Optimal Monetary Interventions in Credit Markets

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Abstract

In an environment based on Lagos and Wright (2005) but with two rounds of pairwise meetings, we introduce imperfect monitoring that resembles operations of unsecured loans. We characterize the set of implementable allocations satisfying individual rationality and pairwise core in bilateral meetings. We introduce a class of expansionary monetary policies that use the seignorage revenue to purchase privately issued debts. We show that under the optimal trading mechanism, both money and debt circulate in the economy and the optimal inflation rate is positive, except for very high discount factors under which money alone achieves the first-best. Our model captures the view that unconventional monetary policy encourages lending while it may create inflation.

1   Introduction

The provision of adequate liquidity has become a prominent issue in monetary policy discussions. In particular, central banks now use unconventional monetary policies, such as the creation of lending facilities and the purchase of private debt, to directly provide liquidity to the private sector. Although the purpose of these policies is to stimulate lending in credit markets hampered by a lack of liquidity, their overall effect, particularly their potential inflationary impact, is still under debate.[1]

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[1]Economists and policymakers have expressed concerns about the inflationary impacts of unconventional monetary policies. For example, John Taylor (2007), in his testimony before the Congress, argued that "This large expansion of reserve balances creates risks. If it is not undone, then the bank reserves will eventually pour out into the economy, causing inflation." In turn, Spencer Dale, chief economist of the Bank of England has said that "with little slack in the economy, businesses would put up prices if extra quantitative easing (QE) found its way into consumers' pockets." http://www.theguardian.com/business/2013/mar/15/bank-england-economist-quantitative-easing

There is a substantial literature on the role of liquidity in the real economy. To a large extent, this literature has emphasized that limited commitment, i.e., the inability of individuals to honor their future obligations, is the key friction that renders liquidity, or their lack thereof, a relevant issue. For example, DSGE models with financial frictions (see, e.g., Gertler and Kiyotaki (2010) and Gertler and Karadi (2011)) show that reduced-form borrowing constraints amplify business cycle fluctuations. These models are particularly useful to assess the quantitative effects of shocks and of policy interventions. However, in their study of unconventional monetary policy, Gertler and Karadi (2011) treat shocks to borrowing constraints independently from shocks to fundamentals, and hence the link between the two is missing.[2] This link is crucial for understanding how changes in primitives (e.g., preference shocks, technological shocks) affect liquidity needs as well as optimal policy interventions related to liquidity provision.

Apart from DSGE models, another strand of literature provides an explicit description of economic environments under which either liquid assets or credit arrangements are necessary to conduct transactions. Again, limited commitment is a crucial friction in these environments (see, e.g., Wallace (2010) and Lagos, Rocheteau, and Wright (2014)). We follow this literature and we contribute to it by studying optimal monetary interventions in the credit market. We are particularly interested in the trade-off between the beneficial impacts of interventions on credit transactions and their potential harmful impact through inflation.

In our framework, both money and debt circulate as means-of-payments, and the debt limit is endogenously determined due to limited commitment. The model is based on the Lagos-Wright (2005) environment (LW henceforth) to keep tractability, but we introduce three key modifications. First, we allow for the use of debt by assuming an imperfect monitoring technology which records some actions made by buyers and can be accessed by sellers in some pairwise meetings. Second, we adopt a mechanism design approach to determine whether money or debt is used depending on the characteristics of the meeting, and also determine the terms of trade. Third, we have three stages in each period: the first two stages correspond to the decentralized market (DM) in LW, and the last stage corresponds to the centralized market (CM).

Our monitoring technology resembles the typical operations of unsecured loans, including credit cards and commercial papers. It only records the identities of the agents involved in the transaction and the debt incurred by the buyer. In particular, it does not keep track of transfers of real balances. The recorded histories of a buyer are updated periodically into credit records (e.g., his FICO score or credit rating) which may be accessed through the monitoring technology only by his future partners. The monitoring technology is imperfect in that only a fraction of sellers have access to it, and buyers can only issue debt when the technology is available.

The mechanism design approach has been used to study a pure currency economy in the LW environment with one DM round. Hu, Kennan, and Wallace (2009) (HKW henceforth) show that a constant money supply can achieve any allocation achievable

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[2]Indeed, Gertler and Karadi (2011) acknowledge the need to endogenize the leverage level in their model.

under perfect monitoring. Gu, Mattesini, and Wright (2013) demonstrate a similar point by showing that either money or credit is sufficient under a class of trading mechanisms. As a result, one round of DM is not sufficient for both money and debt to circulate in a meaningful way.[3] In contrast, our model with two rounds of DM, together with limited availability of the monitoring technology, features a relevant role for both money and debt in the optimal trading mechanism.

In the presence of the imperfect monitoring technology, the mechanism-design problem includes the decision of which meetings are monitored so debt can be issued, and which meetings rely on money. Moreover, the trading mechanisms are also set optimally in each meeting, subject to the limited commitment friction. In particular, the amount of debt buyers can issue is limited by this self-enforcing constraint.

Our main result shows that unconventional monetary policy, meant as public provision of liquidity to the private sector, is generically optimal in our environment. It is optimal among all monetary interventions which respect voluntary participation and incentive compatibility, and are constrained by the information released by the monitoring technology.[4] In particular, our policy uses monetary expansions in the CM to purchase private debt issued by buyers in monitored DM meetings. This policy has two implications. First, it alleviates the borrowing constraints faced by the buyers in monitored meetings, which may increase the welfare. Second, it creates inflation which tightens liquidity in non-monitored meetings. The optimal mechanism trades off these implications but generically finds positive inflation optimal.

