, we allow agents to choose how to form links and analyze which networks are stable.

There is a growing literature studying trading in a network (e.g. Kranton and Minehart (2001), Gale and Kariv (2007), Condorelli and Galeotti (2012), Choi, Galeotti and Goyal (2015), Babus and Kondor (2013), Malamud and Rostek (2014), Nava (2014)). These papers typically model trades that take place either sequentially or in a spot market.

Either way, trading relationships are not considered. In contrast, the role of repeated interactions is at the core of our analysis.

The literature on contract enforcement is substantial. The general aim of this literature is to show that repeated interactions alleviate problems that arise when there is limited enforcement of contracts. Allen and Gale (1999), Kletzer and Wright (2000) and Levin (2003) propose models where contracts are incomplete, either because transaction costs make it too costly to write explicit contracts or simply because the terms of the contract are not verifiable by a third party (i.e. a court). However, when two parties interact repeatedly, they can implement the first-best contract. Shapiro (1984) adds a new angle by assuming that consumers in the market observe with delay the quality of the product being supplied, and hence reputation rewards high quality only with a lag. Other papers depart from the assumption that the same two parties interact with each other, and consider a large population of agents that are matched at random to interact every period. In this case, whether contracts can be enforced or not depends crucially on how much information is available to each agent. Klein (1992) approaches this issue in a model of repeated interaction between businesses that decide whether to give credit, and consumers who decide whether to pay her bill. The author suggests that a credit bureau can hold a record of whether each consumer has ever defaulted or not. Greif (1993) and Tirole (1996) propose an enforcement mechanism based on community reputation. Similarly, Klein and Leffler (1981) rely on costless communication between consumers to enforce that firms supply a high quality product to the market. In this paper we also study whether it is possible to enforce first-best contracts through repeated interactions when agents are randomly matched to trade. However, we consider that agents have access to information via a network of bilateral relationships. We provide conditions under which agents rely on their network to trade the efficient contracts. In addition, we allow agents to choose how to form these relationships and analyze which networks structures are stable.

The paper is organized as follows. The next section introduces the model set-up. In Section 3 we describe in detail the trading protocol and analyze when unsecured trade is implementable, as well as the efficiency of trading through networks. We propose concepts for network formation and show which networks are stable in Section 4. Section 5 concludes.

2 The Environment

Consider an infinite-horizon economy in which a set $\mathcal{N} = \{1, ..., 2n\}$ of agents participate in the market at each date t. All agents are risk-neutral, infinitely lived, and discount the future with the discount factor $\beta = 1/(1+\phi)$, where ϕ is the discount rate. At the beginning of each period, uniformly at random, half of the agents are assigned a liquidity surplus, and the other half are assigned an investment opportunity. Let $\mathcal{L}^t$ be the set of agents with liquidity surpluses in period t (henceforth, *liquidity agents*), and $\mathcal{I}^t$ be the set of agents with investment opportunities in period t (henceforth, *investment agents*). A liquidity agent is endowed with one unit of cash, which can be stored at no cost until the end of the period. An investment agent is endowed with a riskless asset which yields a return of $r > 1$ at the end of the period. In addition, an investment agent has an opportunity to invest in a risky asset. The investment in the risky asset is scalable: if an amount $q \in [0, 1]$ is invested, the risky asset yields a return $\theta_k^t \in \{R(q), 0\}$ by the end of the period with probability p and $(1-p)$, respectively. The returns of the risky asset are independently distributed across agents, as well as over time. We assume that $R' > 0$ and $pR'(1) \geq 1$, $R'' < 0$, $R(0) = 0$, $R(1) = \overline{R}$ with $p\overline{R} > r$.

To exploit the investment opportunity, an investment agent $i \in \mathcal{I}^t$ needs to borrow funds from some liquidity agent $\ell \in \mathcal{L}^t$ at the beginning of each period, t.[2] Typically, in OTC markets parties trade customized contracts. To capture this feature, we assume that once agents have been assigned a type (liquidity or investment), liquidity and investment agents are matched uniformly at random, and each liquidity agent can only lend to the investment agent he is matched with. The debt must be repaid at the end of the period.

Formally, a matching $\mathbf{m}^t$ is a subset of $\mathcal{L}^t \times \mathcal{I}^t$ such that for each liquidity agent $\ell \in \mathcal{L}^t$, there is a unique investment agent $i \in \mathcal{I}^t$ for which the pair $m^t = (\ell, i) \in \mathbf{m}^t$. At each date t, a matching $\mathbf{m}^t$ is randomly drawn from the set of all possible matches at date t. Then, the probability that a pair of agents $(k, k') \in \mathcal{N} \times \mathcal{N}$ is matched at date t is[3]

$$\Pr[(k, k') \in \mathbf{m}^t] = \frac{1}{2(2n-1)}.$$

[2] The implicit assumption is that liquidating the riskless asset at the beginning of the period to self-finance the investment is too costly.

[3] This is because the probability that k is a liquidity agent is $\frac{1}{2}$. Then, conditional on being a liquidity agent, the probability that he is matched with k' as an investment agent is $1/(2n-1)$.

For the remainder of the paper, we refer to a pair of agents before any uncertainty is realized as (k, k'), and to a matched pair of liquidity and investment agents as (ℓ, i).

In this environment, we consider two frictions. First, we assume that there is limited commitment, and that agents can renege on due payments at the end of the period. Second, we consider that agents have limited access to information about other agents' past behavior. In particular, we consider that agents are connected through an informational *network* that allows each agent to observe the unilateral actions that his neighbors take. A network, g^t, is a graph $(\mathcal{N}, \mathcal{E}^t)$, where $\mathcal{N}$ is the set of nodes, and $\mathcal{E}^t \subset \mathcal{N} \times \mathcal{N}$ is the set of links that exist between agents at date t. The set of agents that have a link with agent k in the network g^t is denoted by $\mathcal{N}_k^t$. The information that agents observe is described in detail in Section 3.1.

To counteract the problem of limited commitment, agents can trade against collateral. A transaction is secured when the agent with an investment opportunity pledges the riskless asset as collateral at the beginning of the period. In this case, if the risky project fails ($\theta_i^t = 0$) then the agent with an investment opportunity cannot make any repayments. At the same time, whenever the agent with a liquidity surplus liquidates the collateral at the end of the period, he only obtains a return of one. Since the value of collateral for the borrower is higher than the value of collateral for the lender, there is a welfare loss when trade is secured.

Alternatively, agents can use the information they access through the network and trade without collateral by relying on self-enforcing contracts. In particular, we consider that agents have the option to trade through the informational network. Given a network g^t and a realization of the matching $\mathbf{m}^t$, the pairs that are matched at date t may or may not be connected by a link. If a matched pair (ℓ, i) has a link in the network g^t, they can trade directly through their link. If a matched pair (ℓ, i) does not have a link in the network g^t, they can trade through a *path of intermediaries*. A path of intermediaries between a pair $(k, k') \in \mathcal{N} \times \mathcal{N}$ in a network g^t is a sequence of agents $(j_1, j_2, ..., j_v)$ such that the links $(k, j_1), (j_1, j_2), ..., (j_v, k') \in \mathcal{E}^t$. We use $\mathbb{P}^t(k, k')$ to denote the set of paths from k to k' in the network g^t, and $\mathcal{P}^t(k, k')$ to denote a generic path. Similarly, once the matching $\mathbf{m}^t$ is realized, we use $\mathbb{P}^t\left(m^t\right)$ to denote the set of paths that can be used to intermediate trade between a matched pair $m^t = (\ell, i)$, and $\mathcal{P}^t(m^t)$ to denote a generic

path. The trading protocol is described in detail in Section 3.1. The network has, thus, both a trading and an informational function.

Links in the network are costly. In particular, each agent, k, incurs a *linking* cost for each link he has in the network that has two components: a recurrent component, c_l, that is paid every period, and an idiosyncratic component, c_m, that is paid only in the periods in which the link is used in a transaction. A link can be used in a transaction when it connects a pair of matched agents, or when it connects agents that intermediate trade between a matched pair. Thus, the total cost that an agent pays in any given period t depends not only on his position in the network, but also on the realized matching $\mathbf{m}^t$ and the path of intermediaries used to trade. The motivation behind the structure of the linking costs is related to the two functions that a network has. The idiosyncratic component, c_m, can be interpreted as a *transaction cost*, while the recurrent component, c_l, can be interpreted as a cost to access information, or *informational cost*.

We study when the first-best allocation can be decentralized, and characterize second-best outcomes as well.

3 The (Repeated) Trading Game

In this section we take the network $g = (\mathcal{N}, \mathcal{E})$ as given and we consider that it is fixed for all periods.[4] We investigate whether trading without collateral is beneficial, and analyze the set of *financial contracts* for which trade between any matched pair takes place without collateral, when the level of investment is $q \in [0, 1]$. We consider that the *level of investment* is q when each investment agent borrows an amount q from the liquidity agent with whom he is matched, and invests it in the risky asset.

We begin by specifying the contracts and the trading game, and define strategies and equilibrium. We characterize the level of investment that is implementable in equilibrium. Then, we proceed to analyze the efficiency of financial networks.

[4] In Section 4 we relax this assumption and analyze issues related to network formation and stability.

3.1 Financial contracts and trading procedure

The financial contract that determines the terms of trade between a matched pair has two components. The first component specifies an amount, $d \in [q, r]$, that an investment agent should repay a liquidity agent with whom he is matched in exchange for borrowing q units of funds. The second component allocates a fee $f \in \mathbb{R}_+$ to any agent that can be an intermediary. Thus, if a pair (k, k') is matched and trade through a path $\mathcal{P}(k, k') = (j_1, j_2, ..., j_v)$ without collateral, then the investment agent should repay in total $d + \sum_{s=1}^{v} f$, such that an intermediary j_s receives f, for any $s = 1, ..., v$.

We consider financial contracts, (d, f), that are independent of the position of the agents in the network. However, a crucial feature of our analysis is that the financial contract depends on the network structure g. Thus, an agent's position in the network is only reflected in the total payoff he expects to receive in a given period. However, by comparing different network structure we highlight the relative advantage that network positions offer some agents over others. We also allow the financial contract to depend, on the amount, q, that an investment agent borrows from the liquidity agent with whom he is matched.

Since there is limited commitment, the incentive of intermediaries to transfer the repayments to the next agent depends on the future benefits they expect to receive from trade. In particular, an agent with a liquidity surplus who is an intermediary may find it optimal to keep the repayments for himself, without the expectation of receiving fees in the future. However, for the fees to serve this purpose agents must use the information obtained from the network adequately.

We define the trading procedure at date t, as follows. First each agent is assigned a type (liquidity or investment), and the matching $\mathbf{m}^t$ realizes. These realizations are common knowledge among all agents.

Then, for each matched pair $m^t = (\ell, i) \in \mathbf{m}^t$, the investment agent i proposes a path $\mathcal{P}(m^t) = (j_1, j_2, ..., j_v)$ through which to trade with ℓ (including the empty path, i.e. trade directly with ℓ). We assume that this proposal is common knowledge to all agents. Each agent on the path then sequentially responds with an *yes* or *no*, starting from j_1 and ending with ℓ. If all agents on the path respond with *yes*, then the liquidity agent,

ℓ, transfers q units of cash to the investment agent, i, through the path without asking collateral. Otherwise, the liquidity agent, ℓ, transfers directly *one* unit of cash to the investment agent, i, and, in exchange, the investment agent transfers the riskless asset as collateral to the liquidity agent.[5]

When trade is unsecured, each agent on the path has a debt obligations to the next one according to the financial contract (d, f), as follows. The agent i is obligated to repay $[d + v \cdot f]$ to j_1. Further, each intermediary $j_{v'}$ is supposed to receive $[d + (v - s' + 1)]$ from $j_{s'-1}$ and is obligated to repay $[d + (v - s') \cdot f]$ to $j_{s'+1}$, with $j_0 = i$ and $j_{v+1} = \ell$.

After the risky project realizes its payoff, each agent on the path decides whether he repays his debt obligation. We assume that the agent either repays in full or repays nothing. This assumption will simplify our notation without losing any insights.

When trade is secured, and a pair of matched agents trade directly, the investment agent has the obligation to repay one unit of cash to the liquidity agent. When the project returns $R(1)$, the investment agent repays his obligations and receives back the collateral. However, when the project fails and returns 0, the investment agent has no resources to repay and the liquidity agent liquidates the collateral, for which he receives 1. Intermediaries are not involved in trade and receive no fees.

