# Indeterminacy in Credit Economies*

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**Abstract**

We characterize the equilibrium set of a two-good pure-credit economy with limited commitment, similar to Sanches and Williamson (2010) and Gu et al. (2013b), under both pairwise and centralized meetings. We show that the set of equilibria derived under "not-too-tight" solvency constraints (Alvarez and Jermann, 2000) commonly used in the literature is of measure zero in the whole set of Perfect Bayesian Equilibria. There exist a continuum of endogenous credit cycles of any periodicity and a continuum of sunspot equilibria, irrespective of the assumed trading mechanism. We uncover empirically relevant equilibria that do not emerge under "not-too-tight constraints" such as those in which credit limits grow endogenously over time and those in which credit shuts down periodically. Moreover, we provide examples of credit cycles that dominate all equilibria with "not-too-tight" solvency constraints.

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# 1 Introduction

The inability of individuals to commit to honor their future obligations is a key friction of decentralized economies that jeopardizes the Arrow-Debreu apparatus based on promises to deliver goods at different dates and in different states. Stark examples are pure currency economies where anonymity and lack of commitment make credit infeasible. Arguably, pure currency economies have become less relevant due to advances in record-keeping technologies that facilitate the use of credit. Yet, monitoring technologies do not purge economies from the limited commitment problem—they do not make individuals entirely trustworthy.

Recent contributions in the New Monetarist literature emphasize the existence of multiple equilibria in economies with limited commitment. Sanches and Williamson (2010) prove the existence of two steady states, one with a positive level of credit and one with a complete credit shut-down. Gu et al. (2013b), GMMW thereafter, generalize this finding by showing the existence of periodic equilibria for some trading mechanisms. A noticeable implication of their theory is that cycles are more prevalent in pure monetary economies than in pure credit economies. Both papers follow Alvarez and Jermann, (2000), AJ thereafter, and impose "not-too-tight" solvency constraints according to which in every period agents can issue the maximum amount of debt that is incentive-compatible with no default. Such constraints, however, arbitrarily restrict the equilibrium set. AJ rationalize them by showing that they implement constrained-efficient allocations for their pure exchange, one-good economy. But there is no presumption that such rationale holds for a production economy with two-goods, such as the New Monetarist environment. Hence, instead of imposing arbitrary solvency constraints, the objective of this paper is to give a complete game-theoretic characterization of the (perfect Bayesian) equilibrium set of a New-Monetarist pure credit economy.[1]

The economy features random matching—in pairwise meetings or in large groups—and incorporates intertemporal gains from trade that can be exploited with one-period debt contracts. In the absence of public record keeping, the environment corresponds to the framework of Lagos and Wright (2005) so that one can easily compare equilibrium allocations in credit and monetary economies. In the presence of a public record-keeping technology the environment is borrowed from the one in GMMW.[2]

We start with a simple mechanism where the borrower in each bilateral match sets the terms of the loan contract unilaterally, which allows us to analyze the economy as a standard infinitely-repeated game with imperfect monitoring. If we impose the AJ "not-too-tight" solvency constraints exogenously—which amounts

[1] In Wicksell's (1936) words about pure credit economies, "a thorough analysis of this purely imaginary case seems to me to be worth while, for it provides a precise antithesis to the equally imaginary case of a pure cash system, in which credit plays no part whatever."

[2] As we discuss later in details, there are differences regarding the timing of production that are inconsequential. While the competitive version of our model is closely related to AJ, it differs in important ways, including preferences over two goods.

to restricting strategies and beliefs such that any form of default is punished with permanent autarky—then there is a unique active steady-state equilibrium and a continuum of equilibria with decreasing output levels, but no equilibrium with endogenous cycles. The set of all perfect Bayesian equilibria is much bigger. It contains a continuum of steady-state equilibria and a continuum of periodic equilibria of any periodicity. Each equilibrium can be reduced to a sequence of debt limits, where the debt limit in a period specifies the amount that agents can be trusted to repay. These results are robust to the choice of the mechanism to determine the terms of the loan contract—Nash or proportional bargaining, or even competitive pricing if agents meet in large groups.

We construct the set of all equilibrium outcomes by using simple automata. Simple strategies specify that borrowers always repay up to some endogenous limit in order to remain in good standing with future lenders. If an agent's debt is larger than this limit (an out-of-equilibrium event), they default partially, whereas if their debt is lower than the limit, they repay in full. Lenders accept all loan contracts that specify principals that are consistent with full repayment. We prove that these simple strategies form a PBE and that *all* equilibrium outcomes can be obtained from such strategies.

Why is this larger set of credit equilibria relevant? First, it is important to characterize the full set of equilibrium of a pure credit economy based on the standard equilibrium notion (essentially, subgame perfection) used in monetary theory and repeated games. By imposing arbitrary "not-too-tight" solvency constraints the literature has focused on a narrow set of equilibrium outcomes of measure zero in the set of all equilibria. However, the large multiplicity of equilibria does not imply that everything goes. Fundamentals, including preferences and market structure, do matter for an outcome to be consistent with an equilibrium. We show that the set of credit-cycle equilibria expands as trading frictions are reduced, agents are more patient, and borrowers have more bargaining power (in the version of the model with bargaining).

Second, focusing on equilibria supported by "not-too-tight" solvency constraints set cannot be justified by standard refinements.[3] These equilibria do not have better normative properties; we provide examples where there are a continuum of equilibria that dominate from a social welfare point of view the best equilibrium under "not-too-tight" solvency constraints.[4] Additionally, all our equilibrium outcomes correspond to outcomes of equilibria that are weakly renegotiation-proof.

Finally, by considering the full set of equilibria, we uncover equilibria that are consistent with empirical evidence. We show there are equilibria featuring a secular growth of unsecured credit, with monotone-

[3] To be clear, we are not advocating the use of any refinement beyond the standard notion of subgame perfection since it is the equilibrium concept used in all monetary theory. Refining the equilibrium set could lead to misleading comparisons of allocations between pure currency and pure credit environments.

[4] We generalize this result in a companion paper (Bethune et al., 2017) where we study, in detail, the planner's problem subject to incentive constraints.

increasing debt limits as seen in the US after 1980. There are equilibria featuring periodic credit booms and busts where credit completely dries up and recovers later on. There are asymmetric equilibria with rich distributions of debt limits across agents. From a theory standpoint, "not-too-tight" constraints exclude outcomes of the pure monetary economy (with fiat money but no record keeping) even though they form a strict subset of the outcomes of the pure credit economy (with record keeping but no fiat money). So the result according to which cycles should be less prevalent in pure credit economies is the direct consequence of the imposition of "not-too-tight" solvency constraints. Indeed, all dynamic allocations of monetary economies (e.g., cycles, chaos) are also outcomes in PBE of pure credit economies.

## 1.1 Related literature

We adopt an environment similar to the pure currency economy of Lagos and Wright (2005) and Rocheteau and Wright (2005), but we replace currency with a public record-keeping technology, as in Sanches and Williamson (2010, Section 4). Our paper extends the analysis of Sanches-Williamson which focuses on steady states and of GMMW which focuses on cycles. In both cases, the equilibrium notion considered imposes the "not-too-tight" solvency constraints of AJ. Instead, we present our model as a repeated game with imperfect monitoring with few restrictions on strategies and beliefs (the same restrictions typically imposed on equilibria of pure currency economies).[5] In addition, we consider both stationary and non-stationary equilibria (including endogenous cycles and sunspots), and various trading mechanisms (ultimatum games, axiomatic bargaining solutions, competitive pricing). Our methods to characterize equilibrium outcomes are related but differ from the ones used by Abreu (1988) and Abreu et al. (1990) as our stage game has an extensive form and only buyers/borrowers are (imperfectly) monitored.

Kocherlakota (1998) shows that the set of implementable outcomes of monetary economies is a subset of the implementable outcomes of pure credit economies.[6] We find a similar result, but in contrast to Kocherlakota, we take the trading mechanism as given and we do not restrict outcomes to stationary ones.

Our paper is part of the literature on limited commitment in macroeconomics. Seminal contributions on risk sharing in endowment economies where agents lack commitment include Kehoe and Levine (1993), Kocherlakota (1996), and AJ. Kocherlakota (1996) adopts a mechanism design approach in a two-agent economy with a single good. In contrast we study a two-good production economy where a continuum of agents search for new partners every period.

[5] Repeated games where agents are matched bilaterally and at random and change trading partners over time are studied in Kandori (1992) and Ellison (1994). A thorough review of the literature is provided by Mailath and Samuelson (2006).

[6] Hellwig and Lorenzoni (2009) study an environment similar to AJ and show that the set of equilibrium allocations with self-enforcing private debt is equivalent to the allocations that are sustained with money.

## 2 Description of the game

Time is discrete and starts with period 0. Each date has two stages. The first stage will be referred to as the DM (decentralized market) while the second stage will be referred to as the CM (centralized market). There is a single, perishable good at each stage and the CM good will be taken as the numéraire. There is a continuum of agents of measure two divided evenly into a subset of buyers, $\mathbb{B}$, and a subset of sellers, $\mathbb{S}$.[7] The labels "buyer" and "seller" refer to agents' roles in the DM: only the sellers can produce the DM good (and hence will be lenders) and only the buyers desire DM goods (and hence will be borrowers). In the DM, a fraction $\alpha \in (0, 1]$ of buyers meet with sellers in pairs. (We consider a version of the model with large meetings later.) The CM will be the stage where agents settle debts.

Preferences are additively separable over dates and stages. The DM utility of a seller who produces $y \in \mathbb{R}_+$ is $-v(y)$, while that of a buyer who consumes $y$ is $u(y)$, where $v(0) = u(0) = 0$, $v$ and $u$ are strictly increasing and differentiable with $v$ convex and $u$ strictly concave, and $u'(0) = +\infty > v'(0) = 0$. Moreover, there exists $\tilde{y} > 0$ such that $v(\tilde{y}) = u(\tilde{y})$. We denote by $y^* = \arg\max\,[u(y) - v(y)] > 0$ the quantity that maximizes the match surplus. The utility of consuming $z \in \mathbb{R}$ units of the numéraire good is $z$, where $z < 0$ is interpreted as production.[8] Agents' common discount factor across periods is $\beta \equiv 1/(1 + r) \in (0, 1)$.

With no loss in generality, we restrict our attention to intra-period loans issued in the DM and repaid in the subsequent CM.[9] The terms of the loan contracts are determined according to a simple protocol whereby buyers make take-it-or-leave-it offers to sellers. We describe alternative mechanisms later in the paper. Agents cannot commit to future actions. Therefore, the repayment of loans in the CM has to be self-enforcing.

There is a technology allowing loan contracts in the DM and repayments in the CM to be publicly recorded. The entry in the public record for each loan is a triple, $(\ell, x, i)$, composed of the size of the loan negotiated in the DM in terms of the numéraire good, $\ell \in \mathbb{R}_+$, the amount repaid in the CM, $x \in \mathbb{R}_+$, and the identity of the buyer, $i \in \mathbb{B}$. If no credit is issued in a pairwise meeting, or if $i$ was unmatched, the entry in the public record is $(0, 0, i)$. The record is updated at the end of each period $t$ as follows:

$$\rho^i_{t+1} = \rho^i_t \circ (\ell_t, x_t, i), \tag{1}$$

[7] The assumption of ex-ante heterogeneity among agents is borrowed from Rocheteau and Wright (2005). Alternatively, one could assume that an agent's role in the DM is determined at random in every period without affecting any of our results.

[8] Kehoe and Levine (1993) and AJ consider pure exchange economies. One could reinterpret our economy as an endowment economy as follows. Suppose that sellers receive an endowment $\bar{y}$ in the DM and $\bar{z}$ in the CM. Buyers have no endowment in the DM but an endowment $\bar{z}$ in the CM. The DM utility of the seller is $w(c)$ where $w$ is a concave function with $w'(\bar{y}) = 0$. Hence, the opportunity cost to the seller of giving up $y$ units of consumption is $v(y) = w(\bar{y}) - w(\bar{y} - y)$.

[9] Under linear payoffs in the CM one-period debt contracts are optimal, i.e., agents have no incentives to smooth the repayment of debt across multiple periods. This assumption will facilitate the comparison with pure monetary economies of the type studied in Lagos and Wright (2005).

where $\rho_0^i = (\ell_0, x_0, i)$. The list of records for all buyers, $\rho_t = \langle \rho_t^i : i \in \mathbb{B} \rangle$, is public information to all agents.[10] Agents have private information about their trading histories that are not recorded; in particular, if $\rho_t^i = (0, 0, i)$, then agents other than $i$ do not know whether $i$ was matched but his offer got rejected (in that case, the offer made is not observed either) or was unmatched. However, as discussed later, this private information plays no role in our construction of equilibria.

# 3 The equilibrium set

For each buyer $i \in \mathbb{B}$, a strategy, $s^i$, consists of two functions $s_t^i = (s_{t,1}^i, s_{t,2}^i)$ at each period $t$ conditional on being matched: $s_{t,1}^i$ maps his private trading history, $h_t^i$, and public records of other buyers, $\rho_t^{-i}$, to an offer to the seller, $(y_t, \ell_t)$; $s_{t,2}^i$ maps $((h_t^i, \rho_t^{-i}), (y_t, \ell_t))$, together with the seller's response, to his CM repayment, $x_t$. For each seller $j \in \mathbb{S}$, a strategy, $s^j$, consists of one function at each period $t$, conditional on being matched with buyer $i$: $s_t^j$ maps the seller's private trading history, $h_t^j$, the buyer's identity and public records, $(i, \rho_t^i, \rho_t^{-i})$, and his current offer, $(y_t, \ell_t)$, to a response, *yes* or *no*. We restrict our attention to perfect Bayesian equilibria (see Osborne and Rubinstein, 1994, Definition 232.1) satisfying the following conditions:

(A1) **Public strategies.** In any DM meeting the strategies only depend on histories that are common knowledge in the match, including the buyer's public trading history, his offer and the seller's response in the current match, but not on private histories (nor the public records of other buyers).[11]

(A2) **Symmetry.** All buyers adopt the same strategy, $s^b$, and all sellers adopt the same strategy, $s^s$. Moreover, the buyer's offer strategy, $s_{t,1}^b$, is constant over all public trading histories of the buyer that are consistent with equilibrium behavior, in particular, equilibrium offers at date-$t$ are independent of matching histories.

(A3) **Threshold rule for repayments.** For each buyer $i$ and each date $t$ following any history, there exists a number, $d_t$, such that $d_t$ is weakly larger than the equilibrium loan amount at date $t$, and $s_{t,2}^b(\rho_t^i, (y_t, \ell_t), yes) = \ell_t$ if $\ell_t \leq d_t$ and if $\rho_t^i$ is consistent with equilibrium behavior.