To resolve this trade-off, the crucial determinant in choosing which meetings to be monitored is the tightness of the meeting in terms of liquidity needs, that is, how restrictive the endogenous borrowing constraint is. To efficiently redistribute liquidity across meetings, it is optimal to monitor the meetings that are tight in liquidity, and to dictate buyers to use money in meetings with abundant liquidity. The optimal mechanism then taxes all buyers through inflation and subsidizes only trades in monitored meetings.

1.1   Related Literature

The use of mechanism design to study optimal monetary policy under both limited commitment and under imperfect monitoring goes back to Cavalcanti and Wallace (1999) and Cavalcanti, Erosa, and Temzelides (1999). Those papers assume indivisible asset holdings and focus on circulation of insider money, while inflation is not emphasized. To allow for divisible asset holdings, we build on later papers that adopt the mechanism design ap-

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[3]Some papers obtain coexistence by introducing additional frictions. For instance, Sanches and Williamson (2010) introduce theft that reduces the benefit of money, and Williamson (2012) considers a cost of using currency.

[4]This implies that enforced lump-sum taxes necessary to implement the Friedman rule is not feasible. Moreover, all taxation or subsidies can only happen in monitored meetings, contingent on recorded information. Therefore, the policy labeled "interest-bearing money" proposed by Andolfatto (2009) and the similar scheme in Wallace (2014) are excluded as well. A similar restriction is also imposed by Gomis-Porqueras and Sanches (2013).

proach to the LW environment. Our construction of optimal mechanisms extend the ones proposed in Hu, Kennan, and Wallace (2009) to an environment with two DM rounds and with credit arrangements. We also borrow the debt-limit construction in Bethune, Hu, and Rocheteau (2014), who study a pure credit economy and relax the "not-too-tight" solvency constraint in Alvarez and Jermann (2001), and extend it to our economy with both money and credit.

A few other papers based on LW also analyze imperfect monitoring and endogenous borrowing constraints in a monetary economy, but with one round of DM. Gu, Mattesini, and Wright (2013) show that money and credit cannot be coessential. Lotz and Zhang (2013) obtain coexistence of money and credit by limiting credit access to a fraction of meetings. However, their result crucially relies on the particular trading mechanism adopted, but would not survive under the optimal trading mechanism, as shown in HKW. In a similar model, Gomis-Porqueras and Sanches (2013) study monetary policies similar to those proposed in Andolfatto (2009), and show the optimality of positive inflation. However, as shown in HKW, under the optimal mechanism, zero inflation rate is optimal in that environment. The endogenous borrowing constraint also appears in Berentsen, Camera, and Waller (2007), who introduce financial intermediaries in the LW model that allow buyers to make deposits or to take a (cash) loan before entering the DM. They find positive inflation optimal to deter buyers from defaulting on their loans. Although we also find positive inflation optimal, in contrast to those papers, our expansionary policy differs significantly from those papers.

There are also other papers with two rounds of DM in the LW framework. Berentsen, Camera, and Waller (2005) study the short-run neutrality of money in a pure currency in such an environment. Guerrieri and Lorenzoni (2009) study the amplification mechanism in a similar model, but introduce credit in one of the DM rounds, with perfect enforcement. Telyukova and Wright (2008) explain the credit card debt puzzle in a model where buyers can use credit in one round of DM but have to use money in the other. Different from our focus, they assume perfect enforcement.

Finally, Deviatov and Wallace (2009) study optimal monetary policy in an environment with debt and money and two rounds of interactions between agents in every period, a DM round and a CM round. In contrast to the LW setup, money is indivisible and the CM round is only used to implement monetary policy. They construct a numerical example where the optimal monetary policy involves loans to monitored agents which is used to fund their purchases in the goods market.[5] These loans bear some resemblance to the optimal policy we obtain in our model. They lack, however, a clear mapping from the primitives of the environment to changes in liquidity and the implied optimal monetary policy.

The paper proceeds as follows. In the next section, we present the environment, define trading mechanisms, strategies and equilibrium. We also present results in the case where the monitoring technology is accessible in all meetings and the case where

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[5]They follow Cavalcanti, Erosa, and Temzelides (1999), and Cavalcanti and Wallace (1999), and assume that the actions of a subset of agents is monitored while the actions of the complementary set are private.

it is never accessible and the supply of money is constant. In section 3 we introduce expansionary monetary policies and characterize the set of implementable allocations under such policies. We consider optimality and relate optimality to the coessentiality of money and debt. We also consider alternative monetary policies. Section 4 presents extensions and section 5 concludes. All the proofs are in the Appendix.

2 Model

In this section, we first introduce the environment. We then define trading mechanisms, strategies, and equilibrium. Finally, we look at implementable allocations in the absence of money and implementable allocations with a constant money supply.

2.1 Environment

Time is discrete and the horizon is infinite. The economy is populated by buyers and sellers. The set of buyers is denoted $\mathbb{B}$ and the set of sellers is partitioned into two subsets, $\mathbb{S}_1$ and $\mathbb{S}_2$ both with measure one. Each period is divided into three stages. Buyers randomly meet sellers in $\mathbb{S}_i$ in stage $i \in \{1, 2\}$, and the probability of a successful meeting is σ_i.[6] There are three goods, one for each stage. At stage $i = 1, 2$, a seller from $\mathbb{S}_i$ can produce x_i units of round 1 good for a buyer at cost $c_i(x_i)$ and the buyer's utility is $u_i(x_i)$. Let x_i^* be the solution to $u_i'(x) = c_i'(x)$. In the last stage, agents meet in a centralized market. In this market, they can all consume and produce, and the utility is linear, represented by z. Agents maximize their life-time expected utility with discount factor δ. We let $\rho = \frac{1-\delta}{\delta}$. We call the first two stages DM rounds and the last stage the CM round.