Next, we explain the information structure in detail. As we discussed earlier, an agent j can observe each of his neighbors' unilateral actions, as well as information that is common knowledge, which is the type of the agents, the matching, and the proposed paths by each investment agent. For each agent k, his unilateral actions in the network g at date t, denoted by a_k^t, include the following elements: (i) his responses on the proposed trading paths that he is involved; (ii) whether he repays in full to each of his neighbors, if he is either an intermediary and/or an agent with an investment opportunity. If two agents trade directly, their repayment decision is not observed by their neighbors. Let $\mathbf{a}_k^t = \left(a_k^0, ..., a_k^t\right)$ be the unilateral actions taken by agent k up to date t, and let $\mathbf{a}_0^t = \left(a_0^0, ..., a_0^t\right)$ be the commonly known information up to date t. Then, the history that an agent k observes

[5] Note that when trade is secured, we assume that the liquidity agent lends one unit of fund to the investment agent instead of q. This assumption captures the idea that no gains from trade should be left on the table. Indeed, when trade is secured, the first best quantity to be invested is 1, which is also incentive compatible. However, all our results are robust to this assumption. That is, we can allow agents to borrow q units of funds and to pledge a fraction of the riskless asset as collateral.

at date t is given by $h_k^t = \left\{\mathbf{a}_j^t : (j,k) \in \mathcal{E}\right\} \cup \{\mathbf{a}_0^t\}$. The realization of the risky project is private information and is observable only to the respective investment agent.

Because an agent may be involved in multiple trading paths, we need to specify a timing for their responses and repayments. For each proposed trading path $\mathcal{P} = (j_1, j_2, ..., j_v)$ between a matched pair $m = (i, \ell)$, agents in position j_1 respond simultaneously first, then agents in position j_2, etc. Similarly, for repayment decisions, investment agents decide first simultaneously, and then agents in position j_1, depending on the resources repaid by investment agents, and then agents in position j_2, etc.

Next we introduce strategies and the equilibrium concept. First we define strategies. For each agent k, his strategy in period t, denoted by s_k^t, has three components:

- $s_k^{t,1}$ maps the history h_k^{t-1} he observes, the realization of agents' type, and the matching $\mathbf{m}^t$ to a proposed path, if he is an investment agent;
- $s_k^{t,2}$ maps the history h_k^{t-1} he observes, the commonly known information $\mathbf{a}_0^t$, and the responses of his neighbors before him on the paths that involve him to his responses, if he is a liquidity agent and/or an intermediary;
- $s_k^{t,3}$ maps the history h_k^{t-1} he observes, the commonly known information $\mathbf{a}_0^t$, and the repayments of his neighbors before him on the paths that involve him to his repayment decisions on all trading paths he is involved, if he is an investment agent and/or an intermediary. Note that his repayment decision is constrained by repayment decisions of agents before him on the trading paths.

We use Perfect Bayesian Equilibrium (PBE) in pure strategies as the solution concept. We restrict attention to equilibria that satisfy the following properties.

(A1) **No default**. Every agent consents to trade the contract (d, f) without collateral and there is no default in equilibrium plays.

(A2) **Shortest path.** The shortest paths in the network g are always proposed in equilibrium. When there are multiple shortest path between a matched pair, they are proposed with equal probabilities in equilibrium.

(A3) **Stationary equilibrium allocation**. The level of investment, q, is constant across realized matches and across periods.

Definition 1 *A PBE equilibrium satisfying (A1)-(A3) is called a* ***simple equilibrium****.*

Condition (A1) is a symmetry requirement, as it rules out the possibility that trading without collateral happens for a subset of agents. Similar considerations motivate condition (A3). Condition (A2) requires that the equilibrium trading paths are the shortest ones. This assumption simplifies our analysis, since in general networks multiple paths may be used to trade, but only the shortest one minimizes the expected transaction cost, c_m. For most of our results, this requirement is not binding.

3.2 Implementation

In this section we explain in detail why trading without collateral is desirable, and explore the role of networks in supporting unsecured trade in equilibrium. We first introduce the gains from unsecured trading relative to trading against collateral. We then characterize the investment level, q, that is implementable in a given network g. Focusing on the level of investment, q, provides a rich metric to differentiate across those network structures in which unsecured trade can be sustained.

Definition 2 *A level of investment, q, is* ***implementable*** *in a network g if it is supported in a simple equilibrium given a financial contract (d, f).*

Abstracting from linking and transaction costs, unsecured trading is beneficial relative to trading against collateral. This is because when trade is secured, agents forego some of the return of the riskless asset in those states of the world in which the risky project fails. Indeed, suppose that the level of investment is q. Then, the average surplus generated at each date by trading without collateral is in expectation

$$pR(q) - q + r.$$

In contrast, the average surplus generated at each date by trading against collateral is in expectation

$$pR(1) - 1 + pr + 1 - p.$$

Therefore, the relative gains from unsecured trading are given by the following function

$$\Delta(q) = [pR(q) - q] - [pR(1) - 1] + (1-p)(r-1). \tag{1}$$

Since the return $R(\cdot)$ is a concave and increasing function, the condition $pR'(1) \geq 1$ ensures that $\Delta(\cdot)$ is increasing in $q \in [0,1]$. The relative gains from unsecured trade are maximized and positive when $q = 1$. This implies that $q = 1$ represents the *first-best* level of investment. At the same time, since $\Delta(1) > 0$, it follows that there are positive gains from trading without collateral even for a level of investment $q < 1$.

Although trading without collateral may generate a higher expected surplus than secured trade, it is not necessarily the case that it can be supported in equilibrium. Even in the least restrictive case of complete information, when all histories are publicly observable, unsecured trade can be supported in equilibrium for an investment level q if and only if[6]

$$\phi q \leq \frac{1}{2}\Delta(q). \tag{2}$$

The intuition is simple. When trade is unsecured, agents weigh the long-term benefit from trading without collateral against the one time gain of retaining all the return of the assets and paying 0. In particular, when an investment agent decides whether to repay at the end of the period, he takes into consideration he will be required to pledge collateral at all future dates as an investment agent, if he defaults on his obligations.

When there is incomplete information, condition (2) is no longer sufficient. In this case, the frequency with which an agent trades with counterparties that have access to his private history affects his incentives to default or not on his obligations. Thus, whether unsecured trade can be supported in equilibrium may depend on the size of the economy. Since when trading in a network agents can access the private histories of their neighbors, networks may facilitate the implementation of an investment level q if there are positive gains from unsecured trade, particularly when the number of market participants grows large.

To understand the role of networks in supporting trade without collateral, we first explore the empty network benchmark. The trading procedure in the empty network is

[6] We do not provide a proof for this statement, as the result is standard.

nested in the trading procedure described in Section 3.1. In particular, once the agents' type has been assigned and the matching has been realized, an investment agent can only propose to trade directly (i.e. the empty path) with the liquidity agent he has been matched with. The liquidity agent can accept, and trade without collateral, or reject, and trade against collateral. Clearly, no agent intermediate trades in the empty network. Aside of the information that is common knowledge, each agent observes the action of his counterparty at a given date t. The following lemma gives a full characterization of the level of investment that is implementable in the empty network.

Proposition 1 *Let agents trade in an empty network.*

(i) A level of investment, q, is implementable if

$$\phi q \leq \frac{1}{2(2n-1)} \Delta(q). \tag{3}$$

(ii) For any level of investment $q > 0$, there exists $\bar{n}$ such that q is not implementable for all $n \geq \bar{n}$.

The proposition shows that the level of investment that is implementable when no information (other than agents' own past trades) is observable depends on how large the economy is. This is because the market size affects how likely it is that two counterparties who trade at date t, meet again in a given future period. When n grows large, the probability of meeting the same agent in future periods is small. Thus, if an agent defaults on his current obligation but repays in future trades with other counterparties, the threat he will be required to trade against collateral as a consequence of this default is not binding as n grows large. Hence, he cannot overcome his temptation to default.

As a result, when the market size increases, no level of investment is implementable in an empty network. In other words, there is no q for which trade takes place without collateral as n grows large, even though there may be positive gains from unsecured trade.

In a stark contrast with the empty network is the level of investment that is implementable in a star network, that we characterize next. A network is a star if there exists an agent k_C such that

$$\mathcal{E} = \{(k_C, j) : j \in \mathcal{N}, j \neq k\}.$$

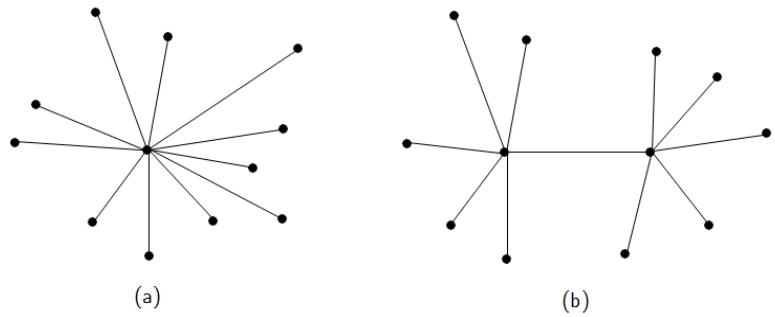

Figure 1: This figure illustrates two types of networks with the same number of agents. Panel (a) shows a star network. Panel (b) illustrates an inter-linked star network.

We refer to agent k_C in a star network the *center agent.* All other agents in the star network are *periphery agents.* A star network with $2n$ agents is denoted g_n^*. Figure 1(a) illustrates a star network.

When analyzing implementation in networks, such as the star or more general structures, we need to consider whether linking costs affect agents' incentives to make repayments as well. Indeed, the transaction cost is consequential for agents' incentives, since an agent incurs it for each of his links that is used in trade in a given period. In contrast, an agent pays the informational cost, c_l, for each link he has in the network every period, independently on whether he trades through the network without collateral or directly against collateral. Thus, the informational cost can be seen as sunk and does not constrain agent's decision to make repayments. While both costs can be taken to zero when we study implementation, they influence significantly welfare, as discussed in Section 3.3, as well the stability of networks, as discussed in Section 4.

To characterize equilibria in networks for the remainder of the paper, we restrict our attention to financial contracts with the property that $d \geq q + c_m$. We use this restriction for simplicity, as it ensures that the liquidity agent is willing to lend to the investment agent through the network, provided that he believes that his counterparties will repay their debts. No insights are lost if we relax the assumption.

The next proposition gives a full characterization of the level of investment that can be implemented under a star network.

Proposition 2 *Let agents trade in a star network g_n^*. Then, a level of investment, q, is implementable if*

$$\phi(q + c_m) + 2c_m \leq \frac{1}{1/2 + \phi}\left[-\phi(q + c_m) + \frac{1}{2}\Delta(q) - c_m\right]. \tag{4}$$

Proposition 2 provides a sufficient condition for a star network to implement a given level of investment q. Condition (4) shows that the level of investment that is implementable in a star network is independent of the number of market participants. Thus, even as n grows large, agents can still trade without collateral when the level of investment q satisfies (4).

We obtain condition (4) by ensuring that both center and periphery agents have the incentive to repay their obligations. We consider first the incentives to repay of a periphery agent. On the one hand, the largest amount that a periphery agent can retain if he reneges on his obligations is $(d + f)$. On the other hand, the expected discounted future benefit of trading without collateral relative to trading against collateral in the star network is at least $\frac{\beta}{1-\beta}\left[\frac{1}{2}\Delta(q) - \frac{1}{2}f - c_m\right]$. Indeed, the first term, $\frac{1}{2}\Delta(q)$, reflects the relative gains from unsecured trading weighted by the probability that the agent is an investment agent. The second term, $\frac{1}{2}f$, reflects the expected fee that an agent must pay to the center agent, when he is an investment agent matched with another periphery agent. The third term reflects the transaction cost. Thus, if

$$-(d + f) + \frac{\beta}{1 - \beta}\left[\frac{1}{2}\Delta(q) - \frac{1}{2}f - c_m\right] \geq 0,$$

or

$$f \leq \frac{1}{1/2 + \phi}\left[-\phi d + \frac{1}{2}\Delta(q) - c_m\right], \tag{5}$$

then a periphery agent has the incentive to make repayments.