We call a perfect Bayesian equilibrium, $(s^b, s^s)$, satisfying conditions (A1)-(A3) above a *credit equilibrium*. A few remarks are in order about these conditions. Our record-keeping technology does not record all actions taken by the agents. Agents have private information about the number of matches they had, quantities they consumed, or offers that were rejected. Because of this private information using perfect Bayesian equilibrium

[10] We could make alternative assumptions regarding what is recorded in a match. For instance, the technology could also record the output level, $y$, together with the promises made by the buyer, i.e., $\rho^i = (y, \ell, x, i)$. Not surprisingly, this would expand the set of equilibrium outcomes. Moreover, we could assume that the seller only observes the record of the buyer he is matched with, $\rho^i$, without affecting our results.

[11] For a formal definition of public strategies see Definition 7.1.1 in Mailath and Samuelson (2006).

(PBE) as the solution concept is both standard and necessary. Alternatively, one may assume that all actions are observable, and PBE is reduced to subgame perfection. Although we prefer our environment, which is closer to the existing literature on monetary economics, our multiplicity result does not rely on the presence of private information. In fact, because of our focus on public strategies, (A1), any PBE we construct is also a subgame-perfect equilibrium (SPE) if all actions were observable.[12] However, agents' beliefs about how other agents will respond to deviations do matter but they are pinned down by equilibrium strategies.

Conditions (A1) and (A2) imply that, for any credit equilibrium, its outcomes are characterized by $\{(y_t, \ell_t)\}_{t=0}^{+\infty}$, the sequence of equilibrium offers made by buyers. Moreover, (A3) implies that $x_t = \ell_t$ for each $t$, and hence the sequence $\{(y_t, \ell_t)\}_{t=0}^{+\infty}$ also determines the equilibrium allocation. Without (A1), equilibrium offers may depend on the buyer's past matching histories.[13] Condition (A3) is not vacuous either. It restricts sellers to believe that buyers will repay their debt when observing a deviating offer with obligations smaller than those in equilibrium.[14] This restriction will rule out inefficiently large trades. As we will see later, taken together the restrictions (A1)-(A3) will allow us to obtain a simple representation of credit equilibria with solvency constraints added to the bargaining problem.

Let $\{(y_t, \ell_t)\}_{t=0}^{+\infty}$ be a sequence of equilibrium offers. Along the equilibrium path the lifetime expected discounted utility of a buyer at the beginning of period $t$ is

$$V_t^b = \sum_{s=0}^{\infty} \beta^s \alpha[u(y_{t+s}) - \ell_{t+s}]. \tag{2}$$

In each period $t+s$ the buyer is matched with a seller with probability $\alpha$ in which case the buyer asks for $y_{t+s}$ units of DM output in exchange for a repayment of $\ell_{t+s}$ units of the numéraire in the following CM and the seller agrees. In any equilibrium $-\ell_t + \beta V_{t+1}^b \geq 0$, which simply says that a buyer must be better off repaying his debt and going along with the equilibrium rather than defaulting on his debt and offering no-trade in all future matches, $(y_{t+s}, \ell_{t+s}) = (0, 0)$ for all $s > 0$. By a similar reasoning the lifetime expected utility of a seller along the equilibrium path is

$$V_t^s = \sum_{s=0}^{\infty} \beta^s \alpha[-v(y_{t+s}) - \ell_{t+s}]. \tag{3}$$

The seller's participation constraint in the DM requires $-v(y_t) + \ell_t \geq 0$ since a seller can reject a trade without fear of retribution (since he is not monitored.) Given that buyers set the terms of trade unilaterally, and the output level is not part of the record $\rho^i$, this participation constraint holds at equality. Our first

[12] In such an equilibrium sellers' beliefs about buyers' private information are irrelevant for their decisions to accept or reject offers. Hence, actions that correspond to agents' private trading histories would not matter even if they were publicly observable.

[13] Obviously, when $\alpha = 1$, the matching-history-independence element in (A1) is vacuous. However, when $\alpha < 1$, it would be difficult to fully characterize all equilibrium outcomes without (A1) but it certainly adds many more equilibria.

[14] Without this restriction one could sustain equilibria in which $y_t > y^*$ for some $t$; to do so, one can adopt a strategy that triggers a permanent autarky for the buyer if his offer $\ell_t$ is smaller than the equilibrium one.

proposition builds on these observations to characterize outcomes of credit equilibria.

**Proposition 1** *A sequence, $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$, is a credit equilibrium outcome if and only if, for each $t = 0, 1, ...,$*

$$\ell_t \leq \sum_{s=1}^{\infty} \beta^s \alpha [u(y_{t+s}) - \ell_{t+s}] \tag{4}$$

$$\ell_t = x_t = v(y_t) \leq v(y^*). \tag{5}$$

As mentioned earlier, a sequence of equilibrium offers, $\{(y_t, \ell_t)\}_{t=0}^{+\infty}$, also determines the sequence of allocations, $\{(y_t, x_t)\}_{t=0}^{+\infty}$, with $x_t = \ell_t$ for each $t$, and hence, Proposition 1 also gives a characterization of allocations that can be sustained in a credit equilibrium. Condition (4), which follows directly from (2) and the incentive constraint $-\ell_t + \beta V_{t+1}^b \geq 0$, is analogous to the participation constraint (IR) in Kehoe and Levine (1993), and the participation constraint in Proposition 2.1 in Kocherlakota (1996). However, while Kehoe and Levine assume the IR constraint from the outset as a primitive condition, (4) is derived as an equilibrium condition in our framework. The condition (5) is the outcome of the buyer take-it-or-leave-it offer and pairwise Pareto efficiency (which follows from the threshold rule A3).[15]

Proposition 1 shows that the conditions (4)-(5) are not only necessary but also sufficient for an equilibrium by constructing a simple equilibrium strategy profile. This strategy profile relies on punishments—the "penal code" in Abreu's (1988) terminology—for both default and excessive lending.[16] Specifically, buyers can be in two states at the beginning of period $t$, $\chi_{i,t} \in \{G, A\}$, where $G$ means "good standing" and $A$ means "autarky" and each buyer's initial state is $\chi_{i,0} = G$. The law of motion of a buyer $i$'s state following a loan and repayment $(\tilde{\ell}, \tilde{x})$ is given by:

$$\chi_{i,t+1}\left(\tilde{\ell}, \tilde{x}, \chi_{i,t}\right) = \begin{cases} A & \text{if } \tilde{x} < \min(\tilde{\ell}, \ell_t) \text{ or } \chi_{i,t} = A \\ G & \text{otherwise} \end{cases}, \tag{6}$$

where $(\tilde{\ell}, \tilde{x})$ might differ from the loan and repayment along the equilibrium path, $\ell_t = x_t$. In order to remain in good standing, or state $G$, the buyer must repay his loan, $\tilde{x} \geq \tilde{\ell}$, if the size of the loan is no greater than the equilibrium loan size, $\tilde{\ell} \leq \ell_t$, and he must repay the equilibrium loan size, $\tilde{x} \geq \ell_t$, otherwise.[17] The autarky state, $A$, is absorbing: once a buyer becomes untrustworthy, he stays untrustworthy forever. Sellers

[15] To derive these conditions formally one has to use the assumption that $y_t$ is not publicly recorded—only the loan contract is—and the threshold property in (A3). See proof of Proposition 1.

[16] There are different approaches for finding equilibria of repeated games. Abreu et al. (1990) introduce the idea of self-generating set of equilibrium payoffs while Abreu (1988) introduces the notion of simple strategies. See Mailath and Samuelson (2006, Section 2.5) for a review of these approaches. We use a related but different approach from the one of Abreu (1988) as our stage game has an extensive form and only a subset of the agents (the buyers) are monitored.

[17] Note that the buyer can remain in state $G$ even if he does not pay his debt in full, and hence default is with respect to the common belief that buyers repay up to the size of the equilibrium loan. Also, notice that there are alternative strategy profiles that deliver the same equilibrium outcome. For instance, an alternative automaton is such that the transition to state $A$ only occurs if $\tilde{x} < \tilde{\ell} \leq \ell_t$. If a loan such that $\tilde{\ell} > \ell_t$ is accepted, then the buyer can default without fear of retribution.

cannot be punished in future periods for accepting a loan larger than $\ell_t$ since their identity is not recorded. However, they are punished in the current period because buyers are allowed to partially default on loans larger than $\ell_t$ while keeping their good standing with future lenders.

The strategies, $(s^b, s^s)$, depend on the buyer's state as follows. The seller's strategy, $s^s_t$, consists of accepting all offers, $(\tilde{y}, \tilde{\ell})$, such that $v(\tilde{y}) \leq \min\{\tilde{\ell}, \ell_t\}$ provided that the buyer's state is $\chi_{i,t} = G$. The buyer repays $s^b_{t,2} = \min\{\ell_t, \tilde{\ell}\}$ if he is in state $G$, and he does not repay anything otherwise, $s^b_{t,2} = 0$. These strategies are depicted in Figure 1 where $(y_t, \ell_t)$ is the offer made by a buyer in state $G$ along the equilibrium path and $(\tilde{y}, \tilde{\ell})$ is any offer. By the one-stage-deviation principle it is then straightforward to show that any $\{(y_t, \ell_t)\}_{t=0}^{\infty}$ that satisfies (4)-(5) is an outcome for the strategy profile $(s^b, s^s)$.

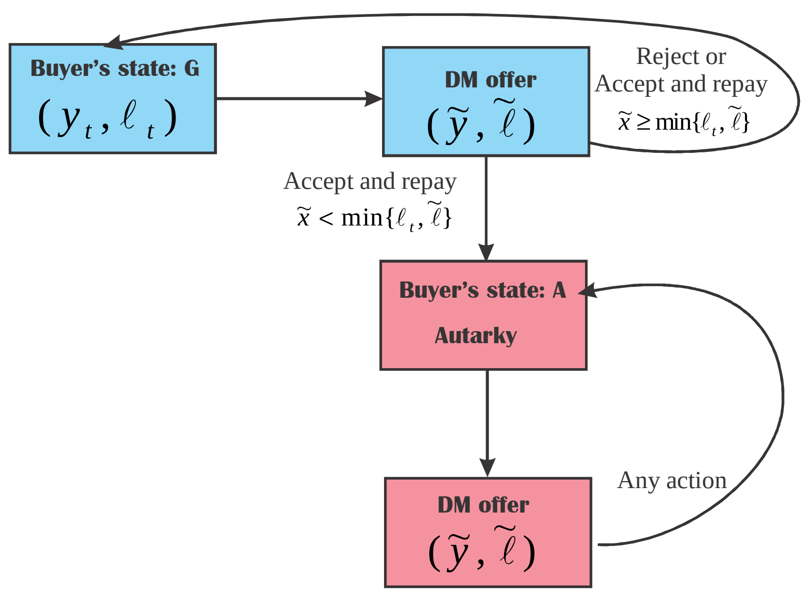


Figure 1: Automaton representation of the buyer's strategy

In the following we propose an alternative formulation of a credit equilibrium in terms of solvency constraints imposed on the bargaining problems in the DM. As in AJ in the context of an economy with competitive trades, a solvency constraint specifies an upper bound—called a *debt limit*—on the quantity of debt an agent can issue, $\ell \leq d_t$. According to this formulation, the buyer in a DM match sets the terms of the loan contract so as to maximize his surplus, $u(y) - \ell$, subject to the seller's participation constraint and the solvency (or borrowing) constraint, $\ell \leq d_t$, i.e.,

$$\max_{y,\ell} \{u(y) - \ell\} \quad \text{s.t.} \quad -v(y) + \ell \geq 0 \quad \text{and} \quad \ell \leq d_t. \tag{7}$$

The solution to (7) is $\ell_t = v(y_t)$ where

$$y_t = z(d_t) \equiv \min\{y^*, v^{-1}(d_t)\}. \tag{8}$$

The solvency constraint is reminiscent to the feasibility constraint in monetary models (e.g., Lagos and Wright, 2005) according to which buyers in bilateral matches cannot spend more than their real balances.

We say that a sequence of debt limits, $\{d_t\}_{t=0}^{\infty}$, is consistent with a credit equilibrium outcome, $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$, if $(y_t, \ell_t)$ is a solution to the bargaining problem, (7), given $d_t$ for all $t \in \mathbb{N}_0$, and the buyer's CM strategy consists of repaying his debt up to $d_t$ provided that his past public histories (up to period $t-1$) are consistent with equilibrium behavior, that is, the sequence $\{d_t\}$ satisfies condition (A3).

It is easy to check from the proof of Proposition 1 that any credit equilibrium outcome, $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$, is consistent with the sequence of debt limits, $\{d_t\}_{t=0}^{\infty}$, such that $d_t = \ell_t$ for all $t \in \mathbb{N}_0$. But the same equilibrium outcome may be implementable by multiple debt limits if (9) is slack and $y_t = y^*$. The following corollary summarizes these results and reduces a credit equilibrium to a sequence of debt limits, $\{d_t\}_{t=0}^{\infty}$, that satisfies a sequence of participation constraints.

**Corollary 1** ***(Equilibrium representation with debt limits)*** *A sequence of debt limits, $\{d_t\}_{t=0}^{\infty}$, is consistent with a credit equilibrium outcome if and only if*

$$d_t \leq \sum_{s=1}^{\infty} \beta^s \alpha [u(y_{t+s}) - v(y_{t+s})] \tag{9}$$

$$v(y_t) = \min\{d_t, v(y^*)\}. \tag{10}$$

Corollary 1 gives a complete characterization of equilibrium outcomes using debt limits. Indeed, by (10), $y_t$ is determined by $d_t$, and hence (9) can be viewed as an inequality that involves $\{d_t\}_{t=0}^{\infty}$ as the only endogenous variables. Without the danger of confusion, we also call a sequence of debt limits, $\{d_t\}_{t=0}^{\infty}$, a credit equilibrium if it satisfies (9) and (10). The next Corollary provides a sufficient condition for a credit equilibrium in recursive form.

**Corollary 2** ***(Recursive sufficient condition)*** *Any bounded sequence, $\{d_t\}_{t=0}^{+\infty}$, that satisfies*

$$d_t \leq \beta \left\{ \alpha \left[ u(y_{t+1}) - v(y_{t+1}) \right] + d_{t+1} \right\}, \tag{11}$$

*where $v(y_t) = \min\{d_t, v(y^*)\}$, is a credit equilibrium.*

The left side of (11) is the cost of repaying the current debt limit while the right side of (11) is the benefit which has two components: the expected match surplus of a buyer who has access to credit and his continuation value given by the debt limit next-period. We represent the right side of (11) by the red curve in Figure 2. Any $\{d_t\}$ located above this curve is a credit equilibrium.