There exists a record-keeping technology, which keeps track of buyers' trading histories in some meetings. We call a meeting a credit meeting if the technology is accessible, and call a meeting a noncredit meeting otherwise. This technology works as follows. For each buyer $b \in \mathbb{B}$, a recorded history at period t is a triple, $h = (h_1, h_2, h_3) \in H$, such that for $i = 1, 2$, $h_i = (b, s_i, z_{i,c})$ keeps track of the buyer's round i DM promise to the seller, where b is the identity of the buyer, s_i is the identity of the seller, and $z_{i,c}$ is the promise-to-pay in terms of CM good, and $h_3 \in \mathbb{R}$ keeps track of the total repayment.[7] Here we assume that the repayment is first used to repay the seller from $\mathbb{S}_1$, if any, before used to repay the seller from $\mathbb{S}_2$.[8] If the buyer does not meet a seller in round-i, or if the buyer meets a seller but there is no trade, h_i is empty. The recorded history h_i is also empty in noncredit

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[6]The meeting pattern that buyers always meet sellers from $\mathbb{S}_i$ at stage i is special. However, in the Appendix we show that our results are robust to more general meeting patterns.

[7]We are assuming that buyers must settle all debt in the same period. Since the utility function in the CM is linear, this assumption is without loss of generality. Moreover, this assumption implies that the IOU's do not circulate.

[8]This assumption is somewhat arbitrary. We could allow the buyer to choose whom to repay to but this would only complicate the notation without adding any insight.

meetings. There also exists a monitoring technology, comprised of a set of records R and a function, $\omega : R \times H \to R$, which updates the record of the buyer based on his recorded history. This technology is only available to sellers in credit meetings and allows the seller to observe the record $r \in R$ of the buyer. Finally, credit may be limited, i.e., the number of total DM rounds with record-keeping is given by $\ell \in \{0, 1, 2\}$. Henceforth, we say that credit is full if $\ell = 2$, credit is limited if $\ell = 1$, and there is no credit if $\ell = 0$.

Lastly, there is an intrinsically useless, divisible, and storable object, called money. The money supply at the end of period t is given by M_t, and new money is introduced in the economy in the CM round. Let π be the net money growth rate. Thus, for each t, $M_{t+1} = (1 + \pi)M_t$.

Our record-keeping technology resembles the typical operations of unsecured loans. It only records the identities of the agents involved in the transaction and the amount that the buyer promised to pay the seller. In particular, it does not record agents' money holdings or refusal to trade. Moreover, the credit records of the buyer are not public information but only observable to matched sellers in credit meetings.[9]

2.2 Implementation

Trading mechanisms

We study outcomes that can be implemented by proposals from a mechanism designer. Although our technology allows for an arbitrary finite set, R, of credit records and arbitrary updating rule, ω, it is without loss of generality to restrict our attention to the case where $|R| = 2$ and to a particular updating rule that uses debt limits as in Bethune, Hu, and Rocheteau (2014). Thus, a proposal consists of the following objects:

(P1) A subset $C \subseteq \{1, 2\}$ of DM rounds which have access to credit records.

(P2) A sequence of debt limits, $\{D_t\}_{t=0}^{\infty}$, two records, G (Good) and B (Bad); and an updating function ω such that: (i) $\omega(r, \emptyset) = r$ and $\omega(B, h) = B$ for all $h \in H$; (ii) $\omega(G, h) = G$ iff $h_3 \geq \min\{D_t, z_{1,c} + z_{2,c}\}$. We also assume that, if $C = \{1, 2\}$, the seller observes the buyer's available debt limit, $D_t - z_{1,c}$, in the second DM round. Intuitively, D_t sets the maximum amount of debt a buyer can credibly commit to pay in a given period. Indeed, if a buyer enters the CM with total amount of debt $z_c > D_t$, he only has incentive to repay D_t as that would be sufficient to keep him in good standing.[10]

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[9]Our record-keeping technology is much weaker than the notion of memory put forth by Kocherlakota (1998), which includes all actions of all direct and indirect partners of an agent. However, as in Kocherlakota (1998), we assume that the record of a buyer can only be observed by his partners, i.e., it is not publicly observable, as in Cavalcanti and Wallace (1999).

[10]The restriction to debt limits is with loss of generality. In particular, one could prevent a buyer from renegotiating and making a different promise by conferring a bad record in case he does so. We do not allow such punishments for two reasons: first, such punishment seems implausible and would not be robust if one introduces heterogeneity; second, and perhaps more importantly, our results do not depend on this restriction but it simplifies the analysis.

(P3) The proposed trades are given by a function o_i^t defined as follows: if $i \in C$, then

$$o_i^t(m, r, d) = (x, z_{i,c}, z_{i,m}),$$

where m is the buyer's announcement of real balance holdings, r is his record, d is his available credit limit, and $(x, z_{i,c}, z_{i,m})$ is the proposed trade—x is the quantity to be produced by the seller, $z_{i,c}$ is the promise the buyer makes to the seller, and $z_{i,m}$ is the transfer of real balances from the buyer to the seller; if $i \notin C$, then

$$o_i^t(m) = (x, z_{i,m}),$$

where m is the buyer's announcement of real balance holdings and $(x, z_{i,m})$ is the trade.

(P4) The price for money ϕ_t in the CM, and an initial distribution of money holdings μ.

We focus only on stationary proposals

$$\mathcal{P} = [C, D, (o_1, o_2), (\phi, \mu)]. \tag{1}$$

The trading protocol in meetings in the DM is as follows. The buyer first announces his real balances, and then both the buyer and the seller respond with *yes* or *no* to the corresponding proposed trade. If both respond with *yes* then they move to the next stage; otherwise, the meeting is autarkic. If they move to the next stage, the buyer makes a take-it-or-leave-it offer, which is implemented if the seller responds with *yes* while the originally proposed trade by the mechanism is carried out otherwise. In turn, the trading mechanism in the CM stage is as follows. Each buyer chooses whether to repay his promises to the mechanism, and agents trade competitively against ϕ to rebalance their money holdings.