Next, we discuss the incentives to repay of the center agent. On the one hand, the largest amount that the center agent can retain if he reneges on his obligations is nd. On the other hand, the expected discounted future benefit of trading without collateral relative to trading against collateral in the star network is $\frac{\beta}{1-\beta}\left[\frac{1}{2}\Delta(q) + (n-1)\,f - (2n-1)\,c_m\right]$. As before, the first term, $\frac{1}{2}\Delta(q)$, reflects the relative gains from unsecured trading weighted

by the probability that the agent is an investment agent. In addition, every period he receives an amount $(n-1)f$ in fees, while his total transaction cost is $(2n-1)c_m$. Thus, the center agent has an incentive to make repayments if

$$-nd + \frac{\beta}{1-\beta}\left[\frac{1}{2}\Delta(q) + (n-1)\,f - (2n-1)\,c_m\right] \geq 0,$$

which holds when

$$f \geq \phi d + 2c_m, \tag{6}$$

since $-\phi d + \frac{1}{2}\Delta(q) - c_m \geq 0$ from (5). Setting $d = q + c_m$, condition (4) ensures that there exists a fee f that satisfies the inequalities (5) and (6) at the same time.

Ensuring the implementation of an investment level, q, in a star network is comparable to the complete information case. Indeed, if we take $f = 0$ and $c_m = 0$ in inequality (5) we obtain condition (2) as when there is complete information. However, an important distinction arises because of the asymmetry in the information that center and periphery agents can access in a star network. While the center agent has information about all other agents in the economy, any periphery agent has information only about the center. Thus, the center agent has the incentive to repay only when he expects to receive a non-zero fee. In fact, condition (6) is a lower bound and condition (5) is an upper bound for the fee that the center agent must receive, in the limit as the number of market participants grows large.

A star network improves on the empty network for large markets as reflected in the level of investment that can be implemented. However, not all networks can implement the first-best, or even lower investment levels if the number of market participants is too large. The following result provides a class of counterexamples.

Proposition 3 *Let agents trade in a connected network g. Suppose that $n > 3$, and let $v_{\max}$ be the maximum number of intermediaries between any pair of agents. Then, the investment level q is implementable only if*

$$\frac{v_{\max}^2}{4n} \leq \frac{\Delta(q)}{\phi(q + c_m)}.$$

Proposition 3 shows that networks in which intermediation paths are too long cannot sustain unsecured trades. When there is a long intermediation path, there can be a realization of the matching so that there are agents on the path who intermediate trade between many matched pairs. This implies that each of them has to transfer at the end of the period a large amount of funds, in fees and repayments to liquidity agents. Hence, the temptation to renege and retain the funds for themselves is stronger than the expected benefits from the future trades.

Proposition 3 together with Proposition 2 suggest that some degree of concentration in intermediating trades is necessary. However, there are networks that can implement the first-best level of investment without relying on concentration in trades. For instance, the complete network can implement the same level of investment as when there is complete information if the linking costs are negligible. While concentration in intermediation may not be necessary, the condition (4) is, nevertheless, necessary for a level of investment q to be implemented in other two classes of networks, at least asymptotically.

Definition 3 *Let $\{g_n\}_n$ be a sequence of networks. Then, a level of investment, q, is* ***asymptotically implementable*** *in $\{g_n\}_n$ if there exists $\bar{n}$ such that q is implementable in g_n for all $n \geq \bar{n}$.*

The two classes of networks we consider allow us to highlight separately two properties of a star network: low average number of links and small average number of intermediaries between pairs. The first class of networks we consider are connected networks that have the same number of links as a star, namely *minimally connected networks.* In a minimally connected network there exists a unique path between any pair of agents. Such network g have the property that the *average number of links* in g is $\eta_g = \frac{2n-1}{2n} \simeq 1$.

The second class of networks we consider are connected networks in which the average number of links, η_g, is bounded, but larger than in a star, while the *average number of intermediaries*, υ_g, is low, close to a star. Given (A2), υ_g is the average number of intermediaries on the shortest path between pairs of agents in g. The following proposition characterizes asymptotically implementable investment levels in these two classes of networks. Note that the star network belongs to both classes.

Proposition 4

(i) *Let $\{g_n\}_n$ be a sequence of minimally connected networks. Then, the level of investment q is asymptotically implementable only if (4) holds.*

(ii) *Let $\{g_n\}_n$ be a sequence of networks. There exist $\bar{\eta} > 1$ and $\bar{\upsilon} > 1$ such that, if $\eta_{g_n} \leq \bar{\eta}$ and $\upsilon_{g_n} \leq \bar{\upsilon}$ for all n, then the level of investment q is asymptotically implementable only if (4) holds.*

For the remainder of the paper, we refer to a network g with $\eta_g \leq \bar{\eta}$ and $\upsilon_g \leq \bar{\upsilon}$, with $\bar{\eta} > 1$ and $\bar{\upsilon} > 1$ as given in Proposition 4, as a *small* network.

Proposition 4 shows that if a level of investment, q, is implementable in a network belonging to either the small or minimally connected class, it must be implementable in a star network as well. This implies that a star network can implement the highest level of investment, at least relative to small and minimally connected networks.

The proof for both part (i) and part (ii) in Proposition 4 follows the same main steps. First we show that there exists an agent whose trades are intermediated with high probability, similar to a periphery agent in the star network. This implies that inequality (5) is asymptotically necessary to ensure that he has incentives to repay. Second, we show that there exists an agent who, at least for some realization of the matching, intermediates a large number of trades, similar to the center agent in the star network. This implies that inequality (6) is asymptotically necessary to ensure that he has incentives to repay.

Another interesting implication that arises from Proposition 4 is related to the fees that the intermediaries receive. In particular, the following result illustrates how the network structure favors some intermediaries with respect to the fees they receive.

Corollary 1 *Let $f_g^{\max}(q)$ be the maximum fee an intermediary can receive in a network g, for a given implementable investment level q. Then, for any sequence $\{g_n\}_n$ of minimally connected networks or small networks,*

$$f_{g_n^*}^{\max}(q) \geq f_{g_n}^{\max}(q)$$

for all asymptotically implementable investment levels q in $\{g_n\}_n$ and for all n sufficiently large.

Corollary 1 shows that the center agent in a star network can receive a higher fee than any intermediary in a minimally connected or small network. While the result holds when the same level of investment is asymptotically implemented in the star network as in a minimally connected or small network, an additional mechanism strengthens this finding. By Proposition 4, the level of investment that is asymptotically implementable in a star network is at least as high as in a minimally connected or small network. This implies that the relative surplus generated by trading without collateral, as defined in (1), is at least as large in a star network. Since the fee represents a division of the surplus between the center the periphery agents, a larger surplus makes it feasible for center agent to receive higher fees.

Moreover, the center agent in a star can receive higher fees than intermediaries in minimally connected or small networks, even for implementable (not only asymptotically) investment levels. For instance, we show in Lemma A.1 in the appendix that maximum fee each of the two intermediaries in an inter-linked star, represented in Figure 1(b), can receive is strictly smaller than the maximum fee the center agent in a star can receive. This is because a periphery agent in an inter-linked star needs to pay with probability half fees to two intermediaries in any period he has an investment opportunity. Thus, ensuring he has incentives to make repayments places a tighter constraint on the fees that each intermediary receives.

3.3 Efficient Networks

In this section we study issues related to welfare and efficiency. Given a network g, an investment level q, the expected aggregate welfare when trading without collateral is given by

$$W(g,q) = \sum_{t=0}^{\infty} \beta^t n \left\{ (pR(q) - q + r + 1) - 4\eta_g c_l - 2(\upsilon_g + 1)c_m \right\} \tag{7}$$

where η_g and υ_g have been defined above as the average number of links and the average number of intermediaries between pairs of agents in g, respectively.

As it is evident from (7), the direct effect of a network structure, g, on welfare can be summarized only by two variables, η_g and υ_g. Given the network g, the total informational cost per period is $|\mathcal{E}| \cdot (2c_l) = (2n) \cdot \eta_g \cdot (2c_l)$. The total transaction cost depends on the

realization of the matching. However, in expectation, in any given period, it only depends on the average number of intermediaries, and hence the total expected transaction cost is $n \cdot (\upsilon_g + 1) \cdot (2c_m)$.

Definition 4 *A network g and an investment level q is a constrained efficient arrangement if it maximizes $W(g, q)$ over the space of connected networks and investment levels such that q is implementable in g.*

Maximizing expected welfare involves a trade-off. On the one hand, the welfare function (7) is increasing in the level of investment, q. On the other hand, there may be high linking costs associated with a network that implements q. For instance, while the complete network can implement the first-best level of investment, the linking costs become infinitely large as the number of market participants grows.

A good candidate for a constrained efficient arrangement is a network that can implement high levels of investment at low linking costs, such as the star network. Indeed, let q^* be the largest investment level that can be implemented asymptotically in the star network, that is

$$q^* = \arg\max \Delta(q)$$
$$\text{s.t. } \phi(q + c_m) + 2c_m \leq \frac{1}{1/2 + \phi}\left[-\phi(q + c_m) + \frac{1}{2}\Delta(q) - c_m\right].$$

Then, as it was anticipated by Proposition 4, q^* is higher than investment levels that can be implemented in small and minimally connected networks. The result is in fact more general, as the next two propositions show.

Proposition 5 *Suppose that $q^* = 1$. Then, the arrangement (g_n^*, q^*) is uniquely constrained efficient, for sufficiently large n.*

Proposition 5 shows that when the first-best level of investment is implementable in a star network, then this is the most efficient network. In other words, concentrated intermediation maximizes social welfare.

The intuition is as follows. By construction, a star network can implement the first-best investment level when (4) holds for $q = 1$. Thus, we only need to show that the star minimizes the linking costs relative to all other connected networks. Since minimally

connected networks have the lowest number of links among connected networks, the informational costs are also minimized. Moreover, transaction costs are the lowest in the star among all minimally connected networks. In any other connected network, informational costs are larger, while transactional costs can be lower. To show that a star network is optimal, we show that it never pays off to decrease the transaction cost while increasing informational costs for n large, independently of c_m and c_l.

When the first-best is not implementable in a star network ($q^* < 1$), the trade-off between the level of investment and linking costs that the welfare function (7) embeds is even more pronounced. We analyze asymptotically the resolution of this trade-off. For this, we first introduce the following definition.

Definition 5 *The sequence $\{g_n\}_n$ and the investment level q is an* ***asymptotically constrained efficient*** *arrangement if for any sequence of connected networks $\{g'_n\}_n$ and any q' asymptotically implementable under $\{g'_n\}_n$, we have that $W(g_n, q) \geq W(g'_n, q')$ for all large n.*

Since any network g can be summarized by the parameters η_g and υ_g, we re-write the welfare function as

$$W(g, q) = W\left(\eta_g, \upsilon_g, q\right),$$

in order to introduce the next result.

Proposition 6 *Suppose that $q^* < 1$. Then, the arrangement (g^*_n, q^*) is asymptotically constrained efficient arrangement if*

$$W(1, 1, q^*) \geq \max\{W(1, \bar{\upsilon}, 1), W(\bar{\eta}, 1, 1)\}, \tag{8}$$

where $\bar{\eta} > 1$ and $\bar{\upsilon} > 1$ are given in Proposition 4.

Essentially, Proposition 6 shows that if q^* is sufficiently close to 1, then the star is still the constrained efficient network. This result follows from the second part of Proposition 4, which shows that an investment level higher than q^* is not implementable in networks in which linking costs are bounded by $\bar{\eta}$ and $\bar{\upsilon}$.[7] Condition (8) ensures that for networks

[7] Since in a "small" network the average number of links and the average number of intermediaries

outside the class of small networks, it does not pay off to increase the investment level if this requires increasing linking costs.

While trading without collateral in a star network yields higher welfare gains than in other connected networks, an interesting questions is whether it may be more beneficial for agents to trade directly against collateral and save on linking costs. The following result addresses this issue.

Corollary 2 *Suppose that q^* is sufficiently close to 1. Then unsecured trade in a star network yields higher welfare than secured trade in the empty network, if linking costs are small.*

The intuition is simple. When trade takes place without collateral in a star network, there is a welfare loss because agents incur linking costs. In addition, when $q^* < 1$, some of the return of the risky asset is forfeited. When trade takes place in the empty network, by Proposition 1, transactions must be secured if the number of market participants is sufficiently large. Trading against collateral involves a welfare loss because of the inefficient liquidation of the riskless asset. The latter effect dominates if linking costs are small or if q^* is close to 1. In contrast, for lower q^*'s, secured trade in the empty network dominates.

4 Network Stability

In this section we investigate whether agents have an incentive to participate in a network and identify structures that are stable when traders are allowed to change their links. For this, we first introduce the network formation game, and the stability concept we are employing. Then we proceed to characterize stable network structures.