The following proposition characterizes all equilibria with "not-too-tight" solvency constraints, meaning $d_t$ is the largest debt limit that solves the buyer's CM participation constraint, (9), at equality. Graphically,

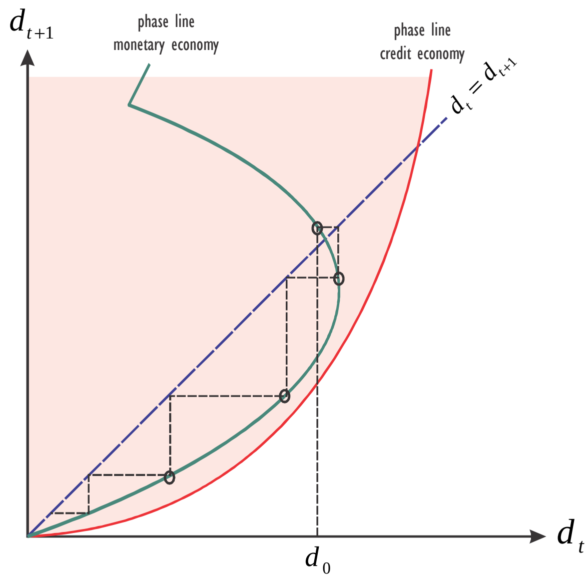


Figure 2: Phase lines

in Figure 2 $\{d_t\}$ must lie exactly on the red curve. Let $d^{\max}$ denote the unique positive root to $rd^{\max} = \alpha\{u[z(d^{\max})] - v[z(d^{\max})]\}$.

**Proposition 2** ***(Symmetric Equilibria under "not-too-tight" constraints.)*** *There are three types of symmetric equilibria under "not-too-tight" solvency constraint: (i)* $d_t = 0$ *for all* $t$*; (ii)* $d_t = d^{\max}$ *for all* $t$*; (iii)* $d_0 \in (0, d^{\max})$ *and* $\{d_t\}$ *is strictly monotone decreasing.*

The restriction to "not-too-tight" solvency constraints does not narrow down the equilibrium set to a single element. As shown in Sanches and Williamson (2010), there are two steady states, one without credit and one with the highest incentive-feasible debt limit, $d^{\max}$. In Figure 2 they correspond to the intersections between the phase line of the credit economy and the 45° line. As shown by Bloise et al. (2013) for a one-good competitive economy, there is a also a continuum of non-stationary equilibria where the initial debt limit is between 0 and $d^{\max}$ and the debt limit falls over time.[18] However, there are no equilibria with cycles or fluctuations, in contrast to equilibrium outcomes of pure monetary economies (e.g., Lagos and Wright, 2003).

[18] Bloise et al. (2013) prove indeterminacy of competitive equilibrium in sequential economies under "not-too-tight" solvency constraints. They show that for any value of social welfare in between autarky and constrained optimality, there exists an equilibrium attaining that value.

# 4 Renegotiation proofness

We show in this section that all the credit equilibrium outcomes we constructed are also outcomes of weakly renegotiation-proof equilibria (Farrell and Maskin, 1989). A WRP equilibrium is a subgame perfect equilibrium where any two continuation payoffs are not Pareto-rankable. Intuitively, if there were such Pareto-dominated equilibrium in any subgame, it seems at some heuristic level that agents should be able to renegotiate and coordinate on the continuation equilibrium that Pareto dominates. Since WRP is formulated for games with perfect information and with two players, to apply this concept, we consider a two-player version of our game, but as we explain later the concepts can be carried over to the general environment. In this version, there is only one buyer and one seller, and they meet at each period with probability $\alpha$. Note that, however, all credit equilibria remain SPE in this two-player version. Finally, since we have two stages at each period, we apply the requirement of WRP to the continuation values at the beginning of the DM's.

Consider a stationary credit equilibrium with debt limit $d$ and DM output, $y$. In our constructed equilibrium strategy, buyers are either in good standing or bad standing with corresponding value functions $V^b$ and $\hat{V}^b$. With autarky as the punishment (which was part of the construction of an equilibrium ), $\hat{V}^b = 0$. In good standing, the continuation value of the buyer at the beginning of DM is $V^b = \alpha\left[u(y) - v(y)\right]/(1-\beta)$ and the continuation value of the seller is $V^s = 0$ since repayment is $x = v(y)$. The repayment constraint is

$$-v(y) + \beta V^b \geq \beta \hat{V}^b \Longleftrightarrow v(y) \leq d \equiv \beta V^b.$$

Recall that we use this simple strategy to recover all equilibrium outcomes. However, this simple equilibrium strategy does not satisfy WRP, which requires that continuation values in different subgames are not Pareto ranked. Indeed, the continuation values following the buyer's bad standing, $\hat{V}^b = 0$ and $\hat{V}^s = 0$, are Pareto dominated by the one following good standing, $V^b > 0$ if $d > 0$ and $V^s = 0$.

Nevertheless, for any stationary credit equilibrium outcome $d \in [0, d^{\max}]$, we can construct a WRP equilibrium with the same equilibrium allocation as follows. A buyer in bad standing can retain a good standing if and only if the following happens. First, the buyer has a successful DM meeting. Second, his offer $(y, x)$ has to satisfy $x \geq d \equiv \beta V^b$ and $y = 0$. Intuitively, the buyer repays at least the value from being in good standing without asking for any consumption. Finally, the seller has to accept the offer and the buyer has to repay $d$ in the CM. In equilibrium, the buyer in bad standing offers $(0, d)$, the seller accepts it, and the buyer repays $d$ and regains a good standing. Strategies on the equilibrium path are as before. The constructed strategies form a SPE. First, the continuation value for a buyer with bad standing, denoted by

$\hat{V}^b$, remains zero if he follows the proposed strategy:

$$\hat{V}^b = \alpha\{-d + \beta V^b\} + (1-\alpha)\beta\hat{V}^b = (1-\alpha)\beta\hat{V}^b,$$

and hence $\hat{V}^b = 0$. This also implies the buyer is weakly better off making the offer $(0, d)$ than any other offer (which will be either rejected or has higher repayment). The seller is also willing to accept the offer. In this proposed equilibrium, no two continuation values can be (strictly) Pareto ranked. When in good standing, the seller's continuation value is $V^s = 0$; in bad standing, the seller's continuation value is $\hat{V}^s = \alpha d > 0$. These results can be extended to our benchmark environment with a continuum of agents if we assume perfect monitoring and use the continuation values right after the DM matches are formed to capture the idea that a pair may renegotiate.[19]

# 5 Equilibria of special interest

In this section we focus on PBE without imposing "not-too-tight" solvency constraints and show that doing so reveals equilibria that are relevant, both theoretically and empirically. We show there exist a continuum of symmetric steady-state equilibria or a continuum of asymmetric steady-state equilibria in which there exists heterogeneity in credit limits across otherwise observationally equivalent borrowers (Section 5.1). We show that the set of credit equilibria contains the set of pure monetary equilibria, including non-stationary ones (Section 5.2). We show that credit booms and busts are ubiquitous; there are a continuum of PBE credit cycles of any periodicity, some of which feature credit completely drying up in certain periods (Section 5.3). Finally, we show that PBE can feature endogenous credit growth (Section 5.4) or sunspot equilibria in which credit limits depend on idiosyncratic states, such as an agent's employment status (Section 5.5).

## 5.1 Symmetric and asymmetric steady states

Until now, we have focused on symmetric equilibria based on restrictions (A2) and (A3). Our framework can be easily extended to allow for asymmetric stationary equilibria where each buyer's debt limit is indexed by his identity. In particular, we use $F : \mathbb{B} \to \mathbb{R}_+$ to denote the debt limit assignment for each buyer. Thus, for each buyer, the incentive-compatibility condition, (9), or, equivalently, (11), can be simplified to read:

$$rd \leq \alpha \{u[z(d)] - v[z(d)]\}, \tag{12}$$

where $z$ given by (8) indicates the DM level of output as a function of $d$.

[19] The construction of equilibrium strategies remains the same, and the argument for WRP is the same: since continuation values are completely pinned down by the buyer standings, and matches with good standing have better payoffs to buyers while matches with bad standing have better payoffs to sellers, no two continuation values can be Pareto ranked.

**Proposition 3** ***(Steady-State Equilibria)*** *There exists a continuum of symmetric steady-state, credit equilibria indexed by $d \in [0, d^{\max}]$ with $d^{\max} > 0$. Additionally, for any distribution $F$ with support $[0, d^{\max}]$ there is an asymmetric equilibrium where the distribution of debt limits across buyers is given by $F$.*

Any debt limit between the two values obtained under "not-too-tight" solvency constraints, 0 and $d^{\max}$, is also part of an equilibrium. There are many ways to generate PBE outcomes with $d \in (0, d^{\max})$. We can adopt the simple strategies described earlier according to which a buyer remains trustworthy as long as he repays $d$. Alternatively, one can consider strategies that punish buyers who default by temporary exclusion from trades instead of permanent autarky. Suppose that a buyer in bad standing recovers his good standing with probability $\kappa$ at the end of a period. (One could also consider punishment of deterministic length.) Then, the value of being in bad standing solves $\hat{V}^b = \kappa\beta V^b + (1-\kappa)\beta\hat{V}^b$ and the highest debt limit consistent with no default solves

$$(r+\kappa)\, d = \alpha \left[ u(y) - v(y) \right].$$

Alternatively, agents who default might benefit from lower, but positive, debt limits in the future. There are now three states: good standing, bad standing, and autarky. Agents transition from good to bad standing the first time they default. If agents default more than once they stay in autarky permanently. The debt limits in good and bad standing, $d$ and $\hat{d}$ respectively, solve

$$\begin{aligned} rd &= \alpha \left\{ [u(y) - v(y)] - [u(\hat{y}) - v(\hat{y})] \right\} \\ r\hat{d} &\leq \alpha \left[ u(\hat{y}) - v(\hat{y}) \right]. \end{aligned}$$

The debt limit $\hat{d}$ is sustained by the simple strategies described earlier. Such equilibrium outcomes are in accordance with the evidence (e.g., Jagtiani and Li, 2014) but do not correspond to equilibria with "not-too-tight" constraints.[20]

Proposition 3 also establishes the existence of asymmetric equilibria with a distribution of debt limits across buyers. Under "not-too-tight" constraints the support of the distribution is limited to $\{0, d^{\max}\}$, i.e., buyers can either borrow at the highest debt limit, $d^{\max}$, or they cannot borrow at all. In contrast, we can generate equilibria with a continuous distribution of debt limits over the interval $[0, d^{\max}]$. Such asymmetric equilibria can help explain various forms of discrimination in loan markets (e.g., Duca and Rosenthal, 1993).

The existence of asymmetric equilibria with $d \in \{0, d^{\max}\}$ is used by Carapella and Williamson (2015) to construct equilibria with default along the equilibrium path. Suppose, for instance, that the extrinsic

[20] Jagtiani and Li (2014) document that despite speedy recovery in their risk scores after bankruptcy filing, most filers have much reduced access to credit in terms of credit limits, and the impact seems to be long lasting. Musto (1999) also documents that individuals regain gradual access to credit after filling for bankruptcy, with a big jump when they lose the bankruptcy flag.

characteristic that determines $d$ is private information in some meetings. In a pooling equilibrium agents with $d = 0$ can receive a loan and find it optimal to default on that loan. Using a similar reasoning, we can use asymmetric equilibria with $d \in [0, d^{\max}]$ to generate partial default in equilibrium.

## 5.2 Pure monetary outcomes

Proposition 2 showed that equilibria with "not-too-tight" solvency constraints do not exhibit fluctuations. This finding is consistent with GMMW according to whom the model must be modified by adding a preference parameter in order to generate cycles (see our Section 6). But it is puzzling given that pure currency economies can generate fluctuations and cycles (e.g., Lagos and Wright, 2003) and we know from Kocherlakota (1998) that there are equivalence results in terms of implementation between pure currency and pure credit economies. In order to reconcile these different findings we now show that the set of credit equilibrium outcomes contains all equilibrium outcomes of a pure monetary economy, including its non-stationary ones, provided that one does not impose "not-too-tight" constraints.[21]

In order to establish this result we shut down the record-keeping technology: individual trading histories are private information in a match. Without any record-keeping technology credit is no longer incentive-feasible as a buyer would find it optimal to renege on his debt. Suppose that all buyers are endowed with $M = 1$ unit of fiat money at time $t = 0$, which makes the environment identical to the one in Lagos and Wright (2003, 2005). The CM price of money in terms of the numéraire good, denoted $\phi_t$, solves the following first-order difference equation (see, e.g., Lagos and Wright, 2003):

$$\phi_t = \beta\phi_{t+1}\left[1 + \alpha\frac{u'(y_{t+1}) - v'(y_{t+1})}{v'(y_{t+1})}\right], \tag{13}$$

where $y_{t+1} = v^{-1}(\phi_{t+1}m)$. A monetary equilibrium is a bounded sequence, $\{(y_t, x_t, \phi_t)\}_{t=0}^{+\infty}$, that solves (13), with $v(y_t) = x_t = \min\{\phi_t, v(y^*)\}$.

**Proposition 4** ***(Monetary vs Credit Equilibria)*** *Let $\{(y_t, x_t, \phi_t)\}_{t=0}^{+\infty}$ be a monetary equilibrium of the economy with no record-keeping. Then, $\{(y_t, x_t, \ell_t)\}_{t=0}^{+\infty}$ where $\ell_t = \min\{\phi_t, v(y^*)\}$ is a credit equilibrium of the economy with record-keeping.*

Given a monetary equilibrium where the value of money is $\phi_t$, one can construct a pure credit equilibrium where buyers are trustworthy to repay $\phi_t$. We illustrate this result in Figure 2, where the green, backward-bending line represents the first-order difference equation for a monetary equilibrium, (13), while the red

[21] This result is related to those in Kocherlakota (1998), but with a key difference: while Kocherlakota (1998) shows that the set of *all* implementable outcomes (allowing for arbitrary trading mechanisms) using money is contained in the set of all implementable outcomes with memory, we compare the equilibrium outcomes for the two economies under a *particular* trading mechanism. See also supplementary appendix S1 for the robustness to other trading mechanisms.

area is the first-order difference inequality for a credit equilibrium, (11). Starting from some initial condition, $d_0$, we represent by a dashed line a sequence $\{d_t\}$ that satisfies the conditions for a monetary equilibrium. This sequence also satisfies the conditions for a credit equilibrium, i.e., all pairs $(d_t, d_{t+1})$ are located in the red area. So the existence of equilibria with endogenous fluctuations in pure credit economies should not come as a surprise given that equivalent equilibria exist in pure monetary economies. As we will show in the following, the reverse of Proposition 4 does not hold; there are equilibria of pure credit economies that are not equilibria of pure monetary economies.