This trading mechanism generalizes the trading protocols considered in Zhu (2008) and Hu, Kennan, and Wallace (2009) (HKW henceforth) to our setting with credit trades. As in those papers, the first stage ensures that the mechanism satisfies individual rationality, and the second stage ensures that it satisfies the pairwise core requirement and hence is coalition-proof.

Strategies and equilibrium

We denote by s_b the strategy of a buyer $b \in B$. In each DM round, s_b maps the buyer's money holding, his record, and the available debt limit, to the buyer's announcement, $m \geq 0$, and to his response $\{yes, no\}$. Obviously, s_b may also differ for credit meetings and non-credit meetings. In turn, conditional on both the buyer and the seller responding with *yes*, s_b gives the buyer offer to the seller. In the CM round s_b maps the buyer's recorded history in the first and second DM rounds to his repayment decisions and to his final money holdings after the CM closes.[11]

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[11]We are assuming that the buyer's strategy does not depend on his private history other than his record, meant as the part of his history which will never be observed by any seller. This assumption is very much in the same spirit as the public perfect equilibrium in the repeated-game literature, and, as far as constrained-efficient allocations are concerned, is without loss of generality.

We denote by s_{s_i} the strategy of a seller $s_i \in \mathbb{S}_i$, where $i \in \{1, 2\}$. In the DM round, the strategy s_i maps the buyer's announced money holding and his record (observable by the seller only in credit meetings) to the seller's response $\{yes, no\}$, and, conditional on both responding yes, another function that maps the buyer's offer to $\{yes, no\}$. We assume that sellers do not carry money across periods.

We restrict attention to symmetric and stationary strategies, and hence a strategy profile may be denoted (s_0, s_1, s_2), where s_0 is the buyer strategy, and s_i is the strategy for sellers from $\mathbb{S}_i$. We define an equilibrium, consisting of a proposal $\mathcal{P}$ and a strategy profile s as follows.

Definition 2.1. An equilibrium is a list

$$E = \langle (s_0, s_1, s_2), [C, D, (o_1, o_2), (\phi, \mu)] \rangle,$$

such that: (i) each strategy is sequentially rational given other players' strategies and the price of money; (ii) the centralized market for money clears at every date; (iii) the number of the total DM rounds with record-keeping per period is limited by ℓ.

Throughout the paper we restrict attention to equilibria with the following characteristics: (1) the buyer always announces the truth about his money holdings, (2) both the buyer and the seller respond with yes in all DM meetings and the buyer always offer the proposed trades; (3) the initial distribution of money across buyers is degenerate - all buyers hold M units of money; (4) buyers in state G repay their promises up to the debt limit at every period. We call such equilibria simple equilibria.

The allocation associated with a simple equilibrium is characterized by a list

$$\mathcal{L}(\mathcal{E}) = [(x_1, x_2^0, x_2^1), (z_1, z_2^0, z_2^1)],$$

where x_1 denotes round-1 DM consumption of the buyer, conditional on meeting a seller; x_2^0 denotes round-2 DM consumption of the buyer, conditional on meeting a seller at round 2 but not meeting a seller at round 1; x_2^1 denotes round 2 DM consumption of the buyer, conditional on meeting a seller at rounds 1 and 2; $z_1 = z_{1,c} + z_{1,m}$ denotes the CM consumption of a round-1 seller, conditional on meeting a buyer; $z_2^0 = z_{2,c}^0 + z_{2,m}^0$ denotes the CM consumption of a round-2 seller, conditional on meeting a buyer who has not matched at round 1; and $z_2^1 = z_{2,c}^1 + z_{2,m}^1$ denotes the CM consumption of a round-2 seller, conditional on meeting a buyer who has matched at round 1.[12]

An allocation $\mathcal{L}$ is implementable if it is implied by the outcome of a simple equilibrium. We remark here that the optimal allocation without taking implementability into account is given by $x_1 = x_1^*$ and $x_2^0 = x_2^* = x_2^1$, where x_1^* and x_2^* solve

$$u_1'(x_1^*) = c_1'(x_1^*) \text{ and } u_2'(x_2^*) = c_2'(x_2^*).$$

To simplify notations and to convey our main insights, in what follows we restrict our attention to the case $\sigma_1 = 1$. In the appendix we consider the case where $\sigma_1 < 1$ and show

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[12]Clearly, the distinction between $(x_2^0, z_{2,c}^0, z_{2,m}^0)$ and $(x_2^1, z_{2,c}^1, z_{2,m}^1)$ is only meaningful if there is match uncertainty in the first DM round, i.e., if $\sigma_1 < 1$.

that our results hold for sufficiently high σ_1's. In this case, an allocation can be denoted by

$$\mathcal{L} = [(x_1, x_2), (z_1, z_2)],$$

where x_i denotes a buyer's round-i DM consumption and z_i denotes CM consumption of a round-i seller. Moreover, we restrict our attention only to allocations that satisfy $z_1 \leq u_1(x_1) \leq u_1(x_1^*)$ and $z_2 \leq u_2(x_2) \leq u_2(x_2^*)$. This restriction is without loss of generality as far as optimal allocations are concerned, but it avoids many uninteresting cases.