4.1 The network-formation game

We consider the following network-formation game. At date 0, fix a network g. At the beginning of each even period $t = 0, 2, ...$, one agent k, selected at random, is allowed to sever one or more of his links. At the beginning of each odd period $t = 1, 3, ...$, one pair

are bounded by $\bar{\eta}$ and $\bar{v}$, respectively, then aggregate informational is at most $4n\bar{\eta}c_l$, while the aggregate transaction cost is at most $2n(\bar{v}+1)c_m$.

of agents (k, k'), selected at random, are given the opportunity to form a link, if they do not have one. If both agents agree, the link is formed. At each period t, agents' linking decisions result in a new network g^t.

After agents make their linking decisions, their types (liquidity or investment) are assigned, and the matching is realized. In the new network g^t, an agent only observes each of his current neighbors' unilateral actions, as well as information that is common knowledge. Thus, we assume that after after an agent severs a link he no longer accesses the history of his former neighbor. Similarly, if two agents form a new link, they have access to their respective complete histories.[8]

Then, the agents trade according to the trading procedure described in Section 3.1. Consistent with the previous section, we allow the financial contract and the level of investment for unsecured trades to depend on the network structure. In particular, we consider a function $\mathcal{C}(g^t)$ that assigns to a network g^t a contract, (d_{g^t}, f_{g^t}), and an investment level q_{g^t}, that specify the terms of trade without collateral. The function $\mathcal{C}(\cdot)$ allows agents to evaluate their continuation payoff for each linking decision they can take at date t, as a function of the distribution of networks that may arise at each future date τ, and given the actions that agents take in the trading game in each possible network g_τ.

We say that the function $\mathcal{C}(\cdot)$ is *tight* if q_{g^t} is the highest level of investment that is implementable in g^t, provided the set of implementable investment levels is non-empty. Given a tight function $\mathcal{C}(\cdot)$, a trading strategy profile is *tight* w.r.t. $\mathcal{C}(\cdot)$ if agents in any connected component of the network g^t trade without collateral among themselves, in each period t when q_{g^t} is implementable in g^t, and after any possible partial history of the network-formation game (both on and off equilibrium paths).

Definition 6 *A network g is **stable** under $[q, (d, f)]$ if there exist a tight function, with $\mathcal{C}(g) = [q, (d, f)]$, and a Nash equilibrium in the network-formation game such that no agent severs a link and no pair of agents forms new links, and agents use a tight trading strategy profile.*

The notion of stability that we propose is consistent with the welfare analysis we

[8] This assumption is without loss of generality. While we could allow an agent to still access the history of a former neighbor up to the date he severed the link, our results remain. This is because agents are forward looking, and they are no able to optimally condition their trading strategy bases on partial histories.

have developed in Section 3.3. In particular, it allows us to check whether constrained efficient networks are also stable. Moreover, focusing on a function that selects the highest implementable level of investment for a given network, we are able to conceptualize the value of a link in a dynamic setting. Indeed, as agents change links, they are still able to extract the maximum surplus in the new network. This implies that the relative benefit an agent obtains by maintaining a link represents a lower bound for the value of the respective link.

In addition, this notion of stability allows us to narrow down the set of stable networks in a meaningful way. For instance, suppose we relax the requirement that agents use a tight trading strategy. Instead, consider that agents ask for collateral in any trades with other agents that have changed their links. Facing a severe punishment, agents may be deterred from severing or forming new links. We conjecture that, in this case, most networks that can implement positive levels of investment are stable. The requirement to use a tight strategy rules out this type of unreasonable punishments.

4.2 Stable network structures

We proceed to characterize stable networks. A natural starting point is to consider the star network. Let q_n^* be the level of investment such that

$$q_n^* = \arg\max \Delta\left(q\right)$$

$$\text{s.t. } \min\left\{\frac{1}{n-1}\left[n\phi(q+c_m) - \frac{1}{2}\Delta(q) + 2nc_m\right], 0\right\} \leq \frac{1}{\frac{n-1}{2n-1}+\phi}\left[-\phi(q+c_m) + \frac{1}{2}\Delta(q) - c_m\right]. \tag{9}$$

In the proof of Proposition 2 we provide in the appendix, we show that (9) is a necessary and sufficient condition for a level of investment q to be implementable in star network with $2n$ agents. Thus, q_n^* is the highest investment level that can be implemented in a star network, for each n. Further, let $f_{g_n^*}^{\max}$ and $f_{g_n^*}^{\min}$ to denote right-hand side and the left-hand side of condition (9) evaluated at q_n^*, respectively. Hence, $f_{g_n^*}^{\max}$ and $f_{g_n^*}^{\min}$ are the upper and lower bounds for the fee that the centre agent in a star with $2n$ agents receives. Note that $f_{g_n^*}^{\max} > f_{g_n^*}^{\min}$ only if $q_n^* = 1$. We have the following result.

Proposition 7 *Suppose that* $0 < c_l \leq \frac{1}{2}\phi(q_n^* + c_m)$. *Then,* g_n^* *is stable under* $[q_n^*, (q_n^* + c_m, f_n^*)]$ *for any* $f_n^* \in [f_n^{\min}, f_n^{\max}]$, *for* n *sufficiently large.*

Proposition 7 shows that a star network is stable, if the informational cost, c_l, is small and the economy size, $2n$, is large. It is indeed expected that if c_l is high, then agents are better off trading against collateral in the empty network. Similarly, if n is small, agents can enforce unsecured in the empty network, and save on linking costs.

The intuition for the stability result is as follows. To see whether a star network is stable, we need to study agents' incentives to sever or form links. In particular, there are two main cases we need to consider. First, we need to show that the center agent has no incentive to delete any link. Second, we need to show that no periphery agent has an incentive to form a new link with another periphery agent.

We start with the center agent, k_C. We illustrate how he evaluates his continuation payoff when he makes a linking decision, given the notion of stability we proposed in Definition 6. Suppose that at the beginning of an even date t, the center agent, k_C, is given the opportunity to sever one or more of his links. If he severs links with a set K_C of his neighbors, the new network is $g_-^t = g_n^* - \{(k_C, k') : k' \in K_C\}$. As usual in an equilibrium analysis, he considers that all other agents respect their equilibrium linking strategy at future dates. We construct equilibrium linking strategies with the property that agents do not sever or form new links after any partial history. This implies that the center agent considers that $g^\tau = g_-^t$ for any $\tau > t$. Further, he understands that the function $\mathcal{C}(\cdot)$ selects the highest level of investment, $q_{g_-^t}$, that is implementable in g_-^t, that $q_{g_-^t}$ is implemented and trade takes place without collateral forever after. Otherwise, if he maintains all his links, the new network is $g^t = g_n^*$, and he reasons in the same way to evaluate his continuation payoff. For the star network to be stable he must find it beneficial to maintain all his links. We show that this is the case by proving that the marginal value of a link for the center is bounded away from zero. Indeed, we find that the highest level of investment in the new network g_-^t can only be lower than q_n^*. In consequence, there exists a function $\mathcal{C}(\cdot)$ that allocates a fee, f, to agent k_C in the new network g_-^t, which ensures a positive lower bound for the marginal value of a link. This implies that the centre has no incentive to sever any link provided that c_l is small.

Next, we discuss the case of periphery agents. When a periphery agent, k, considers whether to form a link with another periphery agent, k', he evaluates his continuation payoff following a similar reasoning process as described above. If, at an odd period t, the agents consent to form a link, the new network is $g_+^t = g_n^* + \{(k, k')\}$. Otherwise, the new network is $g^t = g_n^*$. In the network g_+^t, both agents k and k' are able to trade directly through their link, without paying a fee to center agents in those periods when they are matched to trade. However, as n grows large, the probability of avoiding the fee diminishes, which makes the link too expensive to maintain. Thus, the new network g_+^t would be more attractive than g_n^* only if the fee paid to agent k_C is lower. We show that there exists a function $\mathcal{C}(\cdot)$ that allocates a fee, f, to agent k_C in the new network g_+^t which is higher than, or arbitrarily close to f_n^*, as n goes infinity. This insures that two periphery agents do not have an incentive to form a new link.

We have shown that a star network is both an efficient as well as a stable outcome. An interesting question is whether other networks that we know are not efficient can nevertheless be stable. We illustrate that this is case by providing conditions under which a interlinked star is stable.

Formally, a network is an interlinked star with two centers if there exist agents k_{C1} and k_{C2} such that

$$\mathcal{E} = \{(k_{C1}, k_{C2}), (k_{C1}, j), (k_{C2}, l) : j \in \mathcal{N}_{k_{C1}}, k \in \mathcal{N}_{k_{C2}}\},$$

where $\{\{k_{C1}, k_{C2}\}, \mathcal{N}_{k_{C1}}, \mathcal{N}_{k_{C2}}\}$ forms a partition of $\mathcal{N}$ with $|\mathcal{N}_{k_{C1}}| = |\mathcal{N}_{k_{C2}}| = n - 1$. We refer to agents k_{C1} and k_{C2} as the center agents. All other agents in the star network are periphery agents. We denote an interlinked star network with $2n$ agents by g_n^{**}. Figure 1(b) illustrates an interlinked star network.

As in the case of the star network, a necessary and sufficient condition for a level of investment to be implementable in an interlinked star is that the fee a center agent receives is bounded from below and from above. Lemma A.1 in the appendix states the precise condition. Let $f_{g_n^{**}}^{\max}$ denote the upper bound for fee that a center can agent receives. Then we can introduce the following result.

Proposition 8 *Suppose that $q = 1$ is implementable under an interlinked star network g_n^{**}. Then, there exists $\bar{c}_l > 0$ such that g_n^{**} is stable under $[(q + c_m, f_n^{**}), q]$, with $q = 1$ and f_n^{**} close to $f_{g_n^{**}}^{\max}$, if $0 < c_l \leq \bar{c}_l$, and n sufficiently large.*

The intuition is as follows. To see whether an interlinked star network is stable, we need to study agents' incentives to sever or form links. In particular, there are three main cases to consider. First, we show that neither of the two center agents has an incentive to delete any link. Second, we show that no periphery agent has an incentive to form a new link with another periphery agent. Third, we check whether a periphery agent that is a neighbor of k_{C1} (k_{C2}) has an incentive to form an additional link with the other center agent k_{C2} (k_{C1}). The argument for the first two cases is very similar to the one provided above for the star network. To rule out the third case, we show that there exists a function $\mathcal{C}(\cdot)$ that allocates a fee, f, to the center agents in the new network (where the periphery has a link with both center agents) which is arbitrarily close for f_n^{**}. This implies, however, that the neither of the center agents is willing to incur the cost for the additional link. Thus, neither of them consents to form the additional link, even though it benefits the periphery agent.

These arguments suggest that the periphery agents may find it desirable to trade in a star network. In other words, they may prefer paying a higher fee to one intermediary, than paying a lower fee to two intermediaries some fraction of the time. Thus, an interlinked star is stable because center agents' incentives are binding.

An alternative, stronger, notion of stability departs from Definition 6 and requires that a network is stable for any function $\mathcal{C}(\cdot)$. Thus, under Definition 6, a tight function can assign any fee, f, to intermediaries in a network g such that the level of investment q is implementable in g. In contrast, under this stronger stability concept, a network is stable only if agents maintain their links for all fees (subject to implementability) that can be assigned in the current network, as well as in networks that arise on the continuation paths that follow deviations.

For instance, consider the star network and suppose that the inequality (4) is slack. Then, there may exist a function $\mathcal{C}(\cdot)$ which assigns fees to intermediaries in the network resulting from a deviation, in a way that favors the agents who are deviating. Thus,

suppose that the center agent deletes one or more of his links. Consider a function $\mathcal{C}(\cdot)$ that assigns to the center agent a higher fee than in the original star network. At the same time, the function $\mathcal{C}(\cdot)$ can assign a lower fee to the center agent when two periphery agents form a link. If the star network is stable, than the center agent must not find it beneficial to delete links, which implies that the fee he receives in the original network is sufficiently high. Similarly, if the star network is stable, the periphery agents must not find it beneficial to form a link, which implies that the fee they pay to the center agent in the original network is sufficiently low. Since a fee that meets both requirement may not exist, a star network may not be stable. A similar line of reasoning can be used to argue that an interlinked star is not stable either. In fact, strong stability of networks is a very demanding property, and we expect that many networks do not satisfy it. Nevertheless, the proof of Proposition 7 suggests that the star is stable when condition (4) holds with equality for $q = 1$, as this narrows the set of fees a function $\mathcal{C}(\cdot)$ can assign.