### 5.3 Credit booms and busts

We now show that credit economies have a much larger set of periodic equilibria than monetary economies and such equilibria can be relevant to explain severe credit crises with large volatility in consumption and investment. For instance, Fulford (2014) document that credit limit volatility is substantially larger than standard estimates of income volatility.[22]

We start with 2-period cycles, $\{d_0, d_1\}$, where $d_0$ is the debt limit in even periods and $d_1$ is the debt limit in odd periods. The incentive-compatibility condition, (9), becomes:

$$rd_t \leq \frac{\alpha\left[u(y_{(t+1) \bmod 2}) - v(y_{(t+1) \bmod 2})\right] + \beta\alpha\left[u(y_t) - v(y_t)\right]}{1+\beta}, \quad t \in \{0,1\}, \tag{14}$$

where we used that $\ell_t = v(y_t) = \min\{d_t, v(y^*)\}$ from (10). The term on the numerator on the right side of (14) is the buyer's expected discounted utility over the 2-period cycle starting in $t+1$. We define, for each $d_0 \in [0, d^{\max}]$,

$$\gamma(d_0) \equiv \max\{d_1 : (d_0, d_1) \text{ satisfies (14) with } t = 1\}, \tag{15}$$

the highest debt limit in odd periods consistent with a debt limit equal to $d_0$ in even periods. A 2-period-cycle equilibrium, or simply a *2-period cycle*, is a pair $(d_0, d_1)$ that satisfies $d_0 \leq \gamma(d_1)$ and $d_1 \leq \gamma(d_0)$. The function $\gamma(d)$ is positive, non-decreasing, and concave.[23] The function $\gamma$ is represented in the two panels of Figure 3. It is non-decreasing because if the debt limit in even periods increases, then the punishment from defaulting gets larger and, as a consequence, higher debt limits can be sustained in odd periods. So there are complementarities between buyers' trustworthiness in odd periods and buyers' trustworthiness in even periods. The function $\gamma(d)$ is always positive because even if credit shuts down in even periods, it can be sustained in odd periods by the threat of autarky in both odd and even periods. For a given $d_0$ we define

[22] In the last quarter of 2008 one fifth of credit card holders had at least one credit card account closed and overall credit limits fell by a quarter from 2008 to 2009.

[23] We prove properties of the function $\gamma$ in the Appendix. In particular, $d^{\min} \equiv \gamma(0) > 0$, $\gamma(d) > d$ for all $d \in (0, d^{\max})$, and $\gamma(d^{\max}) = d^{\max}$. If $v(y^*) < d^{\max}$, then $\gamma(d) = d^{\max}$ for all $d \in [v(y^*), d^{\max}]$.

the set of debt limits in odd periods that are consistent with a 2-period cycle by

$$\Omega(d_0) \equiv \{d_1 : d_0 \leq \gamma(d_1), d_1 \leq \gamma(d_0)\}. \tag{16}$$

In Figure 3 the set of credit cycles is the area between $\gamma$ and its mirror image with respect to the 45$^{\rm o}$ line.

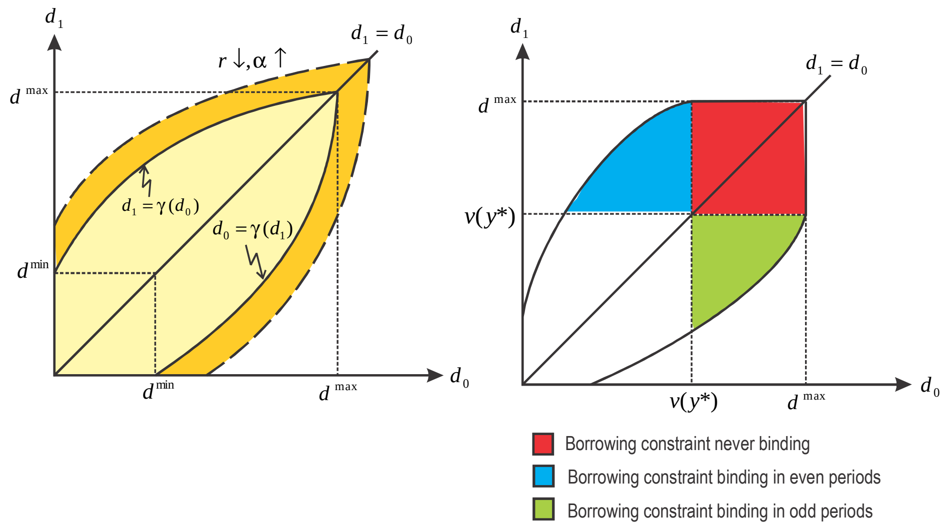


Figure 3: Set of two-period credit cycles

**Proposition 5** ***(2-Period Credit Cycles)*** *For all $d_0 \in [0, d^{\max})$ the set of 2-period cycles with initial debt limit, $d_0$, denoted $\Omega(d_0)$, is a nondegenerate interval.*

If agents are sufficiently impatient, as in the left panel of Figure 3, then the debt limit binds and output is inefficiently low in every period for all credit cycles. However, if agents are patient, then there are equilibria where the debt limit binds periodically. Such equilibria are represented by the blue and green areas in the right panel of Figure 3.

There are equilibria where credit dries up periodically. In the left panel of Figure 3 such equilibria correspond to the case where $d_0 = 0 < d_1 < \gamma(0) = d^{\min}$, i.e., even-period IOUs are believed to be worthless while odd-period IOUs are repaid. If a seller extends a loan in an even period, the buyer defaults, in accordance with equilibrium beliefs, but remains trustworthy in subsequent odd periods. Such outcomes are ruled out by backward induction in pure-currency economies. In contrast a credit economy has IOUs issued at different dates (and by different agents), and hence agents can form different beliefs regarding the terminal value of these different securities.

The result according to which there are a continuum of equilibria does not imply that everything goes. Fundamentals, such as preferences and matching technology, do matter for the outcomes that can emerge.

If agents become more patient, i.e., $r$ decreases, then $\gamma$ shifts upward, as the discounted sum of future utility flows associated with a given allocation increases, and the set of 2-period cycle equilibria expands. The expansion of the equilibrium set is represented by the dark yellow area in the left panel of Figure 3. Similarly, if the frequency of matches, $\alpha$, increases, then $d^{\max}$ increases as permanent autarky entails a larger opportunity cost, and the set of credit cycles expands.

One can generalize the above arguments to $T$-period cycles, $\{d_j\}_{j=0}^{T-1}$. In Figure 4 we represent the set of 3-period cycles for a given $d_2$. The outer edge of this set, which has positive measure in $\mathbb{R}^2$, is represented by a thick black curve. One can also see from the right panel that there is a non-empty set of 3-period cycles (the pink area) where credit shuts down periodically, once ($d_2 = 0$) or twice (e.g., $d_1 = d_2 = 0$) every three periods. Also, for our parametrization the first best is implementable, i.e., there are equilibria in the purple area with $d_t \geq d^* = 1$ for all $t \in \{0, 1, 2\}$. All these examples show that it is not hard to generate cycles in pure credit economies once one abandons restrictive "not-too-tight" solvency constraints.

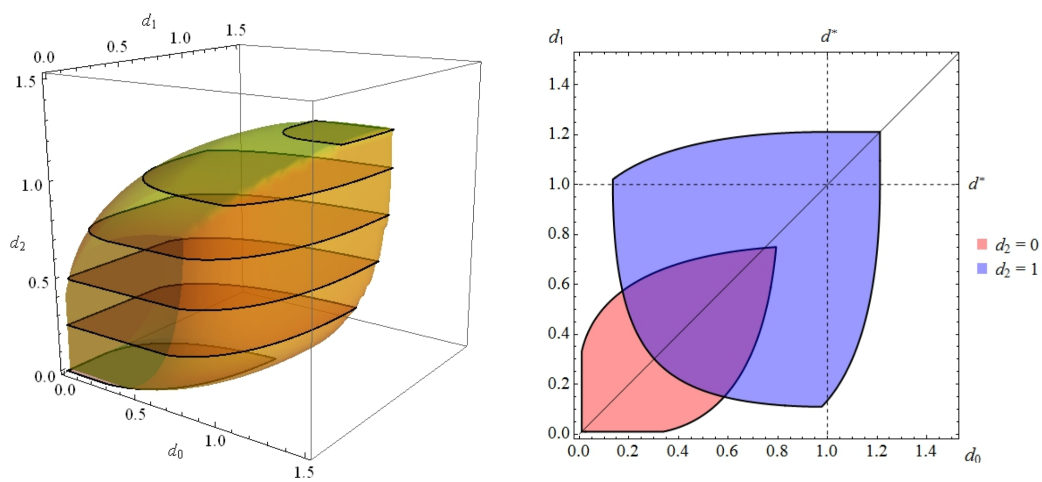


Figure 4: Set of three-period credit cycles: $u(y) = 2\sqrt{y}, v(y) = y, \beta = 0.9, \alpha = 0.25$

## 5.4 Secular credit growth

Not only can our model explain fluctuations in consumption driven by changes in credit conditions, it can also explain the dramatic growth of the share of consumption financed with unsecured credit.[24] Indeed, there are PBE equilibria that feature monotone-increasing debt limits and increasing consumption financed with unsecured credit. In contrast, under "not-too-tight" solvency constraints $\{d_t\}$ is always weakly monotone decreasing.

[24] See, e.g., the evidence discussed in Bethune, Rocheteau, and Rupert (2015).

**Proposition 6** ***(Secular Credit Growth)*** *For all $T > 1$ and $d_T \in (0, d^{\max}]$, there exists equilibria such that $\{d_t\}_{t=0}^{T}$ is monotone increasing.*

As an example, suppose that for all $t \geq T$ the debt limit is equal to its maximum, $d^{\max}$. One can think of the economy as maturing at period $T$ in that it achieves the highest level of trust and the highest level of consumption financed with unsecured credit. In order to obtain a strictly increasing sequence, we impose that $d_{T-1} \leq d^{\max}$. Similarly, at time $T-2$,

$$d_{T-2} < \beta\alpha[u(y_{T-1}) - \upsilon(y_{T-1})] + \beta^2 d^{\max}.$$

We can iterate until we reach $d_0$. Moreover, we can pick $d_0 = 0$ so at the initial date there is no unsecured credit as agents are not trusted to repay their debt. In subsequent periods debt limits increase over time as buyers become more trustworthy.

## 5.5 Sunspot equilibria with applications to the labor market

We now describe equilibria where individual debt limits are subject to idiosyncratic shocks. Suppose that agents are heterogeneous in terms of some state $\chi \in \mathbb{X}$. We assume that $\chi$ does not affect DM preferences or matching probabilities, but it could affect the buyer's endowment or income in the CM. This characteristic could be the name of the individual, a physical characteristic, or his employment status in the context of a model with unemployment. Let $\pi(\chi'|\chi) > 0$ denote the Markov transition function of the state for all $\chi' \in \mathbb{X}$ given some initial state $\chi$. A sunspot credit equilibrium is then a vector, $\langle d_\chi : \chi \in \mathbb{X}\rangle$, of debt limits indexed by the states. Given the vector of debt limits, $\mathbf{d} = \langle d_\chi : \chi \in \mathbb{X}\rangle$, the value of a buyer along the equilibrium path satisfies

$$V_\chi^b(\mathbf{d}) = \alpha\left[u(y_\chi) - \upsilon(y_\chi)\right] + \beta \int V_{\chi'}^b(\mathbf{d}) d\pi(\chi'|\chi), \tag{17}$$

for all $\chi \in \mathbb{X}$ and $y_\chi = \min\{y^*, \upsilon^{-1}(d_\chi)\}$. For any $\mathbf{d} = \langle d_\chi : \chi \in \mathbb{X}\rangle$, it is straightforward to show that there is a unique vector $\langle V_\chi^b(\mathbf{d}) : \chi \in \mathbb{X}\rangle$ that satisfies (17) by the Blackwell sufficient condition and the contraction mapping theorem. As before, the lifetime utility of a buyer is the expected discounted sum of the surpluses coming from DM trades. It follows that a sunspot credit equilibrium is a vector, $\langle d_\chi; \chi \in \mathbb{X}\rangle$, that satisfies

$$d_\chi \leq \beta \int V_{\chi'}^b(\mathbf{d}) d\pi(\chi'|\chi) \quad \forall \chi \in \mathbb{X}. \tag{18}$$

We have the following Proposition.

**Proposition 7** ***(Sunspot equilibria)*** *Suppose that $\mathbb{X}$ has at least two elements and let $\pi(\chi'|\chi)$ be a transition function over $\mathbb{X}$ with a full support. For a given $(\chi, d_\chi) \in \mathbb{X} \times (0, d^{\max})$, the set of sunspot credit equilibria with debt limit $d_\chi$ in state $\chi$, denoted by $\Omega_{\mathbb{X},\pi}(\chi, d_\chi)$, has a positive Lebesgue measure in $\mathbb{R}^{|\mathbb{X}|-1}$.*

This version of the model can easily be integrated in a model of unemployment, following Berensten, Menzio, and Wright (2010) and its version with unsecured credit by Bethune, Rocheteau, and Rupert (2015). Suppose that $\chi \in \{0, 1\}$ is the buyer's employment state. One can construct equilibria where $d_0 < d_1$ and $y_0 < y_1$, unemployed workers have lower debt limits than employed ones even if they have enough resources to repay their debt in the CM. The fact that the employment status matters for the access to credit has implications for the determination of the wage and firms' decision to open vacancies. It can also help explain excess volatility puzzles in labor markets.

# 6 Extensions

In the following we show that our results regarding the equilibrium set of pure credit economies are robust to trading mechanisms other than take-it-or-leave-it offers by buyers. We also extend our model in order to parametrize buyers' temptation to renege on their debt. This assumption plays a key role in GMMW to generate cycles.