2.3 Implementable allocations under full credit and no money

Here we consider the case where credit is full and money has no value. Given an allocation, $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$, the buyer's future surplus at the CM is given by

$$\sum_{t=0}^{\infty} \delta^t \{[u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_1]\} = \frac{1+\rho}{\rho}\{[u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_1]\}.$$

For $\mathcal{L}$ to be implementable, it is then necessary that repaying the promises, which would be $z_1 + z_2$ in equilibrium, and continuing with the equilibrium future payoffs, is preferred to repaying nothing and receiving no trade in all future periods, that is

$$-\rho(z_1 + z_2) \leq [u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_1]. \tag{2}$$

Similarly, for a seller from $\mathbb{S}_i$ to be willing to produce in the DM, his production cost must be covered by his payoffs in the CM, that is,

$$z_1 \geq c_1(x_1) \text{ and } z_2 \geq c_2(x_2). \tag{3}$$

Finally, to ensure the pairwise core requirement, the proposed pairwise round-1 surplus plus the buyer's round-2 surplus should be higher than the pairwise round-1 surplus for the output level $\hat{x}_1$ given by

$$\hat{x}_1 = \min\{x_1^*, c_1^{-1}(z_1 + z_2)\}.$$

Note that the buyer has enough liquidity to induce the seller to produce $\hat{x}_1$ because $c_1(\hat{x}_1) \leq z_1 + z_2$. Formally, the condition is given by

$$u_1(x_1) - c_1(x_1) + \sigma_2[u_2(x_2) - z_2] \geq u_1(\hat{x}_1) - c_1(\hat{x}_1). \tag{4}$$

Indeed, if (4) does not hold, then the buyer would deviate to offering $(\hat{x}_1, \hat{z}_1)$ for some promise $\hat{z}_1$ to make both agents better off than the proposed trade.

The following theorem shows that these three necessary conditions are also sufficient.

Theorem 2.1 (Implementability under Full Credit). *Let $\ell = 2$. An allocation $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$ is implementable if and only if it satisfies (2), (3), and (4).*

To prove the sufficiency of the three conditions, we take $D = z_1 + z_2$ to be the debt limit, and the buyer can keep a good record as long as he pays back at least D from obligations made in the two stages. Conditions (2) and (3) ensure that buyers are willing to repay D in the CM and sellers are willing to produce. To ensure that the buyer has no profitable deviating offers other than the one involving $\hat{x}_1$, we use the HKW mechanism to implement the round-2 allocation so that the buyer can receive a positive round-2 surplus only if his available debt limit when entering round-2 DM is at least z_2. Then, we show that (4) is sufficient to ensure that the deviating offer with $\hat{x}_1$ is not profitable.

2.4 Implementable allocations under no credit

In this section we consider implementable allocations with a constant money supply and without credit. Since there is no credit, money is necessary to implement any positive production. First we remark that conditions (2) and (3) are still necessary for individual rationality: without (2) buyers are better off not participating in any trades; without (3) sellers will not produce. Similarly, (4) is still necessary for the pairwise core requirement. Indeed, if it does not hold, the buyer can deviate and offer $\hat{x}_1$ as output and some monetary payment to make both agents better off. However, with money alone, one more condition is necessary, because the buyer can hold real balances that are only sufficient for round-2 DM trade but skip round 1. Without the record-keeping technology such deviation is not detectable. This leads to the following condition:

$$-\rho z_1 + [u_1(x_1) - z_1] \geq 0. \tag{5}$$

According to condition (5), the surplus for the buyer in round-1 DM, $u_1(x_1) - z_1$, has to be sufficiently large to compensate for his cost of holding z_1 units of real balances across periods.

The following theorem shows that these necessary conditions are also sufficient.

Theorem 2.2 (Implementability under No Credit). *Let $\ell = 0$ and assume that the money supply is constant. An allocation $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$ is implementable if and only if (2), (3), (4), and (5).*

We extend the HKW mechanism to show the sufficiency. However, while in HKW the implementation can be achieved by a mechanism that punishes the buyer with zero surpluses unless he brings at least the right amount of real balances, here in round-2 trades we use a continuous mechanism to ensure a continuous continuation value at the round-1 DM. This is useful because continuity guarantees existence of the proposed trades as the solution to a maximization problem.

Compared to Theorem 2.1, Theorem 2.2 requires an additional condition, (5). As a result of this additional constraint, implementation using a constant money supply and no credit is more restrictive than implementation with full credit.[13] This is necessary for

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[13]Hu, Kennan, and Wallace (2009) show that a constant money supply and no credit achieves the same set of allocations than full credit in the Lagos and Wright model (2009). This underpins the argument that one needs to add more rounds of decentralized trade to deliver a meaningful role for credit.

a theory with coessentiality of money and credit, but not sufficient. In the next section we demonstrate that coessentiality is a feature of our model when we introduce monetary policies.

3   Monetary Interventions and Coessentiality

In this section we focus on the case where $\ell = 1$. Note that, as always, we assume that $\sigma_1 = 1$. We start with a constant money supply. We then consider a class of expansionary policies, and show that, except for very high discount factors, the introduction of this class allows to achieve better allocations.

3.1   Constant money supply

Here we consider constant money supply under limited credit. First we show that money is necessary to implement any allocation that has positive production in both rounds.

Lemma 3.1. *Let $\ell = 1$ and assume that $\phi = 0$. In every implementable allocation, positive production can only occur in credit meetings.*

Lemma 3.1 shows that money is necessary to achieve positive production in noncredit meetings.[14] It turns out that, if credit is limited, money is also sufficient to achieve desirable allocations. In particular, (5) is still necessary in the presence of limited credit, irrespective of whether $C = \{1\}$ or $C = \{2\}$. When $C = \{1\}$, (5) is necessary to ensure that the buyers are willing to repay their debts. Indeed, in the CM the minimum repayment is z_1 and the future surpluses from credit trades is

$$\sum_{t=1}^{\infty} \delta^t [u_1(x_1) - z_1] = \frac{1}{\rho}[u_1(x_1) - z_1],$$

and this implies that (5) is necessary for repayment to be individually rational. Similarly, if $C = \{2\}$, then money is necessary to finance the trades in round-1 DM meetings. Hence, (5) is necessary for otherwise the buyer would prefer to skip round-1 meetings. We have the following lemma.