5 Conclusion

Our results demonstrate that intermediation can be welfare-improving when OTC trades take place through neworks if the market size is large. In our model, the efficiency gains arise when trade against collateral is costly, as networks can provide adequate monitoring to sustain unsecured trade. We characterize the set of investment levels that are implementable when trade is unsecured for two broad classes of networks, namely the minimally connected and the small networks. We also show that trade without collateral is not sustainable in networks with long intermediation chains. Hence, we infer that a certain degree of concentration in intermediation is necessary. In our analysis intermediaries must receive fees to ensure they have the incentive to sustain unsecured borrowing. We characterize an upper and lower bound for the fees intermediaries receive in various networks. We also show that the fee the intermediary receives in the star network is the highest relative to how intermediaries in various other networks can be compensated. The way the compensation of intermediaries is determined in our model may provide an explanation for the rents intermediaries receive in OTC markets.

Our analysis of the constrained efficient arrangement highlights a trade-off between the

cost of maintaining and using a network, and the investment level that is implementable in a network. We provide conditions under which the star is the constrained efficient arrangement among *all* possible arrangements. We obtain this result when the market size is large, and when the star can implement a level of investment that is close to the first-best level. Finally, we show that various networks structures are stable, including star and interlinked star.

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A Appendix

Proof of Proposition 1

(i) First we prove sufficiency. Set $d = q$. We construct a strategy profile and show that it is a simple equilibrium, as follows. For each possible match $m = (\ell, i)$, we summarize the observed history of the match at the end of period t (which is observable to the match) with a state $s_{m,t} \in \{G, B\}$. We use m^r to denote the match with the same pair of agents but with their roles reversed, i.e., $m^r = (i, \ell)$. The state is such that $s_{m,t} = s_{m^r,t}$ for all t, and it evolves as follows: $s_{m,0} = s_{m^r,0} = G$; $s_{m,t+1} = G$ if $s_{m,t} = G$ and if either one of the following two conditions holds: (a) neither m or m^r realizes at period $t+1$, (b) either m or m^r is realized, and the agent assigned to the investment role repays his debt if traded without collateral; $s_{m,t+1} = B$ otherwise. Note that for any match $m = (\ell, i)$ at period t, the pair's actions have no effect on states $s_{m',t}$ with m' having agents other than the pair.

For any realized match $m = (\ell, i)$ at period t, the strategy for the pair only depends on $s_{m,t-1}$ as follows: ℓ accepts the proposed trade without collateral from i if $s_{m,t-1} = G$ and rejects it otherwise; i repays his debt if $s_{m,t-1} = G$ and does not repay otherwise.

Now we show that this strategy is sequentially optimal. Consider a realized match $m = (\ell, i)$ at period t. Because state B is self-absorbing, if $s_{m,t-1} = B$, i has no incentive to repay his debt and hence it is optimal for ℓ to reject his proposal. Now, suppose that $s_{m,t-1} = G$. By the equilibrium strategy of i, he will repay if his trade if accepted. Moreover, accepting or rejecting the trade has no impact on future states of the match. Thus, the current-period payoff for ℓ to accept the trade without collateral is $(d + (1-q))$ while the current-period payoff to reject the trade is $p(d-1) + 1 \leq d + (1-q)$ since $d \geq q$. Hence, it is optimal for agent ℓ to accept the trade without collateral. Finally, assuming that the proposed trade without collateral from i was accepted by ℓ, by not repaying the debt, i's equilibrium strategy follows the deviation considered in the proof of necessity. Thus, by (3), it is optimal for him to repay his debt.

(ii) Now we show that, for any given $q > 0$, it is not implementable for large n's. Recall that $\Delta(q) = 1/2\{p[R(q)+r] + (1-p)r + (1-q)\}$ is the expected surplus from trades without collateral. Here we assume that $\beta > \frac{1}{2}$; the other case can be proved in a similar

fashion. Let N be so large that if $K = \log_2(2N-1) - 1$, then

$$\frac{\beta^K}{2\beta - 1} + \frac{\beta^K}{1-\beta} < \frac{q}{\Delta(q)}. \tag{A.1}$$

Suppose, by contradiction, that q is implementable with $2n \geq 2N$ agents. Now, at period zero, consider an agent with the investment role at the end of period 0 and is supposed to repay his promise, $d \geq q$.

Consider the deviation to default now and, in all future period, behave as a non-defector. The worst scenario for this deviation would be that his current trading partner defects in all future periods, and all who are defected also defect. Thus, at period t, the probability of meeting a defector is at most

$$p_t \equiv \frac{\min\{2^{t-1}, 2n-1\}}{2n-1}.$$

Hence, the expected payoff, relative to secured trades all the time, is at least $\sum_{t=1}^{\infty} \beta^t (1 - p_t)\Delta(q)$, and, for the agent to prefer the equilibrium action than this deviation, it must be the case that

$$-d + \sum_{t=1}^{\infty} \beta^t \Delta(q) \geq \sum_{t=1}^{\infty} \beta^t (1-p_t)\Delta(q),$$

that is,

$$d \leq \sum_{t=1}^{\infty} \beta^t p_t \Delta(q). \tag{A.2}$$

Now, for any $n \geq N$ (recall that N is defined by (A.1)) and for $k = \log_2(2n-1) - 1$, we have

$$\begin{aligned} \sum_{t=1}^{\infty} \beta^t p_t &\leq \frac{(2n-1)\beta^k - 1}{(2n-1)(2\beta-1)} + \frac{\beta^k}{1-\beta} \\ &\leq \frac{\beta^K}{2\beta-1} + \frac{\beta^K}{1-\beta} < \frac{q}{\Delta(q)} \leq \frac{d}{\Delta(q)}, \end{aligned}$$

a contradiction to (A.2). ■

Proof of Proposition 2

We claim that q is implementable under star if and only if (9) holds. Note that (4) implies (9) for any $n > 0$: first,

$$\frac{1}{\frac{1}{2}+\phi}\left[-\phi(q+c_m)+\frac{1}{2}\Delta(q)-c_m\right] \leq \frac{1}{\frac{n-1}{2n-1}+\phi}\left[-\phi(q+c_m)+\frac{1}{2}\Delta(q)-c_m\right]$$

since $\frac{1}{2}+\phi \geq \frac{n-1}{2n-1}+\phi$ for any n; second,

$$n\phi(q+c_m)-\frac{1}{2}\Delta(q)+(2n-1)c_m \leq (n-1)\left[\phi(q+c_m)+2c_m\right]$$

since $-\phi(q+c_m)+\frac{1}{2}\Delta(q)-c_m \geq 0$.

First we prove necessity of (9). Let k_C be the center agent. Suppose that (d, f) with $d \geq q + c_m$ implements q under the star. Consider an agent assigned to the investment role in the periphery, deciding whether to repay his debt, $f + d$. We consider two choices: (a) repay the debt and follow the equilibrium strategies; (b) do not repay and propose direct trades in all following periods. By (A1), the choice (a) has to be better than the choice (b), and hence we have

$$\begin{aligned} & -(d+f)+\frac{\beta}{1-\beta}\left\{\frac{1}{2}\left(pR(q)+r+(1-q)-\frac{2n-2}{2n-1}f\right)-c_m\right\} \\ \geq \quad & \frac{\beta}{1-\beta}\frac{1}{2}\left[p(R(1)-1+r)+1\right], \end{aligned} \tag{A.3}$$

that is,

$$f \leq \frac{1}{\frac{n-1}{2n-1}+\phi}\left[-\phi d+\frac{1}{2}\Delta(q)-c_m\right]. \tag{A.4}$$

Now consider the center agent, k_C, assigned to the investment role and who is at the moment deciding whether to repay his debt, nd. Again, we consider two choices: (a) repay all the debts and follow the equilibrium strategies; (b) do not repay (to any debt) and propose direct trades in all following periods. By (A1), the choice (a) has to be better

than the choice (b), and hence we have

$$\begin{aligned} & -nd + \frac{\beta}{1-\beta}\left\{\frac{1}{2}[pR(q) + r + (1-q)] + (n-1)f - 2nc_m\right\} \\ \geq \quad & \frac{\beta}{1-\beta}\frac{1}{2}\left[p(R(1) - 1 + r) + 1\right], \end{aligned}$$

that is,

$$f \geq \frac{1}{n-1}\left[n\phi d - \frac{1}{2}\Delta(q) + 2nc_m\right]. \tag{A.5}$$

Combining (A.4) and (A.5) and the fact that $d \geq q + c_m$, we obtain (9).

Now we prove sufficiency. Suppose that (9) holds. First we specify the financial contracts. For unsecured trades on the equilibrium path, let $d = q + c_m$ and let $f \geq 0$ satisfy

$$\frac{1}{n-1}\left[n\phi d - \frac{1}{2}\Delta(q) + 2nc_m\right] \leq f \leq \frac{1}{\frac{n-1}{2n-1} + \phi}\left[-\phi d + \frac{1}{2}\Delta(q) - c_m\right].$$

For secured trades on off-equilibrium paths, as mentioned in the main text, the investment level is $q = 1$, and that the investment agent repays 1 to the liquidity agent when the return is $\theta = R(1)$. Given the contracts, the liquidity agent is indifferent between secured and unsecured trades, assuming that the investment agent will repay in unsecured trades. We construct equilibrium strategies as follows. Each periphery agent can be one of the two states, G and B. At date 0, all agents are in state G. A periphery agent stays in state G if and only if he repays all his debts to k_C when assigned to the investment role in the previous periods; otherwise, he enters state B. An agent who enters state B stays there forever. The state of these agents is only observable to the center agent, k_C. Note that if a periphery agent trades directly then this choice does not affect his state and a periphery agent's action in liquidity role does not affect his state. Similarly, the center agent, k_C, can also be in of the two states, G and B. He stays in state G if and only if he repays all his debts in the previous periods; otherwise, he enters state B. His state is then observable to all agents.

The strategy of a periphery agent j assigned to the liquidity role in state G is as follows: if k_C is in state G and if j is in state G, then he accepts any trade through k_C; otherwise,

he demands collateral for any trade. Moreover, he always asks for collateral if asked to trade directly. A periphery agent j assigned to the liquidity role in state B always asks for collateral. The strategy of a periphery agent j in investment role is as follows: if both himself and j_C are in state G, then he propose to trade through k_C and repay his debt; otherwise, he proposes to trade directly and, if his trade without collateral is accepted, he does not repay anything. Finally, the strategy of k_C is as follows: if he is in state B, then he never repays anything; otherwise, he accepts trades from a match $m = (\ell, i)$ if and only if both ℓ and i are in state G and rejects it otherwise, and he repays all debts if and only if it is feasible and the number of periphery agents in state G who repays at the current period, denoted by K_1, and the number of loans k_C has, denoted by K_2 (including his own), satisfy

$$-K_2 d + \frac{\beta}{1-\beta}\frac{K_1}{2(2n-1)}\left[pR(q) + r + (1-q) + (K_1-1)f\right] \tag{A.6}$$

$$+\frac{\beta}{1-\beta}\frac{2n-1-K_1}{2(2n-1)}\left[p(R(1)+r) + (1-p)\right] \tag{A.7}$$

$$\geq \frac{\beta}{1-\beta}\frac{1}{2}\left[p(R(1)+r) + (1-p)\right]. \tag{A.8}$$

Note that when there are still K_1 periphery agents in state G, the expected fees for each such agent is $(K_1-1)f/2(2n-1)$ and since any such fee is paid to k_C, the expected fee revenue is $K_1(K_1-1)f/2(2n-1)$. Moreover, only with those agents k_C can expect to have unsecured trades.

We also need to construct equilibrium beliefs. As agent k_C has complete information, his belief is the actual history. For an periphery agent j, his belief is such that if k_C is in state G, then he believes that all other agents are also in state G. Note that once k_C enters state B, his belief does not matter to his equilibrium strategy any more.