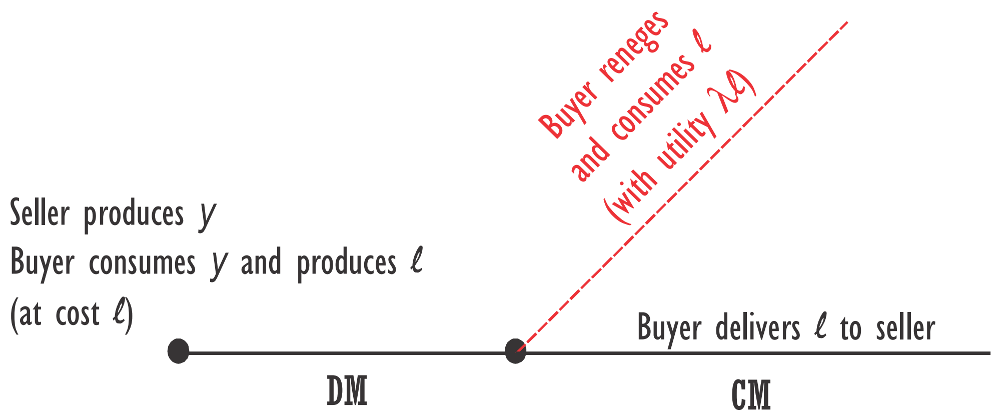


Figure 5: Timing of the extended model with temptation to renege

Suppose from now on that a buyer who promises to deliver $\ell$ units of goods in the next CM incurs the linear disutility of producing at the time he is matched in the DM. This new timing is illustrated in Figure 5.[25] The effort exerted by the buyer in the DM, $\ell$, is perfectly observable to the seller. At the time of delivery, at the beginning of the CM, the disutility of production has been sunk and the buyer has the option to renege on his promise to deliver the good. The buyer's utility from consuming his own output is $\lambda\ell$ with $\lambda \leq 1$. A buyer has no incentive to produce more output than the amount he promises to repay to the seller since the net utility gain from producing $x$ units of the good for oneself is $(\lambda - 1)x \leq 0$. Although the physical environment is different, mathematically speaking, the model of the previous section can be

[25] The description of the buyer's incentive problem is taken from GMMW.

regarded as a special case with $\lambda = 1$. As before we will focus on symmetric perfect Bayesian equilibria that satisfy (A1)-(A3).[26]

Let $\{(d_t, y_t, \ell_t)\}_{t=0}^{\infty}$ be the sequence of equilibrium debt limits and trades. A necessary condition for the repayment of $d_t$ to be incentive feasible is $\beta V_{t+1}^b \geq \lambda d_t$, where the left side is a buyer's continuation value from delivering the promised output and the right side of the inequality is the utility of a buyer if he keeps the output for himself, in which case he enjoys a utility flow $\lambda d_t$, and goes to autarky. Following the same reasoning as before, a credit equilibrium is reduced to a sequence, $\{d_t\}_{t=0}^{\infty}$, that satisfies

$$\lambda d_t \leq \beta V_{t+1}^b = \alpha \sum_{s=1}^{+\infty} \beta^s \left[ u(y_{t+s}) - \ell_{t+s}) \right], \qquad t \in \mathbb{N}_0, \tag{19}$$

where the relationship between $y_t$, $\ell_t$, and $d_t$ will depend on the assumed trading mechanism.

## 6.1 Bargaining

It is standard in the literature on markets with pairwise meetings to determine the outcome of a meeting by an axiomatic bargaining solution. In this section we consider the generalized Nash solution.[27] We adopt the representation of the equilibrium with solvency constraints, $\ell_t \leq d_t$, in order to obtain a convex bargaining set.[28] For a given sequence of debt limits, $\{d_t\}_{t=0}^{+\infty}$, the buyer repays $\min\{\ell_t, d_t\}$ if his date-$t$ obligation from his DM trade is $\ell_t$. Due to the linearity of the CM value functions, the buyer's surplus from a DM trade, $(y_t, \ell_t)$ with $\ell_t \leq d_t$, is $u(y_t) - \ell_t$ and the seller's surplus is $-v(y_t) + \ell_t$. Under generalized Nash bargaining the terms of the loan contract are

$$(y_t, \ell_t) = \arg\max \left[ u(y) - \ell \right]^{\theta} \left[ \ell - v(y) \right]^{1-\theta} \quad \text{s.t.} \quad \ell \leq d_t.$$

The solution is given by

$$y_t = z(d_t) \equiv \min\{y^*, \eta^{-1}(d_t)\} \text{ and } \ell_t = \eta[z(d_t)]. \tag{20}$$

where

$$\eta(y) = \Theta(y)v(y) + \left[1 - \Theta(y)\right] u(y) \text{ and } \Theta(y) = \theta u'(y) / \left[\theta u'(y) + (1-\theta) v'(y)\right]. \tag{21}$$

A sequence, $\{d_t\}_{t=0}^{+\infty}$, is a credit equilibrium under generalized Nash bargaining if and only if

$$\lambda d_t \leq \alpha \sum_{i=1}^{+\infty} \beta^i \left[ u(y_{t+i}) - \eta(y_{t+i}) \right], \quad \forall t \in \mathbb{N}_0, \tag{22}$$

[26] GMMW also introduce an imperfect record-keeping technology as follows. At the end of the CM of period $t$ the repayments are recorded for a subset of buyers, $\mathbb{B}_t^r \subset \mathbb{B}$, chosen at random among all buyers. The set, $\mathbb{B}_t^r$, of monitored buyers is of measure $\pi$, and the draws from $\mathbb{B}$ are independent across periods. So in every period, while his promise is always recorded, a buyer has a probability $\pi$ of having his repayment decision being recorded. Any equilibrium of our model with $\pi < 1$ is also an equilibrium with $\pi = 1$. Hence, setting $\pi = 1$ is with no loss in generality.

[27] We also characterized equilibria under the Kalai proportional solution. See our working paper for details.

[28] Even though the bargaining solution is axiomatic we could consider a simple game where upon being matched the buyer and the seller receive a proposal that they can either accept or reject. The focus here, however, is not on strategic foundations for axiomatic bargaining solutions.

where $y_t$ is the solution to (20).

We denote $\hat{y} = \arg\max \{u(y) - \eta(y)\}$ the output level that maximizes the buyer's surplus. Unlike the proportional solution $\hat{y} < y^*$ for all $\theta < 1$. As a result the buyer's surplus, $u(y) - \eta(y)$, in the right side of the participation constraint, (22), is non-monotonic with the debt limit provided that $\theta < 1$.[29] It follows that the function $\gamma(d)$ is hump-shaped, reaching a maximum at $d = \hat{d} \equiv \eta(\hat{y})$ and it is constant for $d > \eta(y^*)$. In Figure 6 we represent the function $\gamma$ and the set of pairs, $(d_0, d_1)$, consistent with a 2-period credit cycle equilibrium. One can see that the results are qualitatively unchanged except for the fact that the credit limits at a periodic equilibrium can be greater than the highest debt limit at a stationary equilibrium. This result will have important normative implications.

The two red stars in the left panel of Figure 6 are the strict two-period cycles under "not-too-tight" solvency constraints that GMMW focuses on. Such cycles are located at the intersection of $\gamma$ and its mirror image with respect to the line $d_1 = d_0$. It should be clear that the non-monotonicity of the trading mechanism is necessary to obtain such cycles. It can also be checked that cycles under "not-too-tight" solvency constraints do not exist when $\lambda = 1$ (see GMMW).

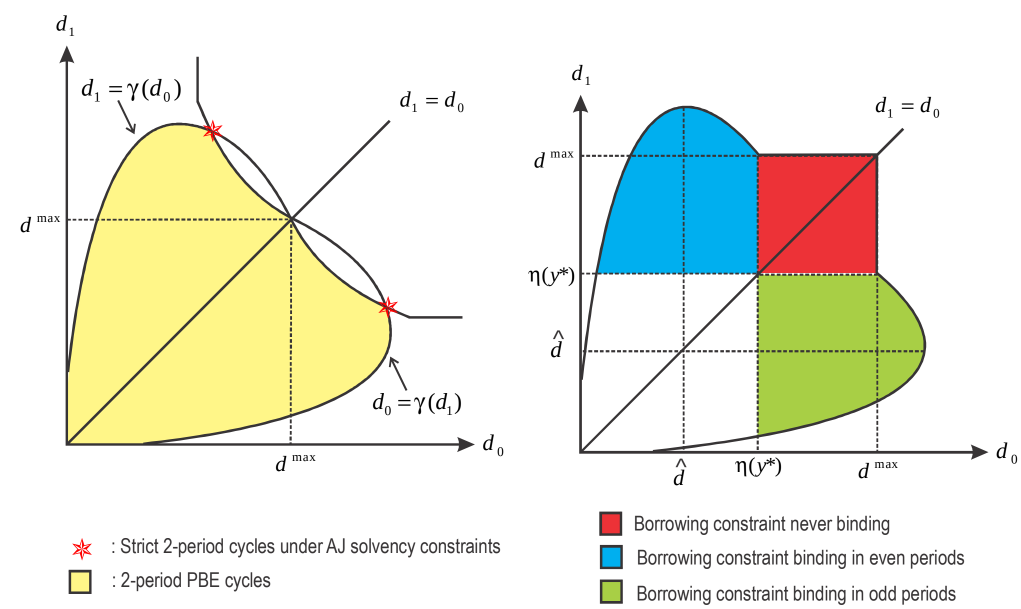


Figure 6: 2-period cycles under Nash bargaining or competitive pricing

In the top panels of Figure 7 we plot the numerical examples in Gu and Wright (2011) under generalized Nash bargaining for the following functional forms and parameter values: $u(y) = [(x+b)^{1-a} - b^{1-a}]/(1-a)$ with $a = 2$ and $b = 0.082$, $v(y) = Ay$, $\beta = 0.6$, $\alpha = 1$, $\theta = 0.01$, and $\lambda = 3/40$. In the top-left panel, $A = 1.1$,

[29] This non-monotonicity property of the Nash bargaining solution and its implications for monetary equilibria is discussed at length in Aruoba et al. (2007).

the two 2-period cycles under "not-too-tight" solvency constraints are such that borrowing constraints bind periodically. In the top-right panel, $A = 1.5$, the borrowing constraint binds in all periods. For both examples there exists a continuum of PBE 2-period cycles, a fraction of which feature borrowing constraints that bind periodically and a fraction of which have borrowing constraints that bind in all periods.

## 6.2 Competitive pricing

Here we follow Kehoe and Levine (1993) and AJ and assume that the terms of the loan contract in the DM are determined by competitive pricing. We reinterpret matching shocks as preference and productivity shocks, i.e., only $\alpha$ buyers want to consume and only $\alpha$ sellers can produce. As in the previous sections, buyers' repayment strategy follows a threshold rule: for a given sequence of debt limits, $\{d_t\}_{t=0}^{+\infty}$, the buyer repays $\min\{\ell_t, d_t\}$ if his date-$t$ obligation from his DM trade is $\ell_t$.[30] Moreover, the overall amount of debt issued by a buyer in the DM of period $t$, $\ell_t$, is known to all agents. Hence, if $p_t$ denotes the price of DM output in terms of the numéraire, the buyer's problem is $\max_y \{u(y) - p_t y\}$ s.t. $p_t y \leq d_t$. The solution is $y_t = \min\left\{u'^{-1}(p_t), d_t/p_t\right\}$. Using that there is the same measure, $\alpha$, of buyers and sellers participating in the market, market clearing implies $p_t = v'(y_t)$. As a result $y_t = y^*$ if $y^* v'(y^*) \geq d_t$ and $\eta(y_t) \equiv y_t v'(y_t) = d_t$ otherwise. The buyer's surplus is $u(y) - py = u(y) - v'(y)y$. For a given $p$, the buyer's surplus is non-decreasing in his borrowing capacity, $d_t$. However, once one takes into account the fact that $p = v'(y)$ then the buyer's surplus is non-monotone in his capacity to borrow, $d_t$. Provided that $v$ is strictly convex, the buyer's surplus reaches a maximum for $y = \hat{y} < y^*$.

A sequence, $\{d_t\}_{t=0}^{+\infty}$, is a credit equilibrium under competitive pricing if and only if (22) holds for all $t \in \mathbb{N}_0$, where $y_t$ is given by

$$y_t = z(d_t) \equiv \min\{y^*, \eta^{-1}(d_t)\} \text{ and } \ell_t = \eta[z(d_t)], \tag{23}$$

A steady state is a $d$ such that

$$r\lambda d \leq \alpha \left\{u\left[z(d)\right] - v'\left[z(d)\right] z(d)\right\}. \tag{24}$$

Under some weak assumptions on $v$ (for example, $\eta(y) = v'(y)y$ is convex), $d^{\max} > 0$, i.e., there exists a continuum of steady-state equilibria. This also implies that there exist a continuum of strict, 2-period, credit cycle equilibria.[31] This result can be contrasted with the ones in GMMW (Corollary 1-3) where conditions on parameter values are needed to generate a finite number (typically, two) of cycles. The right panel of

[30] If a buyer repays $x_t \neq \ell_t$ in the CM, then each unit of IOU issued by that buyer has a payoff equal to $x_t/\ell_t$ units of numéraire to its owner.

[31] Under competitive pricing, the function $\gamma$ (analogous to (15)) may not be monotone or concave, but the logic for Proposition 5 does not depend on those properties. See also the supplementary appendix S2 for a formal proof of the existence of 2-period cycles.

Figure 6 illustrates these differences. Under "not-too-tight" solvency constraints credit cycles are determined at the intersection between $\gamma(d)$ and its mirror image with respect to the 45$^{\text{o}}$ line. These cycles are marked by a red star. If we allow for slack buyers' participation constraints, cycles are at the intersection of the area underneath $\gamma(d)$ and its mirror image with respect to the 45$^{\text{o}}$ line—the blue area in the figure. Finally, Proposition 4 on the equivalence result between monetary equilibria and credit equilibria holds for Walrasian pricing as well. (See the Supplementary Appendix S1 for a formal proof).

We now review the numerical examples in GMMW in the case where the DM market is assumed to be competitive. The functional forms are $u(y) = y$, $v(y) = y^{1+\gamma}/(1+\gamma)$, and there are no idiosyncratic shocks, $\alpha = 1$. The first example in the bottom left panel of Figure 7 is obtained with the following parameter values: $\gamma = 2.1$, $\beta = 0.4$, $\lambda = 1/6$. GMMW identify two (strict) two-period cycles under "not-too-tight" solvency constraints , $(d_0, d_1) = (0.477, 0.936)$ and its converse, marked by red dots in the figure. The second example in the bottom right panel is obtained with the following parameter values: $\gamma = 0.5$, $\beta = 0.9$, $\lambda = 1/10$. The credit cycles under "not-too-tight" solvency constraints, $(d_0, d_1) = (0.933, 1.037)$ and its converse, are such that period allocations fluctuate between being debt-constrained and unconstrained. We find a much bigger set of PBE credit cycles represented by the blue colored region. There are a continuum of cycles such that the allocations fluctuate between being debt-constrained and unconstrained and a continuum of cycles such that agents are debt-constrained in all periods. In the second example, the credit cycle under "not-too-tight" solvency constraints is such that $(y_0, y_1) = (0.96, 1.00)$ while the most volatile PBE is $(y_0, y_1) = (0.96, 0.00)$.