Lemma 3.2. *Suppose that $\ell = 1$. An allocation $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$ is implementable under a constant money supply only if it satisfies (2), (3), and (5).*

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[14]The key for the result in Lemma 3.1 is the assumption, akin to Kocherlakota (1998), that the record of a buyer who participates in a credit meeting can only be observed by his partners. If his record was publicly observable, as in Kocherlakota and Wallace (1998) and Cavalcanti and Wallace (1999), one could provide conditions under which there exist equilibria where a deviation by seller s in a non-credit meeting with buyer b eventually leads to an action by some agents which reveals the initial deviation to the entire population (see Araujo and Camargo (2013)).

Under $C = \{1\}$, (4) is also necessary because in round-1 DM, the buyer can offer money and issue debt to the seller, and hence the previous logic applies. Under $C = \{2\}$, however, this condition is not necessary. Indeed, if we propose the buyer to issue debt to finance his round-2 trades, then this liquidity cannot be used in round-1 DM's.[15]

3.2 Expansionary monetary policy

Here we introduce monetary interventions and show that it increases the set of implementable allocations. We confine our consideration to schemes without taxation, and discuss taxes later. In the absence of taxes, any monetary intervention increases the money supply and the real effects of such intervention are determined by how the seigniorage revenue is used. In what follows we consider interventions that use the seigniorage revenue to purchase private debt, which we label expansionary monetary (EM) policies. Later, we will compare EM policies with other feasible policies that are consistent with the record-keeping technology and limited commitment frictions, including those usually considered in the literature.[16]

Consider a mechanism under $\ell = 1$ with $C = \{i\}$, that is, round-i DM is has credit meetings. In credit meetings, buyers may issue debts to sellers. The EM policy sets a maximum amount k of debts (in terms of the CM good) that the mechanism will redeem using newly printed money. Precisely, for any recorded promise at period t, $(b, s_i, z_{i,c})$, the mechanism will purchase $\min\{k, z_{i,c}\}$ from the seller. Let π be the net money growth rate. Thus, for each t, $M_{t+1} = (1 + \pi)M_t$ and we focus only on proposals with constant real balances, that is, $\phi_{t+1}M_{t+1} = \phi_t M_t$. Recall that we use M_t to denote the aggregate money stock at the end of period t. Then, if, for each buyer b, $z_{i,c}^b$ is the amount of debt that b has for his stage-i trade (which, obviously, would be zero if the buyer did not meet a seller at stage i), feasibility requires a corresponding inflation rate π such that

$$\int_{b \in \mathbb{B}} \min\{z_{i,c}^b, k\} db = \pi \phi_t M_{t-1}. \tag{6}$$

For implementation, we require that, for every t, the equilibrium issuance of private debts is consistent with the inflation rate and with the amount of debt purchases set by the EM. Formally, a proposal now includes

$$\mathcal{P} = [C, D, (o_1, o_2), (\phi, \mu)],$$

and a EM policy (k, π). We say that a EM policy is active if $k > 0$. An allocation,

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[15]This may be surprising as it is necessary under $\ell = 2$, but it is because of the credit-record updating procedure. In principle, one could punish renegotiation in round-1 trades by imposing a bad record on the buyers and avoid the condition (4). However, such punishment may be unrealistic and, as it turns out, the condition (4) is never binding as far as the constrained-efficient allocation is concerned, regardless of ℓ and C.

[16]For instance, in Lagos and Wright (2005) the seigniorage revenue is returned in a lump-sum fashion. In Andolfatto (2010) and Wallace (2013) the seigniorage revenue is returned conditional on the agent's money holding.

$\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$, is implementable with EM if it is implementable under a proposal $\mathcal{P}$ and an EM policy (k, π).[17]

Here we give give a full characterization of implementable allocations with EM under $\ell = 1$. We distinguish two cases: the first uses $C = \{1\}$ while the second uses $C = \{2\}$.

Theorem 3.1 (Expansionary Monetary Policies). *Suppose that $\ell = 1$.*

(i) An allocation, $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$, is implementable with EM and $C = \{1\}$ if and only if (2), (3), and (4).

(ii) An allocation, $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$, is implementable with EM and $C = \{2\}$ if and only if (3), (5), and

$$\{-\rho z_1 + [u_1(x) - z_1]\} + \frac{(\rho + 1)\sigma_2}{\rho + \sigma_2}\{-\rho z_2 + \sigma_2[u_2(x_2) - z_2]\} \geq 0. \tag{7}$$

Theorem 3.1 shows that, if $C = \{1\}$, full credit and the EM policy implement exactly the same set of allocations. If (5) holds, the EM policy is inactive and, as shown in Theorem 2.2, a constant money supply suffices even if credit is not used. However, if (5) does not hold, an active EM policy is necessary to implement the allocations achieved with full credit. We also obtain that, if $C = \{2\}$, an active EM policy can implement allocations which cannot be implemented with full credit. Indeed, there are allocations which do not satisfy (2) but satisfy (3), (5), and (7). As we shall see later, these may include constrained-efficient allocations. The intuition for this result runs as follows. Absent the EM policy, a buyer who participates in the second DM round incurs a cost z_2 in the CM round in order to redeem the debts issued in exchange for the round-2 good. This is true irrespective of whether credit is full or limited. In the presence of the EM policy, the cost associated the issuance with the same number of debts is given by $z_2 - k + \pi z_1$, i.e., the direct cost of redeeming part of the debts issued by the buyer himself, and the indirect cost of holding z_1 real balances to participate in the first DM round when the inflation rate is given by π. Feasibility of the EM policy implies $\pi z_1 = \sigma_2 k$, and we can rewrite the cost $z_2 - k + \pi z_1$ is equal to $z_2 - (1 - \sigma_2)k$, which is lower than z_2 whenever $\sigma_2 < 1$. In other words, the EM policy allows for a redistribution of the production costs in the second DM round.[18]