To show that these strategies form a simple equilibrium, first notice that (A1)-(A3) are satisfied. Moreover, the agents' beliefs are consistent with equilibrium strategies. In particular, when a proposed trade is rejected with k_C in state G, it is believed to be a mistake and agents are all in state G and will continue to accept trades and repay from next period on. We use the one-shot-deviation principle to verify sequential rationality. By (A.4) and (A.5) and the previous discussion no agent has incentive to deviate along

the equilibrium path. On the off-equilibrium path, the history is summarized by the configuration of states. For a periphery agent assigned to the liquidity role, because k_C will not accept any trade from an investment agent in state B, he is indifferent between accepting a unsecured trade with k_C and having a secured trade so long as k_C is in state G and it is optimal to reject any other trade (note that a periphery investment agent will not repay any debt incurred through direct trading). For a periphery agent in investment role, as their state only depends on whether they repay k_C, their incentive is determined by (A.4). Note that as they believe all other agents are in state G, the continuation payoff is given by the left side of (A.3). Finally, for k_C, (A.6) determines whether he has incentive to remain in state G or not. ■

Proof of Proposition 3

Let g be a given network with $2n > 6$ agents. For any pair of agents, (j, j'), let $\text{dist}(j, j')$ be the distance between them. Then, if the pair is matched, by (A2), the number of intermediaries between them is $\text{dist}(j, j') - 1$. Define

$$D = \llcorner \frac{\max\{\text{dist}(j, j') : j, j' \in \mathcal{N}\}}{2} \lrcorner,$$

i.e., either $2D$ or $2D + 1$ is the longest distance between any two nodes in g. Thus, $2D - 1 \leq v_{\max} \leq 2D$. Here we assume that the path corresponding to this distance is given by $\mathcal{P} = (j_1, ..., j_{2D})$; the other case is similar. Suppose that the investment level q is implementable under g with financial contract (d, f), $d \geq q + c_m$. Note that by (A2), the investment agent always choose the shortest path to intermediate trades, and hence, when (j_1, j_{2D}) forms a match, they use the path $\mathcal{P}$ with a positive probability. Moreover, if $(j_k, j_{k'})$ forms a match with $1 \leq k < k' \leq 2D$, they use the path $(j_k, ..., j_{k'})$ with a positive probability.

Thus, there exists a realization of matching and proposed paths such that, for each $k = 1, ..., D$, the maximum debt the agent j_k has is at least kd. By symmetry, the maximum sum of total debts $j_1, ..., j_{2D}$ have are at least

$$2 \sum_{k=1}^{D} kd = (D + 1)Dd.$$

However, because all the fees cannot exceed the total future gains from trade relative to secured trades, $\frac{1}{\phi}n\Delta(q)$, we have

$$-D(D+1)d + \frac{1}{\phi}n\Delta(q) \geq 0$$

for the first-best to be implementable, i.e.,

$$\frac{D(D+1)}{n} \leq \frac{\Delta(q)}{\phi d}.$$

The result follows from the fact that $d \geq q + c_m$. Now, note that $\nu_{\max} \leq 2D$ and we obtain the desired result. ■

Proof of Proposition 4

(i) Let g be a given minimally connected network with $2n$ agents under which q is implementable under g with financial contract (d, f), $d \geq q + c_m$. We show that

$$\phi d - \frac{3}{4n}\Delta(q) + 2c_m \leq f \leq \frac{-\phi d + \frac{1}{2}\Delta(q) - c_m}{\frac{n-1}{2n-1} + \phi}. \tag{A.9}$$

By taking n to infinity in the above inequality and replacing d with $q + c_m$, we obtain (4).

First we show the necessity of the first inequality in (A.9). We show this by finding an agent who intermediates "many" unsecured trades.

For each agent j and a neighbor j' to j, define $L_{(j,j')}$ as the number of agents who have a path to j through j', and let $L_j = \max_{j' \text{ links to } j} L_{(j,j')}$. Let χ be the largest integer less than $2n/3$.

Claim 1. There exists an agent j with $L_j < 2n - \chi$.

Proof. We prove the claim by contradiction. Suppose such an agent j does not exist, that is, for any non-leaf agent j, $L_j \geq 2n - \chi$. Then we construct a path that is arbitrarily long. Take an arbitrary non-leaf agent j_1. Then, there exists an agent j_2, a neighbor of j_1, such that $L_{(j_1,j_2)} \geq 2n - \chi$. Then, remove all the agents who have a path to j_1 without passing through j_2, and call the remaining graph g_1. g_1 is then still a minimally connected network with at least $2n - \chi > 4n/3$ agents. Moreover, since $L_{(j_1,j_2)} \geq 2n - \chi > n$, and since $L_{j_2} \geq 2n - \chi$, there exists j_3 such that j_3 is in g_1 and $L_{(j_1,j_2)} \geq 2n - \chi$. Then, remove

all the agents who have a path to j_2 without passing through j_3, and call the remaining graph g_2. Suppose that we have constructed $j_1, ..., j_\nu$ and $j_\nu \in g_{\nu-1}$, with $g_{\nu-1}$ has at least $2n - \chi$ agents. Then, j_ν has a neighbor $j_{\nu+1}$ with $L_{(j_\nu, j_{\nu+1})} \geq 2n - \chi$. Remove all the agents who have a path to j_v without passing through j_{v+1}, and call the remaining graph g_v; note that $j_{\nu+1}$ is in g_v. Therefore, we may continue the process indefinitely, a contradiction to the finiteness of the graph g. $\square$

By Claim 1, there exists an agent j with $L_j < 2n - \chi$. Let $(j_1, ..., j_\nu)$ be his neighbors, ordered in a way such that

$$L_{(j,j_1)} \geq L_{(j,j_2)} \geq .... \geq L_{(j,j_\nu)}.$$

Then we claim that we can find a realization of matching such that j intermediates at least χ matches. Consider two cases.

(a) $2n - \chi > L_{(j,j_1)} \geq \chi$. In this case we can have χ agents that has a path to j through j_1 with investment role, and match each of them to an agent that has a path to j without going through j_1 with liquidity role.

(b) $L_{(j,j_1)} < \chi$. Then, we can find ν^* such that

$$2\chi > L_{(j,j_1)} + ... + L_{(j,j_{\nu^*})} > \chi.$$

Then, we can take χ agents that has a path to j through $j_1, ..., j_{\nu^*}$ with investment role, and match each of them to an agent that has a path to j without going through $j_1, ..., j_{\nu^*}$ with liquidity role.

Now we consider the incentive to repay for the agent j. Let $K \geq \chi$ be the largest number of trades j intermediates. Then, the expected number of fees for j is at most K. Hence, we have

$$-Kd + \frac{\beta}{1-\beta}\left[\frac{1}{2}\Delta(q) + Kf - 2Kc_m\right] \geq 0,$$

that is,

$$f \geq \phi d - \frac{1}{2K}\Delta(q) + 2c_m > \phi d - \frac{3}{4n}\Delta(q) + 2c_m,$$

which gives the first inequality in (A.9).

To prove the necessity of the second inequality in (A.9), consider the incentive for a leaf agent. Since the leaf agent has degree one, he has to pay the fee f with probability $\frac{n-1}{2n-1}$ and he never serves as an intermediary. Thus, it is optimal to repay his largest possible debt, $d+f$, when assigned to the investment role, it must be the case that

$$-(d+f)+\frac{\beta}{1-\beta}\left[\frac{1}{2}\Delta(q)-c_m-\frac{n-1}{2n-1}f\right]\geq 0,$$

and hence

$$f\leq\frac{-\phi d+\frac{1}{2}\Delta(q)-c_m}{\frac{n-1}{2n-1}+\phi}.$$

This proves the second inequality in (A.9).

(ii) Here we choose $\overline{\eta}=1.05$ and $\overline{\nu}=1.05$. Let

$$\widetilde{\Lambda}_n=(2n-1)\frac{2-\overline{\nu}}{6\overline{\eta}}-1. \tag{A.10}$$

For $n\geq 100$, $\widetilde{\Lambda}_n\geq 0.3n$. Let g be a given network with $2n$ agents with $\eta_g\leq\overline{\eta}$ and $\nu_g\leq\overline{\nu}$ under which q is implementable under g with financial contract (d,f), $d\geq q+c_m$. We show that, for $n\geq 100$,

$$\phi d-\frac{1}{0.15n}\Delta(q)+2c_m\leq f\leq\frac{-\phi d+\frac{1}{2}\Delta(q)-c_m}{\frac{n-1}{2n-1}+\phi}. \tag{A.11}$$

By taking n to infinity in the above inequality and replacing d with $q+c_m$, we obtain (4).

To show the first inequality in (A.11), we find an agent whose incentive is similar to that of the center agent in the star network. We first need a claim about existence of an agent j with large degrees.

Claim 2. Let Λ be the maximum degree in g. We show that

$$\Lambda\geq(2n-1)\frac{2-\nu_g}{6\eta_g}-1. \tag{A.12}$$

Proof. Let δ_j be the degree of agent j. Then,

$$\begin{aligned} 2n(2n-1)(\nu_g+1) &\geq 3 \times \left[\sum_{j \in \mathcal{N}} \left(2n-1-\delta_j - \sum_{j' \text{ linked to } j} \delta_{j'} \right) \right] \\ &\geq 3 \times \{2n[2n-1] - 2|\mathcal{E}(g_n)| - 2|\mathcal{E}(g_n)|\Lambda\} \\ &\geq 3 \times (2n) \times [2n-1-2\eta_g(1+\Lambda)]. \end{aligned}$$

Then, (A.12) follows directly by rearranging terms. □

Since $\eta_g \leq \overline{\eta}$ and $\nu_g \leq \overline{\nu}$, (A.12) also implies that $\Lambda \geq \widetilde{\Lambda}_n$. Hence, we can find an agent j who has degree at least $\widetilde{\Lambda}_n \geq 0.3n$. Now, consider, S, the set of j's neighbors. Since by deleting all the links between agents in S the network is still connected, it follows that the number of those links has to be at most

$$2n\overline{\eta} - (2n-1) = 2n(\overline{\eta}-1) + 1 \leq 0.1n + 1.$$

Thus, there are at least $0.15n$ agents in the set S who has no link with any other agent in S. Thus, the maximum number of intermediations for agent j is at least $K \geq 0.15n$. Note that the expected number of fees for j is less than K. To ensure that a simple equilibrium exists, considering j's incentive, it must be the case that

$$-Kd + \frac{\beta}{1-\beta}\left[\frac{1}{2}\Delta(q) + Kf - 2Kc_m\right] \geq 0. \tag{A.13}$$

Since $K \geq 0.15n$, (A.13) implies the first inequality in (A.11).

Now we show the second inequality in (A.11). Since $|\mathcal{E}(g_n)| = 2n\eta_g \leq 2n\overline{\eta} = 2.1n$ and hence the sum of all agents' degrees is less than $4.2n$, and since there exists one agent with degree at least $0.3n$, there exists an agent with degree less than $(4.2-0.3)/2 = 1.95$. Hence, there exists some agent with only one link. Since he has only one link, he cannot serve as an intermediary but has to go through an intermediary with probability at least $\frac{n-1}{2n-1}$, that is, his incentive to repay is exactly the same as a leaf agent in the proof of (i). Hence, his incentive requires

$$-(d+f) + \frac{\beta}{1-\beta}\left[\frac{1}{2}\Delta(q) - \frac{n-1}{2n-1}f - c_m\right] \geq 0. \tag{A.14}$$

By rearranging terms, (A.14) implies the second inequality in (A.11). ■

Proof of Corollary 1

By Proposition 4, if investment level q is asymptotically implementable in $\{g_n\}$, then (4) holds for q, and, by Proposition 2, it is also implementable in $\{g_n^*\}$. Moreover, by the proof of Proposition 4 (the second inequality in (A.9) and (A.11)),

$$f_{g_n}^{\max}(q) \leq \frac{-\phi(q+c_m)+\frac{1}{2}\Delta(q)-c_m}{\frac{n-1}{2n-1}+\phi},$$

and, by the the proof of 2 (the second inequality in (9)),

$$f_{g_n^*}^{\max}(q) = \frac{-\phi(q+c_m)+\frac{1}{2}\Delta(q)-c_m}{\frac{n-1}{2n-1}+\phi}.$$

Thus, $f_{g_n}^{\max}(q) \leq f_{g_n^*}^{\max}(q)$. ■

Proof of Proposition 5

First note that under the star network, g_n^*, with $2n$ agents, $\eta_{g_n^*} = 1 - 1/2n$ and $\nu_{g_n^*} = 1 - 1/n$. Since $q^* = 1$ and hence the first-best level of investment is implementable, the average welfare is given by

$$W^* = \frac{1}{2}\left\{[R(1)-1]+(1+r)-4\left(1-\frac{1}{2n}\right)c_l - 2\left(1-\frac{1}{n}\right)c_m\right\}.$$

Since the star network already implements the first-best level of investment, it remains to show that it minimizes linking costs (both recurrent and idiosyncratic) among all connected networks.

First it is easy to verify that transaction costs are minimized under star among all minimally connected networks.