## 6.3 Credit cycles and welfare

We now show that the imposition of "not-too-tight" solvency constraints not only reduces drastically the equilibrium set, but it does so by eliminating equilibria with good normative properties. To see this, we consider the set of two-period cycles represented in the left panel of Figure 7 ($\gamma = 2.1$, $\beta = 0.4$, $\lambda = 1/6$), and we measure society's welfare by $u\left[y(d_0)\right] - v\left[y(d_0)\right] + \beta\left\{u\left[y(d_1)\right] - v\left[y(d_1)\right]\right\}$. In the left panel of Figure 8 we highlight in red and green the set of 2-period cycles, $(d_0, d_1)$, that dominate the equilibria under "not-too-tight" solvency constraints (marked by stars). There exist a continuum of such cycles that feature slack participation constraints. Hence, the imposition of "not-too-tight" solvency constraints eliminates good equilibria. The right panel of Figure 8 increases $\lambda$ from $\lambda = 1/6$ to $\lambda = 1/4$. There is no credit cycle under the "not-too-tight" solvency constraints, but there are a continuum of PBE cycles where credit constraints are "too-tight" a fraction of which dominate the highest steady state in social welfare.

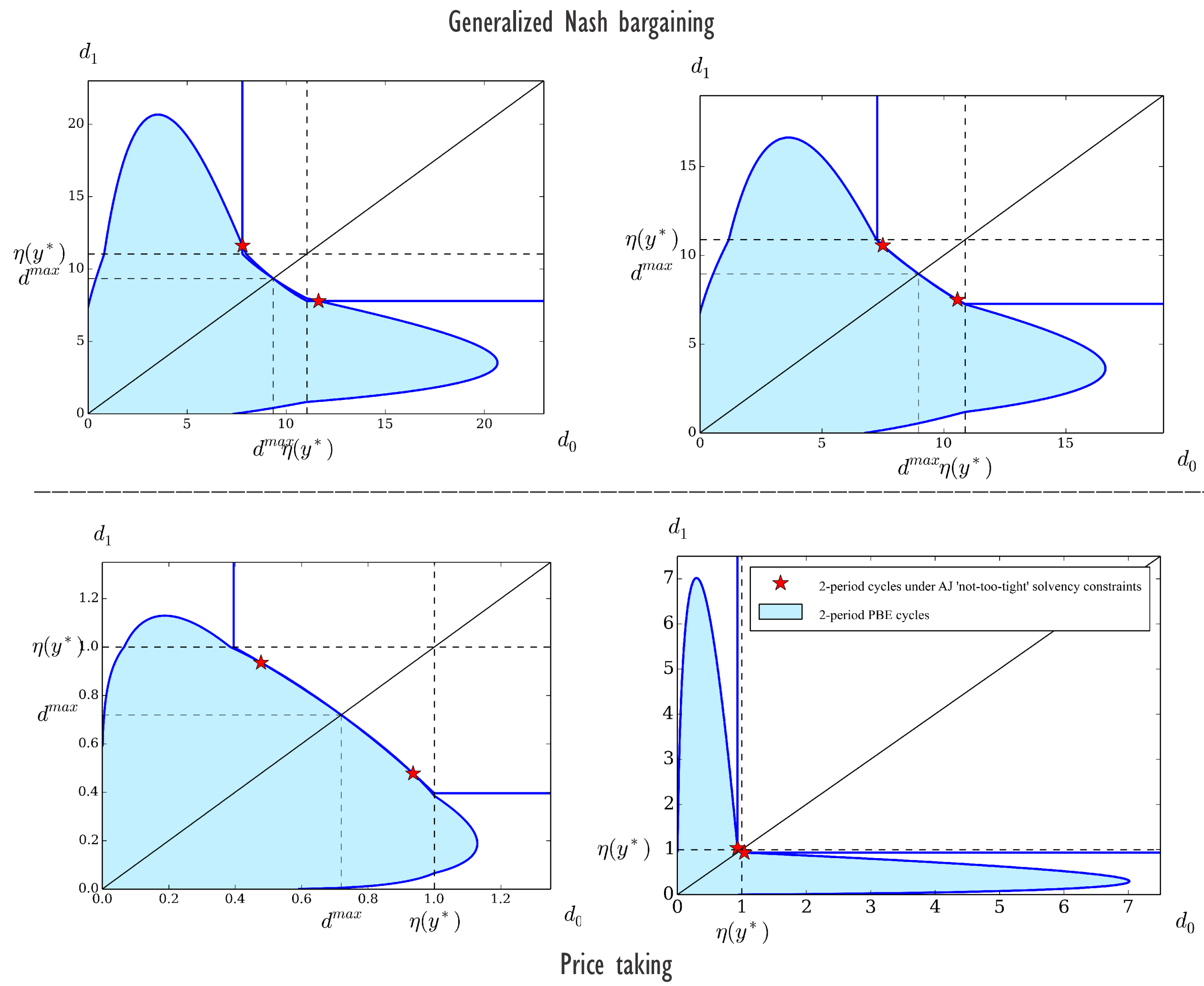


Figure 7: The blue area is the set of all PBE credit cycles. The red dots are credit cycles under AJ "not-too-tight" solvency constraints. The top panels are obtained under generalized Nash bargaining while the bottom panels are obtained under price taking.

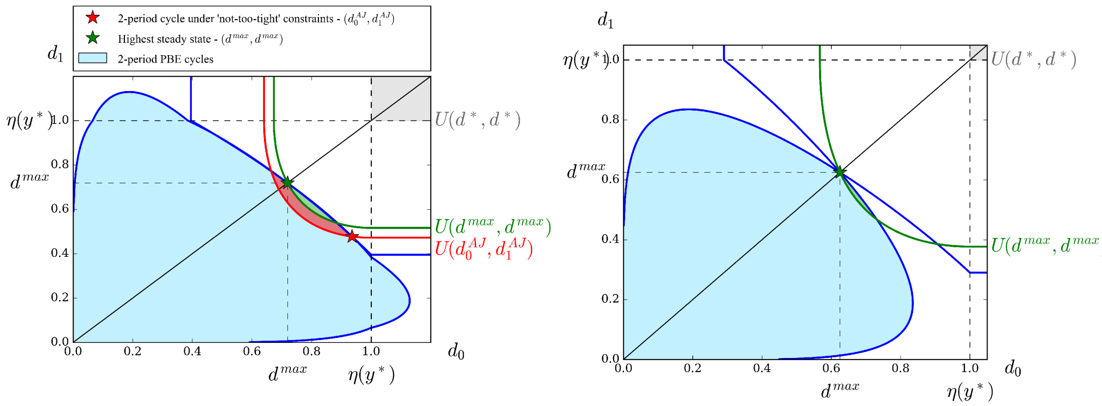


Figure 8: The blue area is set of all 2-period cycles. The red star is the 2-period cycle in GMMW and the green star is the highest steady state. Left panel: $\lambda = 1/6$; Right panel: $\lambda = 1/4$.

# 7 Conclusion

We have characterized the set of perfect Bayesian equilibrium outcomes of a pure credit economy under limited commitment. The economy features inter-temporal gains from trade that can be exploited with one-period loan contracts. Such contracts and their execution are publicly recorded. Agents interact either through random, pairwise meetings under various trading mechanisms, as in the New-Monetarist literature, or in competitive spot markets, as in AJ. The set of equilibria can be characterized by a set of solvency constraints (that can vary through time or across agents).

We showed that the equilibrium set is much larger than the one obtained under the "not-too-tight" solvency constraints imposed throughout the literature and that the new equilibria are relevant both theoretically and empirically. For instance, we found a continuum of endogenous cycles and sunspot equilibria with credit booms and busts. Some credit cycles are such that credit shuts down periodically and some credit cycles generate larger welfare than stationary equilibria under "not-too-tight" solvency constraints. There are also equilibria where endogenous debt limits increase over time, consistent with the secular growth of the consumption share financed with unsecured credit. Finally, equilibrium outcomes of the pure currency economy are outcomes of the pure credit economy, but the reverse is not true. In summary, imposing "not-too-tight" solvency constraints in New Monetarist environments entails a severe loss in generality for positive analysis with consequences for normative analysis.

## References

[1] Abreu, Dilip (1988). "On the theory of infinitely repeated games with discounting," Econometrica 56, 383-396.

[2] Abreu, Dilip, David Pearce, Ennio Stacchetti (1990). "Toward a theory of discounted repeated games with imperfect monitoring," Econometrica 58, 1041-1063.

[3] Alvarez, Fernando and Urban Jermann (2000). "Efficiency, equilibrium, and asset pricing with risk of default," Econometrica 68, 775-797.

[4] Aruoba, Boragan, Guillaume Rocheteau, and Christopher Waller (2007). "Bargaining and the value of money," Journal of Monetary Economics 54, 2636-2655.

[5] Berensten, Aleksander, Guido Menzio, and Randall Wright (2011). "Inflation and unemployment in the long run," American Economic Review 101, 371-398.

[6] Bethune, Zachary, Tai-Wei Hu, and Guillaume Rocheteau (2017). "Optimal Credit Fluctuations," Working Paper.

[7] Bethune, Zachary, Guillaume Rocheteau, and Peter Rupert (2015). "Aggregate unemployment and household unsecured debt," Review of Economic Dynamics 18, 77-100.

[8] Bloise, Gaetano, Pietro Reichlin, and Mario Tirelli (2013). "Fragility of Competitive Equilibrium with Risk of Default," Review of Economic Dynamics 16, 271-295.

[9] Carapella, Francesca, and Stephen Williamson (2015). "Credit markets, limited commitment, and government debt," Review of Economic Studies 82, 963-990.

[10] Duca, John, and Stuart Rosenthal (1993). "Borrowing constraints, household debt, and racial discrimination in loan markets," Journal of Financial Intermediation 3, 77-103.

[11] Ellison, Glenn (1994). "Cooperation in the prisoner's dilemma with anonymous random matching," Review of Economic Studies 61, 567-588.

[12] Farrell, Joseph, and Eric Maskin (1989). "Renegotiation in repeated games," Games and Economic Behavior 1, 327-360.

[13] Fulford, Scott (2015). "How important is variability in consumer credit limits?," Journal of Monetary Economics 72, 42-65.

[14] Gu, Chao and Randall Wright (2011). “Endogenous credit cycles,” Federal Reserve Bank of Minneapolis, Working Paper 689.

[15] Gu, Chao, Fabrizio Mattesini, Cyril Monnet, and Randall Wright (2013a). “Banking: A New Monetarist approach,” Review of Economic Studies 80, 636-662.

[16] Gu, Chao, Fabrizio Mattesini, Cyril Monnet, and Randall Wright (2013b). “Endogenous credit cycles,” Journal of Political Economy 121, 940 - 965.

[17] Hellwig, Christian and Guido Lorenzoni (2009). “Bubbles and self-enforcing debt,” Econometrica 77, 1137-1164.

[18] Jagtiani, Julapa, and Wenli Li (2014). “Credit access after consumer bankruptcy filing: new evidence,” Federal Reserve Bank of Philadelphia Working paper 14-25.

[19] Kandori, Michihiro (1992). “Social norms and community enforcement,” Review of Economic Studies 50, 63-80.

[20] Kehoe, Timothy and David Levine (1993). “Debt constrained asset markets,” Review of Economic Studies 60, 865-88.

[21] Kocherlakota, Narayana (1996). “Implications of efficient risk sharing without commitment,” Review of Economic Studies 63, 595-609.

[22] Kocherlakota, Narayana (1998). “Money is memory,” Journal of Economic Theory 81, 232-251.

[23] Lagos, Ricardo, and Randall Wright (2003). “Dynamics, cycles, and sunspot equilibria in ‘genuinely dynamic, fundamentally disaggregative’ models of money,” Journal of Economic Theory 109, 156-171.

[24] Lagos, Ricardo and Randall Wright (2005). “A unified framework for monetary theory and policy analysis,” Journal of Political Economy 113, 463-484.

[25] Mailath, George, and Larry Samuelson (2006). Repeated Games and Reputations. Oxford University Press: Oxford, UK.

[26] Musto, David (2004). “What happens when information leaves a market? Evidence from post-bankruptcy consumers,” Journal of Business 77, 725-748.

[27] Orsborne, Martin and Ariel Rubinstein (1994). A Course in Game Theory. MIT Press.

[28] Rocheteau, Guillaume, and Randall Wright (2005). "Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium," Econometrica 73, 175-202.

[29] Sanches, Daniel and Stephen Williamson (2010). "Money and credit with limited commitment and theft," Journal of Economic Theory 145, 1525-1549.

[30] Wicksell, Knut (1936). Interest and Prices: A Study of the Causes Regulating the Value of Money. Augustus Kelley, New York.

# Appendix: Proofs of lemmas and propositions

**Proof of Proposition 1** **($\Rightarrow$)** Here we prove necessity. Suppose that $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$ is an equilibrium outcome in a credit equilibrium, $(s^b, s^s)$.

(i) Here we show condition (4). Because the worst payoff to buyers at each period is 0 (autarky) while the equilibrium payoff at period $t$ is $u(y_t) - x_t$, condition (4) is necessary for buyers to repay their promises at each period.

(ii) To show condition (5), we first show that $x_t = v(y_t)$ for all $t$. Note that (A3) implies that $x_t = \ell_t$ for all $t$. If $x_t < v(y_t)$, then the seller would not accept the offer. Suppose, by contradiction, that $x_t > v(y_t)$. Then, the buyer may deviate and offer $(y', \ell_t)$ with $v(y') \in (v(y_t), \ell_t)$. Because this deviation does not affect the buyer's public record and the buyer has the same incentive to repay his debt, it is dominant for the seller to accept it. It then is a profitable deviation because $y' > y_t$.

Next, to show that $y_t \leq y^*$ for all $t$, suppose, by contradiction, that $y_t > y^*$ and hence $u(y_t) \geq x_t \geq v(y_t) > v(y^*)$. Then there exists an alternative offer, $(y', \ell') = (y', x')$, such that $u(y') - x' > u(y_t) - x_t$ and $-v(y') + x' > -v(y_t) + x_t$ and $\ell' \leq \ell_t$. It is dominant for the seller to accept this alternative offer. The seller's payoff at the current period is 0 if he rejects. However, if he accepts, then by (A3), the threshold rule for repayment, the buyer will repay his promise $\ell' = x'$. Then, by accepting the offer the seller obtains $-v(y') + x' > 0$. Thus, $(y', \ell')$ is a profitable deviation for the buyer.