3.3   Optimal monetary policy and coessentiality

Here we study the optimal allocations among implementable allocations. Our main focus is on necessity of the EM policy to achieve such optimal allocations. We first define the

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[17]Alternatively, we can formulate the expansionary monetary policy as a proportional subsidy. More precisely, the policy sets a fraction κ for credit stage i, and the mechanism commits to purchase κ fraction of the debts issued by buyers to stage-i sellers. To avoid buyers from overissuing their debts one can choose the debt limit appropriately and it can be shown that, in terms of implementability, this scheme is equivalent to the scheme considered above.

[18]We are assuming that the record keeping technology and the monitoring technology cannot implement such redistribution schemes. If this assumption is relaxed, then the EM policy cannot do better than full credit.

social welfare function. Again, here we focus on the case where $\sigma_1 = 1$ and $\ell = 1$. For a given allocation, $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$, its welfare is given by

$$\mathcal{W}(\mathcal{L}) = \sum_{t=0}^{\infty} \delta^t \left\{ [u_1(x_1) - c_1(x_1)] + \sigma_2[u_2(x_2) - c_2(x_2)] \right\} \tag{8}$$

$$= \frac{1+\rho}{\rho} \left\{ [u_1(x_1) - c_1(x_1)] + \sigma_2[u_2(x_2) - c_2(x_2)] \right\}. \tag{9}$$

We say that an allocation is *constrained efficient* if it maximizes the social welfare given by (8) among all implementable allocations under $\ell = 1$ with EM. Although our focus is on constrained-efficient allocations under $\ell = 1$, results about constrained-efficient allocations under $\ell = 0$ and 2 will be useful to understand the constrained-efficient allocations under $\ell = 1$.

We remark here that to maximize the social welfare, it is without loss of generality to have the constraint (3) binding, i.e., to consider only allocations of the form $\mathcal{L} = [(x_1, x_2), (c_1(x_1), c_2(x_2))]$, and, hence, we also say that a pair, (x_1, x_2), is a constrained-efficient allocation if $[(x_1, x_2), (c_1(x_1), c_2(x_2))]$ is a constrained-efficient allocation. Note that this applies to all cases: $\ell = 0, 1, 2$.

In particular, if the first-best allocation is implementable under $\ell = 0$ and a constant money supply, then credit is not essential in the sense that it is not needed to implement the constrained-efficient allocation. By Theorem 2.2, to determine whether a first-best allocation, (x_1^*, x_2^*), is implementable under $\ell = 0$ amounts to check whether the conditions (2) and (5) hold under that allocation, and we have the following corollary. Note that (4) is trivially satisfied for any first-best allocation.

Corollary 3.1. *The first-best allocation, (x_1^*, x_2^*), is implementable under $\ell = 0$ and with a constant money supply if and only if*

$$\rho \leq \rho^M \equiv \min \left\{ \frac{[u_1(x_1^*) - c_1(x_1^*)] + \sigma_2[u_2(x_2^*) - c_2(x_2^*)]}{c_1(x_1^*) + c_2(x_2^*)}, \frac{u_1(x_1^*) - c_1(x_1^*)}{c_1(x_1^*)} \right\}. \tag{10}$$

The first term inside the min operator corresponds to (2) with $((x_1, x_2), (z_1, z_2)) = ((x_1^*, x_2^*), (c_1(x_1^*), c_2(x_2^*)))$, and the second term corresponds to (5). By Corollary 3.1, when $\rho \leq \rho^M$, credit is not essential, and we are only interested in the case where $\rho > \rho^M$. To study the constrained-efficient allocations for $\rho > \rho^M$, it is useful to first consider the constrained-efficient outcomes under $\ell = 2$.

Lemma 3.3. *The allocation (x_1^C, x_2^C) is the constrained-efficient allocation under $\ell = 2$ if and only if it maximizes the social welfare, (8), subject to (2) with $(z_1, z_2) = (c_1(x_1), c_2(x_2))$.*

Lemma 3.3 shows that in order to find the constrained-efficient allocation, only the participation constraint (2) is relevant, but the pairwise core requirement (4) is not binding.

As shown below, (x_1^C, x_2^C) is always implementable under $\ell = 1$ with EM. Thus, whenever such allocation is not implementable under $\ell = 0$, coexistence of money and

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credit is necessary to implement (x_1^C, x_2^C), and, by Lemma 3.2, EM is also necessary. Hence, both money and credit are coessential and EM is essential. Moreover, we will show that even when (x_1^C, x_2^C) *is* implementable under $\ell = 0$, as long as it is not the first-best, EM can actually implement an even better allocation achieved by setting $C = \{2\}$. Let (x_1^{C2}, x_2^{C2}) be the allocation that maximizes (8) subject to (5) and (7) with $(z_1, z_2) = (c_1(x_1), c_2(x_2))$, that is, (x_1^{C2}, x_2^{C2}) is the optimal allocation among those implementable with EM and with $C = \{2\}$, as shown in Theorem 3.1 (ii). Compared against conditions for implementability under $\ell = 0$, although (5) is the same, (7) relaxes (2), and hence, as we will see later, it helps implement better allocations.