Next, we show that for any connected network g_n with $2n$ agents,

$$\nu_{g_n} + 1 \geq 2 - \frac{2}{2n-1}\eta_{g_n}.$$

To see this, note that for each agent j, any agent who is directed connected to him has

distance 1 but every other agent has distance at least 2, and hence

$$\begin{aligned}\nu_{g_n}+1 &\geq \frac{\sum_{j\in\mathcal{N}}\{\deg(j)+2[2n-1-\deg(j)]\}}{2n(2n-1)}\\ &= \frac{4n(2n-1)-2(2n)\eta_{g_n}}{2n(2n-1)}\\ &= 2-\frac{2}{2n-1}\eta_{g_n},\end{aligned}$$

where $\deg(j)$ is the degree of agent j.

Thus, the network costs of g_n per capita, denoted by C_n, satisfies

$$C_n = 4\upsilon_{g_n}c_l + 2\eta_{g_n}c_m \geq 4\upsilon_{g_n}c_l + 2\left[2-\frac{2}{2n-1}\eta_{g_n}\right]c_m.$$

Now, let $C_n^* = 4(1-\frac{1}{2n})c_l - 2(2-1/n)c_m$ be the corresponding cost for the star network, we have

$$C_n - C_n^* \geq S(\eta_{g_n},n) \equiv 4\left\{\left[\eta_{g_n}-\left(1-\frac{1}{2n}\right)\right]c_l + \left(\frac{1}{2n}-\frac{1}{2n-1}\eta_{g_n}\right)c_m\right\}.$$

Now, for each n, $S_1(\eta_{g_n},n) = 4\{c_l - \frac{1}{2n-1}c_m\}$. Then, for all $n > N_2$, $S_1(\eta_{g_n},n) > 0$ and hence is strictly increasing in η_{g_n}. Since we are only concerned with networks other than the minimally connected one, we may assume that $\eta_{g_n} \geq 1$. Now, for all $n > N_2$,

$$S(1,n) \equiv 4\left\{\frac{1}{2n}c_l + \left(\frac{1}{2n}-\frac{1}{2n-1}\right)c_m\right\} > 0.$$

This implies that $C_n - C_n^* > 0$. ■

Proof of Proposition 6

Let $\{g_n\}$ be a sequence of networks and let q^* be defined in the main text. Consider two cases. First, suppose that $\eta_{g_n} \leq \overline{\eta}$ and $\nu_{g_n} \leq \overline{\nu}$ infinitely often. Then, by Proposition 4 (ii), $q \leq q^*$. Since, by the arguments in Proposition 5, the star minimizes the linking costs among all connected networks for large n's, the candidate arrangement dominates that sequence. Next, suppose that $\eta_{g_n} > \overline{\eta}$ or $\nu_{g_n} > \overline{\nu}$ for all sufficiently large n. Since $W(1,1,q^*) \geq \max\{W(\overline{\eta},1,1), W(1,\overline{\nu},1)\}$, q^* and the star network performs better for

large n's. ■

Proof of Corollary 2

The aggregate welfare under secured trade and empty network with $2n$ agents is given by

$$W_n^e = \sum_{t=0}^{\infty} \beta^t n \left\{ p[R(1) + r] + (1-p) \right\},$$

and hence

$$W(g_n^*, q^*) - W_n^e \geq \frac{n}{1-\beta} \left\{ \Delta(q^*) - 4c_l - 4c_m \right\},$$

and the last term is strictly positive if q^* is sufficiently close to 1 and if $(c_l + c_m)$ is sufficiently small. ■

Proof of Proposition 7

We consider the following strategies and show they constitute a Nash equilibrium and use a tight trading strategy profile. First, agents never sever existing links or form new links in equilibrium. After any deviation, they also never sever existing links or form new links in equilibrium. In the trading game, all connected agents accept unsecured trades from other connected agents as long as the set of implementable investment levels under g^t at period t is non-empty (and conduct secured trades otherwise), both on and off equilibrium paths.

First we show that the center agent has no incentive to delete any link. We begin with a claim about the implementable investment levels in networks where the center has deleted some of his links.

Claim 1. Let g_K be the resulting network by deleting K links from the star. If the set of implementable investment levels is non-empty in g_K, then, the highest level implementable under g_K, denoted by q_K, satisfies $q_K \leq q_n^*$ for n large.

Proof. First we give necessary conditions for implementability. Fix some candidate investment level and contract, $[q, (d, f)]$. Consider the incentive of a connected periphery:

$$-(d+f) + \frac{\beta}{1-\beta} \left\{ \frac{2n-K-1}{2(2n-1)} \Delta(q) - \frac{2n-K-2}{2(2n-1)} f - \frac{2n-K-1}{2n-1} c_m \right\} \geq 0.$$

This implies that

$$G(f,K) \equiv -\left\{\phi + \frac{2n-K-2}{2(2n-1)}\right\} f - \phi d + \frac{2n-K-1}{2(2n-1)}[\Delta(q) - 2c_m] \geq 0,$$

and that the upper bound for f given q is given by the implicit function $f = g(K)$ such that $G(g(K), K) = 0$. Now, for $K \leq 2n-2$,

$$G_f = -\left[\phi + \frac{2n-K-2}{2(2n-1)}\right] \leq 0$$

and

$$G_K = \frac{1}{2(2n-1)} f - \frac{1}{2(2n-1)}[\Delta(q) - 2c_m].$$

Since $g'(K) = -G_f/G_K$, to show that $g'(K) \leq 0$, it suffices to show that $G_K \leq 0$, that is, $g(K) \leq \Delta(q) - 2c_m$, which in turn is equivalent to

$$-\phi d + \frac{2n-K-1}{2(2n-1)}[\Delta(q) - 2c_m] \leq \left[\phi + \frac{2n-2-K}{2(2n-1)}\right][\Delta(q) - 2c_m].$$

Rearranging the terms and taking $d = q + c_m$, it suffices to show that

$$-\phi(q + c_m) + \frac{1}{2(2n-1)}[\Delta(q) - 2c_m] \leq \phi[\Delta(q) - 2c_m].$$

Note that if $-\phi(q + c_m) + 1/2[\Delta(q) - 2c_m] < 0$, then trade without collateral is not implementable. Let $\underline{q}$ be the lowest q for which $-\phi(q + c_m) + 1/2[\Delta(q) - 2c_m] \geq 0$. Given that $q \geq \underline{q}$, we may replace the right-side with zero and the above inequality holds if

$$2(2n-1)\phi(q + c_m) - [\Delta(q) - 2c_m] \geq 0,$$

which holds for large n and $q \geq \underline{q}$.

Now, consider the incentive for the center agent. We have

$$-\frac{2n-K}{2} d + \frac{\beta(2n-1-K)}{(1-\beta)(2n-1)}\left\{\frac{\Delta(q)}{2} + \frac{2n-2-K}{2} f - (2n-1-K)c_m\right\} \geq 0.$$

This implies that

$$H(f,K) \equiv (2n-2-K)f + \Delta(q) - \frac{(2n-K)(2n-1)}{(2n-1-K)}\phi d - 2(2n-1-K)c_m \geq 0.$$

The lower bound for f given q is then given by the implicit function $f = h(K)$ such that $H(h(K),K) = 0$, and $h'(K) = -H_K/H_f$. Now,

$$H_f = 2n-2-K \geq 0$$

and

$$H_K = -f - \frac{-(2n-1)(2n-1-K) + (2n-K)(2n-1)}{(2n-1-K)^2}\phi d + 2c_m.$$

Note that $h'(K) \geq 0$ if $H_K \leq 0$, which holds if

$$h(K) \geq -\frac{2n-1}{(2n-1-K)^2}\phi d + 2c_m.$$

This holds if

$$-\Delta(q) + \frac{(2n-K)(2n-1)}{2n-1-K}\phi(q+c_m) + 2c_m \geq 0.$$

Again, we have this inequality if $q \geq \underline{q}$ and n large.

Combining these incentives, we have

$$\frac{-\Delta(q) + \frac{(2n-K)(2n-1)}{(2n-1-K)}\phi d + 2(2n-1-K)c_m}{2n-2-K} \leq \frac{-\phi d + \frac{2n-K-1}{2(2n-1)}[\Delta(q) - 2c_m]}{\phi + \frac{2n-K-2}{2(2n-1)}}. \quad \text{(A.15)}$$

By taking derivatives with respective to K, we have verified that the left-hand side is increasing in K while the right-hand side is decreasing in K. Thus, q_K, defined as the maximizer to $\max_q \Delta(q)$ subject to (A.15) with $d = q + c_m$, must satisfy $q_K \leq q_0 = q_n^*$. □

Now, for each K where the set of investment levels is non-empty under g_K, let $\mathcal{C}(g_K) = [q_K, (q_K + c_m, f_K)]$, where f_K corresponds to the left-hand side of (A.15) with $q = q_K$. Note that when q_K may not be sustainable as its benefit may be lower than secured trades; in that case, we have agents trade with collateral and the center receives no fee.

Then, the benefit per period by deleting K links (relative to the star network) is less

than

$$\begin{aligned}
& \left\{ \frac{2n-1-K}{2(2n-1)}\Delta(q_K) + \frac{(2n-1-K)(2n-2-K)}{2(2n-1)} f_K - \frac{(2n-1-K)^2}{2n-1} c_m \right\} \\
- & \left\{ \frac{1}{2}\Delta(q) + (n-1) f_n^{\min} - (2n-1) c_m \right\} + K c_l \\
= & \frac{2n-K}{2}\phi(q_K + c_m) - n\phi(q_n^* + c_m) + K c_l.
\end{aligned}$$

Now, by Claim 1, $q_K \leq q_n^*$; hence,

$$\frac{2n-K}{2}\phi(q_K + c_m) - n\phi(q_n^* + c_m) + K c_l \leq K \left[-\frac{\phi}{2}(q_n^* + c_m) + c_l \right] \leq 0,$$

provided that $c_l \leq \frac{1}{2}\phi(q_n^* + c_m)$. This shows that the center agent does not want to sever any link.

Next, we show that when a pair of two leaf agents are chosen in the linking stage, they have no incentive to form a link. Let g' denote the network by having exactly two periphery agents forming a new link between them.

Claim 2. Let g' be the network by having exactly two periphery agents forming a new link between them, and let q' be the highest level of investment implementable under g'.
(a) Suppose that (4) holds for $q = 1$ with strict inequality. Then, for n large, $q' = 1$ and there is a corresponding fee $f' \geq f_n^*$.
(b) Suppose that (4) does not hold for $q = 1$ with strict inequality. Then, for any fee f' corresponding to q', both $|f' - f_n^*|$ and $|q' - q_n^*|$ converge to zero as n goes to infinity.

Proof. Fix a candidate contract, $[q, (d, f)]$ with $d = q + c_m$. Consider the center agent. His incentive requires

$$-nd + \frac{\beta}{1-\beta}\left\{ \frac{1}{2}\Delta(q) + (2n-3)\frac{2n-2}{2(2n-1)} f + 2\frac{2n-3}{2(2n-1)} f - c_m - \frac{n(2n-3)}{2n-1} c_m \right\} \geq 0.$$

This implies that

$$f \geq \frac{\phi n d - \frac{1}{2}\Delta(q) + c_m + \frac{n(2n-3)}{2n-1} c_m}{\frac{n(2n-3)}{2n-1}}.$$

Consider the two periphery agents who are linked. Their incentives require

$$-(d+f)+\frac{\beta}{1-\beta}\left\{\frac{1}{2}\Delta(q)-\frac{2n-3}{2(2n-1)}f-c_m\right\}\geq 0,$$

and hence

$$f\leq\frac{-\phi(q+c_m)+\frac{1}{2}\Delta(q')-c_m}{\phi+\frac{2n-3}{2(2n-1)}}.$$

Thus, $[q,(d,f)]$ is implementable if and only if

$$\frac{\phi n(q+c_m)-\frac{1}{2}\Delta(q)+c_m+\frac{n(2n-3)}{2n-1}c_m}{\frac{n(2n-3)}{2n-1}}\leq\frac{-\phi(q+c_m)+\frac{1}{2}\Delta(q')-c_m}{\phi+\frac{2n-3}{2(2n-1)}}. \tag{A.16}$$

(a) Note that by taking n to infinity, (A.16) coincides with (4). Hence, if (4) holds for $q=1$ with a strict inequality, (A.16) holds for $q=1$ for large n. Thus, $q'=1$ for large n. However, note that

$$\frac{-\phi(q+c_m)+\frac{1}{2}\Delta(q)-c_m}{\phi+\frac{2n-3}{2(2n-1)}}\geq f_n^{\max}$$

for $q=1$, and hence we can pick a fee $f'\geq f_n^{\max}$.