**($\Leftarrow$)** Here we show sufficiency. Let $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$ be a sequence satisfying (4) and (5). Consider $(s^b, s^s)$ given as follows. Buyers can be in two states, $\chi_{i,t} \in \{G, A\}$, and each buyer's initial state is $\chi_{i,0} = G$. The law of motion of the buyer $i$'s state are given by:

$$\chi_{i,t+1}\left[(\ell', x', i), \chi_{i,t}\right] = \begin{cases} A & \text{if } x' < \min(x_t, \ell') \text{ or } \chi_{i,t} = A \\ G & \text{otherwise} \end{cases}. \tag{25}$$

The strategies are such that $s^b_{t,1}(\rho^i_t) = (y_t, \ell_t)$ if the state for $\rho^i_t$ is $G$ and $s^b_{t,1}(\rho^i_t) = (0, 0)$ otherwise; $s^b_{t,2}(\rho^i_t, (y', \ell'), yes) = \min\{\ell', \ell_t\}$ if the state for $\rho^i_t$ is $G$ and $s^b_{t,2}(\rho^i_t, (y', \ell'), yes) = 0$ otherwise; $s^s_t(\rho^i_t, (y', \ell')) = yes$ if the state for $\rho^i_t$ is $G$ and $v(y') \leq \min\{\ell', \ell_t\}$, and $s^s_t(\rho^i_t, (y', \ell')) = no$ otherwise. We show that $(s^b, s^s)$ is a credit equilibrium.

Given $s^b$, $s^s$ is optimal: the seller expects a buyer in state $G$ to repay up to $\ell_t$ at period $t$ and hence he accepts an offer, $(y', \ell')$, if $v(y') \leq \min\{\ell', \ell_t\}$; with buyers in state $A$ he expects no repayment at all and hence rejects any offer. Next, we show that $s^b$ is optimal given $s^s$. Consider a buyer with state $A$ at the beginning of period $t$. Any offer to the seller is rejected and therefore it is optimal for the buyer to offer $(0, 0)$. Similarly, for such a buyer at the CM stage at period $t$ with a promise $\ell'$, his state will remain in $A$, independent of his repayment decision and hence it is optimal to repay nothing.

Now consider a buyer with state $G$ at the CM stage of period $t$, with a promise $\ell'$ made to the seller. The buyer has to pay $\min\{\ell_t, \ell'\}$ to maintain state $G$. By (4), paying this amount is better than becoming an $A$ person, whose continuation value is 0. Finally, consider a buyer with state $G$ at the beginning of period $t$. Note that under $s^b$, his continuation value from period $t+1$ onward is independent of his offer at period $t$. Moreover, for any offer $(y, \ell)$, the seller accepts the offer if and only if $v(y) \leq \min\{\ell, \ell_t\}$. Thus, a buyer's problem is

$$\max_{(y,\ell)} u(y) - \min\{\ell, \ell_t\} \text{ s.t. } v(y) \leq \min\{\ell, \ell_t\}.$$

Because $\ell_t = v(y_t) \leq v(y^*)$, $(y_t, \ell_t)$ is a solution to the problem.

**Proof of Corollary 1** **(⇐)** Here we show sufficiency. Let $\{d_t\}_{t=0}^{\infty}$ be a sequence satisfying (9) and (10). Then, we can determine the outcome, $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$, consistent with $\{d_t\}_{t=0}^{\infty}$ by the solution to the bargaining problem, (8), that is, $x_t = \ell_t = v(y_t) = \min\{v(y^*), d_t\}$ for each $t$. It remains to show that $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$ is the outcome of a credit equilibrium, $(s^b, s^s)$, with buyers' repayment strategy consistent with $\{d_t\}_{t=0}^{\infty}$. As in the proof of Proposition 1, the strategy follows a simple finite automaton with two states, $\chi_{i,t} \in \{G, A\}$, and each buyer's initial state is $\chi_{i,0} = G$. The law of motion of the buyer $i$'s state are given by:

$$\chi_{i,t+1}\left[(\ell', x', i), \chi_{i,t}\right] = \begin{cases} A & \text{if } x' < \min(d_t, \ell') \text{ or } \chi_{i,t} = A \\ G & \text{otherwise} \end{cases}. \tag{26}$$

This law of motion is the same as (25), where $d_t$ replaces $x_t$. The strategies are analogous to those constructed in the proof of Proposition 1, but with $d_t$ as the maximum amount of debt the buyer repays: at date $t$, the buyer offers $(y_t, \ell_t)$ in state $G$, the seller accepts the offer $(y', \ell')$ iff $v(y') \leq \ell' \leq d_t$ and the buyer's state is $G$, and the buyer repays $\min(\ell', d_t)$ in the CM in state $G$ if $\ell'$ is the loan issued in DM. Following exactly the same logic as in the proof of Proposition 1, (9) and (10) ensure that $(s^b, s^s)$ is a credit equilibrium.

**(⇒)** Here we show necessity. Let $\{d_t\}_{t=0}^{\infty}$ be a sequence consistent with a credit equilibrium outcome, $\{(y_t, x_t, \ell_t)\}_{t=0}^{\infty}$. By definition, $\{d_t\}_{t=0}^{\infty}$ satisfies (10). To show (9), consider a buyer at period-$t$ CM with a loan size $\ell' = d_t$ (perhaps on an off-equilibrium path). For repayment of $d_t$ to be optimal in state $G$, (9) must hold, i.e., the buyer prefers repaying $d_t$ to permanent autarky.

**Proof of Corollary 2** Rewrite the incentive-compatibility constraint (11) at time $t+1$ and multiply it by $\beta$ to obtain:

$$\beta d_{t+1} \leq \beta^2 \left\{\alpha \left[u(y_{t+2}) - v(y_{t+2})\right] + d_{t+2}\right\}. \tag{27}$$

Combining (11) and (27) we get:

$$d_t \leq \beta \left\{ \alpha \left[ u(y_{t+1}) - v(y_{t+1}) \right] \right\} + \beta^2 \left\{ \alpha \left[ u(y_{t+2}) - v(y_{t+2}) \right] \right\} + \beta^2 d_{t+2}.$$

By successive iterations we generalize the inequality above as follows:

$$d_t \leq \sum_{s=1}^{T} \beta^s \left\{ \alpha \left[ u(y_{t+s}) - v(y_{t+s}) \right] \right\} + \beta^{t+T} d_{t+T}. \tag{28}$$

By assumption, $\{d_t\}$ is bounded, $\lim_{T \to \infty} \beta^{t+T} d_{t+T} = 0$. Hence, by taking $T$ to infinity, it follows from (28) that $\{d_t\}$ satisfies (9).

**Proof of Proposition 2** We characterize symmetric equilibria according to $d_0$. We consider three cases.

(a) Suppose that $d_0 = 0$. Then, (9) at equality implies that $u(y_t) - v(y_t) = 0$ for all $t > 0$, which, in turn, by (10), implies that $d_t = 0$ for all $t > 0$.

(b) Suppose that $d_0 = d^{\max}$. Note that "not-too-tight" solvency constraint also implies that (11) holds with equality. We show by induction that $d_t = d^{\max}$ for all $t$. Suppose that $d_t = d^{\max}$. Then, (11) at equality, $d^{\max} = \beta \left\{ \alpha \left[ u(y_{t+1}) - v(y_{t+1}) \right] + d_{t+1} \right\}$, and hence, by by (10),

$$\frac{1}{\beta} d^{\max} - d_{t+1} = \alpha \left[ u(z(d_{t+1})) - v(z(d_{t+1})) \right]. \tag{29}$$

By definition, $d_{t+1} = d^{\max}$ is a solution to (29), and, since the right side of (29) is increasing in $d_{t+1}$ and the left-side is strictly decreasing, it is also the unique solution. This proves $d_t = d^{\max}$ for all $t$.

(c) Suppose that $d_0 \in (0, d^{\max})$. We show by induction that there is a unique sequence $\{d_{t+1}\}_{t \geq 0}$ that satisfies (11) at equality with $y_{t+1}$ pinned down by (10) and that $d_{t+1} < d_t$ for all $t \geq 0$. To see this, for any given $d_t \in (0, d^{\max})$, consider the following equation

$$\frac{1}{\beta} d_t - d_{t+1} = \alpha \left[ u(z(d_{t+1})) - v(z(d_{t+1})) \right]. \tag{30}$$

Since $d_t < d^{\max}$, for $d_{t+1} = d_t$,

$$\frac{1}{\beta} d_t - d_t = r d_t < \alpha \left[ u(z(d_t)) - v(z(d_t)) \right],$$

while for $d_{t+1} = 0$,

$$\frac{1}{\beta} d_t > \alpha \left[ u(z(0)) - v(z(0)) \right].$$

Hence, there is a unique solution $d_{t+1} \in (0, d_t)$ to the equation (30). Moreover, by Corollary 2, such a sequence corresponds to a credit equilibrium.

**Proof of Proposition 3** Define the right side of (12) as a function

$$\Psi(d) = \alpha \left\{ u\left[z(d)\right] - \upsilon\left[z(d)\right] \right\}. \tag{31}$$

$\Psi$ is continuous in $d$ with $\Psi(0) = 0$ and $\Psi(d) = \alpha\left[u\left(y^*\right) - \upsilon\left(y^*\right)\right]$ for all $d \geq \upsilon(y^*)$. Moreover, it is differentiable with

$$\Psi'(d) = \alpha \left\{ \frac{u'[z(d)] - \upsilon'[z(d)]}{\upsilon'[z(d)]} \right\} \text{ if } d \in (0, \upsilon(y^*)], \text{ and } \Psi'(d) = 0 \text{ if } d > \upsilon(y^*).$$

This derivative is decreasing in $d$ for all $d \in (0, \upsilon(y^*))$. Hence, $\Psi$ is a concave function of $d$, and the set of values for $d$ that satisfies (12) is an interval $[0, d^{\max}]$, where $d^{\max} \geq 0$ is the largest number that satisfies $\Psi(d^{\max}) = rd^{\max}$. Moreover, $d^{\max} > 0$ if and only if $\Psi'(0) > r$, which is always satisfied since $\Psi'(0) = \infty$ by assumption on preferences.

Since in our framework each buyer's future gains from trade and hence his debt limit is unrelated to other buyers, the existence of asymmetric equilibria in which a buyer $i$'s debt limit is such that $d^i \in [0, d^{\max}]$ follows immediately.

**Proof of Proposition 4** Replace $d_t = \phi_t$ into the buyer's optimality condition in a monetary economy, (13), to get

$$d_t = \beta d_{t+1} \left[ 1 + \alpha \frac{u'(y_{t+1}) - \upsilon'(y_{t+1})}{\upsilon'(y_{t+1})} \right]. \tag{32}$$

The right side of (32), $\left[u'(y_{t+1}) - \upsilon'(y_{t+1})\right]/\upsilon'(y_{t+1})$, is the derivative of the function, $u[\upsilon^{-1}(d_{t+1})] - d_{t+1}$, with respect to $d_{t+1}$. From the strict concavity of the function and the fact that it is equal to 0 when evaluated at $d_{t+1} = 0$,

$$\frac{u'(y_{t+1}) - \upsilon'(y_{t+1})}{\upsilon'(y_{t+1})} d_{t+1} < u(y_{t+1}) - \upsilon(y_{t+1}). \tag{33}$$

From (32) and (33),

$$d_t < \beta\alpha \left[u(y_{t+1}) - \upsilon(y_{t+1})\right] + \beta d_{t+1}. \tag{34}$$

Iterating (34),

$$d_t < \sum_{j=1}^{J} \beta^j \alpha \left[u(y_{t+j}) - \upsilon(y_{t+j})\right] + \beta^J d_{t+J}. \tag{35}$$

Applying the transversality condition, $\lim_{J\to\infty} \beta^J d_{t+J} = 0$ to (35), we prove that the sequence, $\{d_t\}$, is a solution to (32) satisfies (9), and hence it is part of a credit equilibrium.

**Proof of Proposition 5** Before the proof proper, we need a lemma about the function $\gamma(d)$.

**Lemma 1** *The function $\gamma(d)$ is positive, non-decreasing, and concave. Moreover, $d^{\min} \equiv \gamma(0) > 0$, $\gamma(d) > d$ for all $d \in (0, d^{\max})$, and $\gamma(d^{\max}) = d^{\max}$. If $\upsilon(y^*) < d^{\max}$, then $\gamma(d) = d^{\max}$ for all $d \in [\upsilon(y^*), d^{\max}]$.*

**Proof.** Define the correspondence $\Gamma : \mathbb{R}_+ \supset \mathbb{R}_+$ as follows:

$$\Gamma(d) = \{x \in \mathbb{R}_+ : r\,(1+\beta)\,x \le \alpha\,\{u\,[z(d)] - \upsilon\,[z(d)]\} + \beta\alpha\,\{u\,[z(x)] - \upsilon\,[z(x)]\}\,. \tag{36}$$

Then, $\gamma(d) = \max \Gamma(d)$. First we show that $\Gamma(d)$ is a closed interval and $\gamma$ is well-defined. By definition, $x \in \Gamma(d)$ if and only if

$$r(1+\beta)x \le \Psi(d) + \beta\Psi(x),$$

where $\Psi(d) = \alpha\,\{u\,[z(d)] - \upsilon\,[z(d)]\}$. Using a similar argument to that in Proposition 3, $\Gamma(d)$ is a closed interval with zero as the lower end point. Thus, $\gamma$ is well-defined, and $\gamma(d)$ is the largest $x$ that satisfies

$$r\,(1+\beta)\,x = \Psi(d) + \beta\Psi(x). \tag{37}$$

Moreover, if $d > d'$, then $\Gamma(d') \subseteq \Gamma(d)$, and hence $\gamma$ is a non-decreasing function: Because $\Psi(d)$ is constant for all $d \ge \upsilon(y^*)$, $\gamma$ is constant for all $d \ge \upsilon(y^*)$, but it is strictly increasing for $d < \upsilon(y^*)$. Now we show that

$$\gamma(0) > 0,\ \gamma(d^{\max}) = d^{\max},$$

where $d^{\max}$ is given in Proposition 3. First, as $\Psi(0) = 0$, and $\Psi(x)$ is a concave function, $\gamma(0) > 0$ if and only if $r(1+\beta) < \Psi'(0) = \infty$, which holds by Inada conditions. Moreover, as the two curves $r\,(1+\beta)\,x$ and $\beta\Psi(x)$ intersect at $\gamma(0) \equiv d^{\min} > 0$, by concavity of $\Psi$ we have $\beta\Psi'(d^{\min}) < r(1+\beta)$. Second, by Proposition 3, $d^{\max} > 0$ and $rd^{\max} = \Psi(d^{\max})$. Therefore, $r\,(1+\beta)\,d^{\max} = \Psi(d^{\max}) + \beta\Psi(d^{\max})$ and hence $\gamma(d^{\max}) = d^{\max}$.

Finally, we show that $\gamma$ is a concave function. Applying the implicit function theorem to (37), for all $0 < d < \upsilon(y^*)$,

$$\gamma'(d) = \frac{\Psi'(d)}{(1+\beta)\,r - \beta\Psi'[\gamma(d)]}.$$

Note that $(1+\beta)\,r - \beta\Psi'[\gamma(0)] = (1+\beta)\,r - \beta\Psi'(d^{\min}) > 0$ and hence $(1+\beta)\,r - \beta\Psi'[\gamma(d)] > 0$ for all $d$. By concavity of $\Psi$, $\gamma'(d)$ is decreasing in $d$. Hence, $\gamma$ is a concave function. ■

**Proof of Proposition 5 Proper:**

Notice that, by definition, any pair $(d_0, d_1)$ that satisfies $d_0 \le \gamma(d_1)$ and $d_1 \le \gamma(d_0)$ also satisfies (14) with $y_0 = z(d_0)$ and $y_1 = z(d_1)$, and hence $(d_0, d_1)$ is a 2-period credit cycle. By Lemma 1, $\gamma$ is a concave function with $\gamma(0) > 0$ and $\gamma(d^{\max}) = d^{\max}$, and hence, $\gamma(d) > d$ for all $d \in [0, d^{\max})$, where $d^{\max}$ is given in Proposition 3. Thus, for each $d_0 \in [0, d^{\max})$, the interval $[d_0,\, \gamma(d_0)]$ is nondegenerate and $\gamma(d_0) < d^{\max}$.

Hence, for each $d_1 \in [d_0, \gamma(d_0)]$, $d_0 \le d_1 < \gamma(d_1)$, where we used that $\gamma(d) > d$ for all $d \le \gamma(d_0) < d^{\max}$, so $(d_0, d_1)$ is a 2-period credit cycle. This gives a full characterization of the set of 2-period cycles with $d_0 \le d_1$, and the set of cycles with $d_1 \le d_0$ is its mirror image with respect to the 45° line. Thus, for each $d_0 \in [0, d^{\max})$, the set $\Omega(d_0)$ is a nondegenerate interval.

**Proof of Proposition 6** The proof is by construction. Let $d_T \in (0, d^{\max}]$ be given. We construct $\{d_{T-1}, d_{T-2}, ..., d_0\}$ inductively as follows. Since $d_T > 0$, there is a $d_{T-1} > 0$ such that

$$d_{T-1} < \min\left\{d_T, \beta\left\{\alpha\left[u(y_T) - \upsilon(y_T)\right] + d_T\right\}\right\},$$

where $y_T = \min\{\upsilon^{-1}(d_T), y^*\}$. Suppose that we have chosen $d_{T-s} > 0$. Then, there exists $d_{T-(s+1)} > 0$ such that

$$d_{T-(s+1)} < \min\left\{d_{T-s}, \beta\left\{\alpha\left[u(y_{T-s}) - \upsilon(y_{T-s})\right] + d_{T-s}\right\}\right\},$$

where $y_{T-s} = \min\{\upsilon^{-1}(d_{T-s}), y^*\}$. This completes the induction.

**Proof of Proposition 7** Here we show that for any transition function over $\mathbb{X}$ with a full support, denoted by $\pi$, we have a continuum of sunspot equilibria indexed by $d \in (0, d^{\max})$. For any $d \in (0, d^{\max})$, we have

$$rd < \alpha\{u[z(d)] - \upsilon[z(d)]\}. \tag{38}$$

Fix an element $\chi_0 \in \mathbb{X}$ and let $\mathbb{X}_{-0} = \mathbb{X} - \{\chi_0\}$. Define the set

$$\Omega_{(\mathbb{X},\pi)}(d_{\chi_0}) = \left\{\mathbf{d}_{\mathbb{X}_{-0}} = \langle d_\chi : \chi \in \mathbb{X}_{-0}\rangle : d_\chi \le \beta \int V^b_{\chi'}(\langle d_{\chi_0}, \mathbf{d}_{\mathbb{X}_{-0}}\rangle) d\pi(\chi'|\,\chi) \text{ for all } \chi \in \mathbb{X}\right\}.$$

Let $d_{\chi_0} \in (0, d^{\max})$. Then, by (38), the sequence $\langle d_\chi : \chi \in \mathbb{X}_{-0}\rangle$ with $d_\chi = d_{\chi_0}$ for all $\chi \in \mathbb{X}_{-0}$ is in $\Omega_{(\mathbb{X},\pi)}(d_{\chi_0})$ where all inequalities in the definition above are strict inequalities. Thus, the set $\Omega_{(\mathbb{X},\pi)}(d_{\chi_0})$ contains an open ball with a positive radius centered at $\langle d_\chi : \chi \in \mathbb{X}_{-0}\rangle$ with $d_\chi = d_{\chi_0}$ for all $\chi \in \mathbb{X}_{-0}$. Hence, it has a positive Lebesgue measure in $\mathbb{R}^{|\mathbb{X}_{-0}|}$ and almost all points in it satisfy $d_\chi \ne d_{\chi'}$ for all $\chi \ne \chi'$. Note that for any $\langle d_\chi : \chi \in \mathbb{X}_{-0}\rangle \in \Phi(d)$, $\langle d_\chi ; \chi \in \mathbb{X}\rangle$ with $d_{\chi_0} = d$ is a sunspot credit equilibrium by (18).

# FOR ONLINE PUBLICATION
# Supplementary Appendices

## S1. Equivalence between monetary and credit equilibria

Here we extend the equivalence result, Proposition 4, to other trading mechanisms. We first consider bargaining in the pairwise meetings and then consider Walrasian pricing for large group meetings. We adopt the environment introduced in Section 4 without record-keeping. The monetary trades follow a similar pattern to that in Section 3.3: buyers who cannot commit to deliver goods in the CM use money to buy DM goods from sellers in the DM. They produce CM goods in the first stage of each period in order to sell them for money in the CM. Notice that the timing of producing CM goods (whether it takes place in the first or second stage of each period) is irrelevant for buyers' behavior because it is only incentive-feasible to sell these goods in the CM for money. Sellers use money obtained from DM sales to buy CM goods. Because $\lambda \leq 1$, buyers never produce CM goods for self-consumption. As a result, the parameter $\lambda$ plays no role in monetary equilbria. So with no loss of generality we set $\lambda = 1$.

**Bargaining** Under a general bargaining solution represented by the function $\eta(y)$, the sequence for the values of money, $\{\phi_t\}$, solves

$$\max_{m \geq 0} \left\{\phi_t m + \beta\alpha \left[u(y_{t+1}) - \eta(y_{t+1})\right]\right\}$$

where $\phi_{t+1} m = \eta(y_{t+1})$ for all $t$. Replace $d_t = \phi_t$ for all $t$ in the above problem and take the FOC, we obtain

$$d_t = \beta d_{t+1} \left\{\alpha \left[\frac{u'(y_{t+1})}{\eta'(y_{t+1})} - 1\right] + 1\right\}, \tag{39}$$

where $\eta(y_t) = d_t$ for all $t$. In the credit economy, the debt limits, $\{d_t\}$, solves

$$d_t \leq \beta \left\{\alpha \left[u(y_{t+1}) - \eta(y_{t+1})\right] + d_{t+1}\right\}. \tag{40}$$

Because $\eta$ is concave, $u \circ \eta^{-1}(d_t) - d_t$ is concave in terms of the value of money. The right side of (39), $\left[u'(y_{t+1}) - \eta'(y_{t+1})\right]/\eta'(y_{t+1})$, is the derivative of the function, $u[\eta^{-1}(d_{t+1})] - d_{t+1}$, with respect to $d_{t+1}$. From the strict concavity of the function and the fact that it is equal to 0 when evaluated at $d_{t+1} = 0$,

$$\frac{u'(y_{t+1}) - \eta'(y_{t+1})}{\eta'(y_{t+1})} d_{t+1} < u(y_{t+1}) - \eta(y_{t+1}). \tag{41}$$

From (39) and (41),

$$d_t < \beta\alpha \left[u(y_{t+1}) - \eta(y_{t+1})\right] + \beta d_{t+1}. \tag{42}$$

Iterating (42),

$$d_t < \sum_{j=1}^{J} \beta^j \alpha \left[u(y_{t+j}) - \eta(y_{t+j})\right] + \beta^J d_{t+J}. \tag{43}$$

Applying the transversality condition, $\lim_{J\to\infty}\beta^J d_{t+J}=0$ to (43), we prove that the sequence, $\{d_t\}$, solution to (39) satisfies (40), and hence it is part of a credit equilibrium.

This concavity of $\eta$ is satisfied for the proportional bargaining solution and for the general Nash bargaining solution under the functional forms for $u$ and $v$ that guarantee the concavity of the buyer's surplus.

**Walrasian pricing** Suppose the DM is competitive and $p_t$ denotes the price of DM goods in terms of CM goods. In a monetary economy the buyer chooses money holdings as the solution to:

$$\max_{m,y_{t+1}\geq 0}\left\{-\phi_t m+\beta\alpha\left[u(y_{t+1})-p_{t+1}y_{t+1}\right]+\beta\phi_{t+1}m\right\}, \tag{44}$$

where, $\phi_{t+1}m\geq p_{t+1}y_{t+1}$. The first-order condition for (44) is

$$\phi_t=\beta\phi_{t+1}\left\{\alpha\left[\frac{u'(y_{t+1})}{p_{t+1}}-1\right]+1\right\}.$$

From the seller's maximization problem, $p_{t+1}=v'(y_{t+1})$ so that $\{\phi_t\}$ solves

$$\phi_t=\beta\phi_{t+1}\left\{\alpha\left[\frac{u'(y_{t+1})}{v'(y_{t+1})}-1\right]+1\right\}. \tag{45}$$

It should be noticed that it is the same first-order difference equation as the one obtained under buyers' take-it-or-leave-it offers. Notice, using $\phi_{t+1}=v'(y_{t+1})y_{t+1}$ by market-clearing (i.e., $m=1$), that

$$\phi_{t+1}\left[\frac{u'(y_{t+1})}{v'(y_{t+1})}-1\right]=u'(y_{t+1})y_{t+1}-v'(y_{t+1})y_{t+1}<u(y_{t+1})-v'(y_{t+1})y_{t+1},$$

from the concavity of $u$. Recall that a sufficient condition for the sequence of debt limits to be a credit equilibrium is

$$d_t\leq\beta\left\{\alpha\left[u(y_{t+1})-v'(y_{t+1})y_{t+1}\right]+d_{t+1}\right\}.$$

This proves that the phase of the monetary equilibrium is located to the left of the phase line of the credit equilibrium. Hence, by the same reasoning as before, any outcome of the monetary economy is an outcome of the credit economy.

## S2. Existence of 2-period cycles under alternative mechanisms

**Walrasian pricing** Under Walrasian pricing, $\eta(y) = v'(y)y$. Here we show existence of a continuum of 2-period cycles when $\eta(y)$ is convex. Recall that $z(d) = \min\{\eta^{-1}(d), y^*\}$. Let $d^{\max}$ be the unique positive solution to

$$r\lambda d = \alpha \{u[z(d)] - \eta[z(d)]\}. \tag{46}$$

**Lemma 2** *Suppose that $\eta(y)$ is convex. For each $d_0 \in [0, d^{\max})$, there is a nondegenerate interval, $\Omega(d_0)$, such that for any $d_1 \in \Omega(d_0)$, $(d_0, d_1)$ is a (strict) 2-period cycle.*

**Proof.** Because $\eta(y)$ is convex, there is a unique positive number, denoted $y^{\max}$, such that

$$r\lambda\eta(y^{\max}) = \alpha[u(y^{\max}) - \eta(y^{\max})].$$

It can be verified that that $d^{\max}$ is given by

$$d^{\max} = \begin{cases} \eta(y^{\max}) \text{ if } y^* \geq y^{\max} \\ \frac{\alpha\{u(y^*) - \eta(y^*)\}}{r\lambda} \text{ otherwise.} \end{cases}$$

Note that any $d \in [0, d^{\max}]$ corresponds to a steady-state equilibrium. Let us turn to 2-period cycles. A pair, $(d_0, d_1)$, is a 2-period cycle if for $t = 0, 1$,

$$r\lambda d_t \leq \frac{\alpha\{u[z(d_{t+1})] - \eta[z(d_{t+1})]\} + \beta\alpha\{u[z(d_t)] - \eta[z(d_t)]\}}{1+\beta}. \tag{47}$$

Hence,

$$\Omega(d_0) = \{d_1 \geq 0 : (d_0, d_1) \text{ satisfies (47)}\}.$$

For all $d \in [0, d^{\max})$, because $r\lambda d < \alpha\{u[z(d)] - \eta[z(d)]\}$, $(d_0, d_1) = (d, d)$ satisfies (47) with a strict inequality. Hence, by continuity, there is a nonempty open set contained in $\Omega(d)$. Moreover, because $\eta$ is concave, the set $\Omega(d)$ is convex and hence is a nondegenerate interval. ■

**Nash bargaining** For all $y \leq y^*$, $u(y) - \eta(y) \geq \theta[u(y) - v(y)]$ and hence $\eta(y) \leq (1-\theta)u(y) + \theta v(y)$. Under proportional bargaining a 2-period cycle solves

$$r\lambda[(1-\theta)u(y_t) + \theta v(y_t)] \leq \frac{\{\alpha\theta[u(y_{t+1}) - v(y_{t+1})] + \beta\alpha\theta[u(y_t) - v(y_t)]\}}{1+\beta}.$$

It implies

$$r\lambda\eta(y_t) \leq \frac{\{\alpha[u(y_{t+1}) - \eta(y_{t+1})] + \beta\alpha[u(y_t) - \eta(y_t)]\}}{1+\beta}.$$

Hence $(y_t, y_{t+1})$, and the associated $(d_t, d_{t+1}) = (\eta(y_t), \eta(y_{t+1}))$, is a credit cycle under generalized Nash bargaining.