(b) If (4) fails for $q=1$ with a strict inequality, then (A.16) fails for $q=1$ for large n. Then, for large n, both constraints are binding for the second-best allocations. Since the two conditions, (A.16) and (9), coincide at the limit, the convergence follows. Now, suppose that (4) holds for $q=1$ with an equality. Then both $f_n^{\max}$ and $f_n^{\min}$ converge to the same limit as f'. □

Now, the benefit of forming this new link per period is then less than

$$-c_l+\frac{1}{2}|\Delta(q_n^*)-\Delta(q')|+\frac{1}{2(2n-1)}f+\frac{2n-2}{2(2n-1)}|f_n^*-f'|\equiv -c_l+T(n).$$

However, Claim 2 implies that we can choose f' such that $T(n)\to 0$ as n goes to infinity.

Finally, for a leaf agent to sever a link and trade with collateral, the gain per period is less than

$$-\left[0.5\Delta(q)-c_m-\frac{n-1}{2n-1}f-c_l\right]\leq 0,$$

as long as $c_l\leq\phi(d+f)$ by (9). ■

Implementability for Interlinked Stars with Two Centers

Here we give a characterization of implementable investment levels in the interlinked star network with two centers, g_n^{**}. Note that by comparing the characterization to (9), it is straightforward to verify that the upper bound for the fees is lower under interlinked star than that under star for any given q.

Lemma A.1 *Consider the interlinked star with two centers and with $2n$ agents. Then, $[(d, f), q]$ is implementable if and only if*

$$\frac{\frac{n}{n-1}\phi d - \frac{1}{2(n-1)}\Delta(q) + \frac{3n-1}{2(2n-1)}c_m}{\frac{3n-2}{2(2n-1)} - \phi} \leq f \leq \frac{-\phi d + \frac{1}{2}\Delta(q) - c_m}{2\phi + \frac{3(n-1)}{2(2n-1)}}. \tag{A.17}$$

Proof. First consider the periphery incentive:

$$-(d + 2f) + \frac{\beta}{1-\beta}\left\{\frac{1}{2}\Delta(q) - \frac{n-1}{2(2n-1)}2f - \frac{n-1}{2(2n-1)}f - c_m\right\} \geq 0,$$

and this gives the upper bound on f. Second, consider the center agent's incentive.

$$\begin{aligned} & -nd - (n-1)f + \frac{\beta}{1-\beta}\left\{\frac{1}{2}\Delta(q)\right\} \\ + \quad & \frac{\beta}{1-\beta}\left\{(n-1)\frac{2n-2}{2(2n-1)}f + n\frac{n-1}{2(2n-1)}f - (n-1)c_m - \frac{n(n-1)}{2n-1}c_m\right\} \geq 0. \end{aligned}$$

Now we prove sufficiency. Suppose that (A.17) holds. The financial contracts are given as follows. For unsecured trades on the equilibrium path, let $d = q + c_m$ and let $f \geq 0$ be in between the two numbers in (A.17). For secured trades on off-equilibrium paths, the investment level is $q = 1$, and the investment agent repays 1 to the liquidity agent when the return is $\theta = R(1)$ (when $\theta = 0$, the liquidity agent liquidates the collateral and obtains payoff 1). Given the contracts, the liquidity agent is indifferent between secured and unsecured trades (both of which give him a zero surplus), assuming that the investment agent will repay in unsecured trades.

We construct equilibrium strategies as follows. Each periphery agent can be in one of the two states, G and B, and each center can also be in one of the two states, G and B. At date 0, all agents are in state G. For $l = 1, 2$, a periphery agent in $\mathcal{N}_{k_{Cl}}$ stays in state

G if and only if he repays all his debts to k_{Cl} when assigned to the investment role in the previous periods; otherwise, he enters state B. The state of an agent in $\mathcal{N}_{k_{Cl}}$ is only observable to the center agent k_{Cl}. Similarly, a center agent k_{Cl} stays in state G if and only if he repays all his obligations, including those to $k_{C(-l)}$ and his debtor when assigned to the investment role in the previous periods; otherwise, he enters state B. Any agent who enters state B stays there forever. Note that if a periphery agent trades directly then this choice does not affect his state and a periphery agent's action in liquidity role does not affect his state.

For $l = 1, 2$, the strategy of a periphery agent j_l in $\mathcal{N}_{k_{cl}}$ assigned to the liquidity role in state G is as follows: if k_{Cl} is in state G, then he accepts any trade through k_{Cl}; if k_{Cl} is in state B, he demands collateral for any trade. Moreover, he always asks for collateral if asked to trade directly. A periphery agent j_l in $\mathcal{N}_{k_{Cl}}$ assigned to the liquidity role in state B always asks for collateral.

The strategy of a periphery agent j_l in $\mathcal{N}_{k_{Cl}}$ assigned to investment role is as follows: if both himself and k_{Cl} are in state G, then he propose to trade through k_{Cl} (and $k_{C(-l)}$, if needed) and repay his debt; if himself or k_{Cl} is in state B (or both), he proposes to trade directly, and, if his trade without collateral is accepted (following off-equilibrium behavior), he does not repay anything.

Finally, for $l = 1, 2$, the strategy of k_{Cl} is as follows: if he is in state B, then he never accepts any trade without collateral and he never repays anything. Instead, if he is in state G, he accepts trades from a match $m = (\ell, i)$ if and only if both ℓ and i are in state G (and, when trade has to go through $k_{C(-l)}$, $k_{C(-l)}$ is in state G as well) and rejects it otherwise, and he repays all debts if and only if it is feasible and (i) $k_{C(-l)}$ is in state G and the number of periphery agents in $\mathcal{N}_{k_{Cl}}$ in state G who repays at the current period, denoted by K, and the total obligation k_{Cl} has to agents in state G, denoted by L (including the fees owe to $k_{C(-l)}$), satisfy

$$\begin{aligned} & -\phi L + \frac{1}{2}\left[p(R(q)+r) + (1-p)\left(\frac{K+n}{2n-1}r + \frac{n-1-K}{2n-1}\right) + (1-q)\right] \\ + & \left(\frac{K(K+n)}{2n-1} + \frac{nK}{2n-1}\right) f \geq \frac{1}{2}\left[p(R(1)-r) + (1-p)\right]; \end{aligned}$$

or, (ii) $k_{C(-l)}$ is in state B and the number of periphery agents in $\mathcal{P}_l$ in state G who repays at the current period, denoted by K', and the total obligation k_{Cl} has to agents in state G, denoted by L', satisfy

$$\begin{aligned} & -\phi L' + \frac{1}{2}\left[p(R(q)+r) + (1-p)\left(\frac{K'}{2n-1}r + \frac{2n-1-K'}{2n-1}\right) + (1-q)\right] \\ + \quad & \frac{K'(K'-1)}{2n-1} f \geq \frac{1}{2}\left[p(R(1)-r) + (1-p)\right]. \end{aligned}$$

We also need to construct equilibrium beliefs. For a central agent k_{Cl}, his belief about a periphery agent in $\mathcal{N}_{k_{Cl}}$ or $k_{C(-l)}$ is the actual history, his belief about an agent in $\mathcal{N}_{k_{C(-l)}}$ is that he is in state G if $k_{C(-l)}$ is in state G. For a periphery agent j_l in $\mathcal{P}_l$, his belief is such that if k_{Cl} is in state G, then he believes that all other agents are also in state G (and hence, he treats any rejection of unsecured trades from k_{Cl} as mistakes).

To show that these strategies form a simple equilibrium, first notice that (A1)-(A3) are satisfied. Moreover, the agents' beliefs are consistent with equilibrium strategies. In particular, when a proposed trade is rejected by k_{Cl} in state G, it is believed to be a mistake and agents are all in state G and will continue to accept trades and repay from next period on. We use the one-shot-deviation principle to verify sequential rationality. No agent has incentive to deviate along the equilibrium path. On the off-equilibrium path, the history is summarized by the configuration of states and we verify sequential rationality as follows.

(i) Consider a periphery agent j_l in $\mathcal{N}_{k_{Cl}}$ assigned to the liquidity role. Since k_{C1} and k_{C2} will not accept any trade from an investment agent in state B, he is indifferent between accepting a unsecured trade with k_{Cl} and having a secured trade so long as k_{Cl} is in state G and it is optimal to reject any other unsecured trade (note that a periphery investment agent will not repay any debt incurred through direct trading). Thus, it is optimal for j_l to accept unsecured trades from k_{Cl} when k_{Cl} has state G and to reject unsecured trades from k_{Cl} when k_{Cl} has state B.

(ii) Consider a periphery agent j_l in $\mathcal{N}_{k_{Cl}}$ assigned to the investment role. Since j_k's state only depends on whether they repay k_{Cl} and since j_l will only have secured trades once entering state B, his incentive to repay is determined by the previous arguments.

(iii) Finally, consider the center agent k_{Cl}. By construction, his strategy is optimal. ■

■

Proof of Proposition 8

We use the same candidate equilibrium strategy profile as that in the proof of Proposition 7.

First we show that either center agent does not want to sever any link.

Let g_K be the resulting network when one center severs K links to the peripheries. To ensure repayments, the periphery incentive requires

$$-(d+2f)+\frac{\beta}{1-\beta}\left\{\frac{2n-1-K}{2(2n-1)}\Delta(q)-\frac{(n-1-K)+2(n-1)}{2(2n-1)}f-\frac{2n-1-K}{2n-1}c_m\right\}\geq 0,$$

and this inequality gives an upper bound on f. Using the same arguments as in the proof of Proposition 7, we can show that this upper bound is decreasing in q and in K. Thus, when f^* is close to $f^{\max}$, we can choose the fee under g_K, f_K, to be lower than f^*. Similar arguments hold when severing the link to the other center.

Now, for the incentive to sever links, we distinguish two cases.

(a) The star severs K links with peripheries. The benefit of doing so is less than

$$\begin{aligned} S(K) &= \left[\frac{(n-1-K)(2n-2-K)}{2(2n-1)}+\frac{n(n-1-K)}{2(2n-1)}\right]f^{\max} \\ &- \frac{(3n-2)(n-1)}{2(2n-1)}f^{\max}+Kc_l+\frac{(n-1)(3n-1)}{2(2n-1)}c_m. \end{aligned}$$

Now, $S''(K)=f^{\max}/(2n-1)>0$, and hence it suffices to show that $0\geq S(n-1)$, which holds if

$$c_l\leq 0.7(f+c_m). \tag{A.18}$$

(b) The star severs K links with peripheries, together with the link to the other star. The benefit of doing so is less than

$$\begin{aligned} S_2(K) &= \frac{(n-1-K)(n-1-K)}{2(2n-1)}f^{\max} \\ &- \frac{(3n-2)(n-1)}{2(2n-1)}f^{\max}+(K+1)c_l+\frac{(n-1)(3n-1)}{2(2n-1)}c_m. \end{aligned}$$

Now, $S_2''(K) = f^{\max}/(2n-1) > 0$, and hence it suffices to show that $0 \geq S_2(n-1)$, which holds by (A.18).

Second, we show that each star k_{Cl} does not want to form a new link with a periphery in $j_{-l} \in \mathcal{N}_{k_{C(-l)}}$. Consider the network after that new link. For n large, it can be shown that the constraints for implementability converges to the same limit as the original network, and hence we can choose a fee that is arbitrarily close to the original one for n large. Moreover, if k_{Cl} is linked with j_{-l}, the expected total fee revenue is also arbitrarily close to that in the original network, since the fee is paid by j_{-l}, in either case, if and only if j_{-l} is linked to someone on $\mathcal{N}_{k_{Cl}}$, but k_{Cl} has to pay the information cost. Thus, forming such link is not profitable for k_{Cl} when n is large.

Third, no two peripheries want to form a new link. Again, the incentives in the new network is arbitrarily close to the original one and hence we can pick a fee f' that is arbitrarily close to f. Hence, the benefit of forming this new link per period is then less than

$$-c_l + \frac{1}{2(2n-1)} f + \frac{2n-2}{2(2n-1)} |f - f'| \equiv -c_l + T(n).$$

However, $T(n) \to 0$ as n goes to infinity, and hence it is not profitable to form such a link for n large.

Finally, each $j \in \mathcal{N}_{k_{Cl}}$ does not want to sever his link. This follows exactly the same reasoning as in the proof of Proposition 7. ■