The essentiality of credit and hence of EM, however, would fail when $u_1(x) = u_2(x)$, $c_1(x) = c_2(x)$ for all x, and $\sigma_2 = 1$. However, it will fail only in such knife-edge cases but credit is essential generically. To define the genericity we need some more notation. For any $\rho > 0$, define $(\bar{x}_1, \bar{x}_2)$ as the unique positive solution to

$$u_1(\bar{x}_1) - (1 + \rho)c_1(\bar{x}_1) = 0 = \sigma_2 u_2(\bar{x}_2) - (\sigma_2 + \rho)c_2(\bar{x}_2). \tag{11}$$

Generically, $(\bar{x}_1, \bar{x}_2) \neq (x_1^C, x_2^C)$, as the latter has to satisfy the FOC's implied by the maximization problem as well. We have the following theorem.

Theorem 3.2. *Suppose that $\ell = 1$ and $\sigma_1 = 1$.*

(1) Both (x_1^C, x_2^C) and (x_1^{C2}, x_2^{C2}) are implementable with EM, and either one of them is the constrained-efficient allocation.

(2) Suppose that $\rho > \rho^M$, $\sigma_2 < 1$, and that $(\bar{x}_1, \bar{x}_2) \neq (x_1^C, x_2^C)$. Then, the constrained efficient allocation can only be implemented with EM but not with constant money supply.

Theorem 3.2 (1) shows that the constrained efficient allocation may be either (x_1^C, x_2^C) or (x_1^{C2}, x_2^{C2}). Later on we provide some examples for which either one of them is the constrained efficient allocation. Theorem 3.2 (2) then show that, except for the case where money alone can implement the first-best, generically, when (x_1^C, x_2^C) is the constrained-efficient allocation, the optimal mechanism has $C = \{1\}$, and when (x_1^{C2}, x_2^{C2}) is the constrained-efficient allocation, the optimal mechanism has $C = \{2\}$. Therefore, in our theory, not only credit and money are coessential, but the determination of the credit sector is endogenous and depends on the economic fundamentals. Moreover, although the EM policy is essential, the nature of the optimal policy will depend largely on the fundamentals as well. We illustrate these results by some comparative statics.

Comparative Statics

Here we give some examples to illustrate how the optimal EM policy may respond to productivity shocks. Here we only focus on comparative statics across steady states, but our model can be extended to allow for persistent shocks. We also restrict attention only to the case where the first-best is implementable, but the insights here extend to the other cases as well.

We consider the following functional forms: for $i = 1, 2$,

$$u_i(x_i) = \theta_i x_i^{a_i} \text{ and } c_i(x_i) = x_i, \ \sigma_i = 1.$$

Under this functional form, the first-best are given by

$$x_i^*(\theta_i) = (\theta_i a_i)^{\frac{1}{1-a_i}}, \ i = 1, 2.$$

Here we consider only $C = \{1\}$. The other case, $C = \{2\}$, can be done in a symmetric manner. By Theorem 3.1 (i), the first-best allocations are implementable under $C = \{1\}$ if and only if (2) holds for $(x_1^*(\theta_1), x_2^*(\theta_2))$, that is,

$$\theta_1^{\frac{1}{1-a_1}} a_1^{\frac{a_1}{1-a_1}} - (1+\rho)(\theta_1 a_1)^{\frac{1}{1-a_1}} + \theta_2^{\frac{1}{1-a_2}} a_2^{\frac{a_2}{1-a_2}} - (\sigma_2 + \rho)(\theta_2 a_2)^{\frac{1}{1-a_2}} \geq 0. \tag{12}$$

Moreover, implementation of $(x_1^*(\theta_1), x_2^*(\theta_2))$ requires an active EM policy if

$$\theta_1^{\frac{1}{1-a_1}} a_1^{\frac{a_1}{1-a_1}} - (1+\rho)(\theta_1 a_1)^{\frac{1}{1-a_1}} < 0,$$

that is, if

$$a_1^{\frac{a_1}{1-a_1}} - (1+\rho)a_1^{\frac{1}{1-a_1}} < 0. \tag{13}$$

Moreover, when this happens, the optimal policy with lowest inflation rate is given by

$$k(\theta_1) = \theta_1^{\frac{1}{1-a_1}} \left\{ a_1^{\frac{1}{1-a_1}} - \frac{1}{1+\rho} a_1^{\frac{a_1}{1-a_1}} \right\}, \ \pi(\theta_1, \theta_2) = \frac{k(\theta_1)}{x_2^*(\theta_2)}.$$

Thus, under this functional form, the minimum optimal inflation rate increases with θ_1 but decreases with θ_2. This implies that, to determine the optimal monetary policy, not only how the shock affects the overall economy matters, but how the shock affects the credit sector relative to the money sector also matters. In particular, if the shock is more beneficial to the credit sector, i.e, if θ_1 increases more, then the optimal inflation rate is pro-cyclical. In contrast, if the shock is more beneficial to the money sector, i.e, if θ_2 increases more, then the optimal inflation rate should be counter-cyclical.

To illustrate these findings, we provide some numerical examples. We use the following numbers: $\rho = 2\%$, $a_1 = 0.99$, $a_2 = 0.97$, $\sigma_2 = 1$, θ_1 and θ_2 are controlled by a parameter η as follows:

$$\theta_1 = 2 \times \eta \text{ and } \theta_2 = \frac{0.17}{1-\eta}.$$

Under this parametrization, both θ_1 and θ_2 increase with η, but the relative increase depends on the value of η. When η is large, θ_2 increases more than θ_1 and vice versa for η small. It can be verified that when $\eta \in [0.49, 0.52]$, both (12) and (13) hold, and hence the first-best is implementable, but the EM is active with $C = \{1\}$. As discussed above, we expect the optimal inflation rate to increase with η when θ_1 increases more than θ_2, that is, when η is relatively small, while it decreases with η when θ_1 increases more than θ_2, that is, when η is relatively large. The following figure illustrates this result: the optimal inflation rate, π, measured in the vertical axis, first increases and then decreases with η, measured in the horizontal axis.
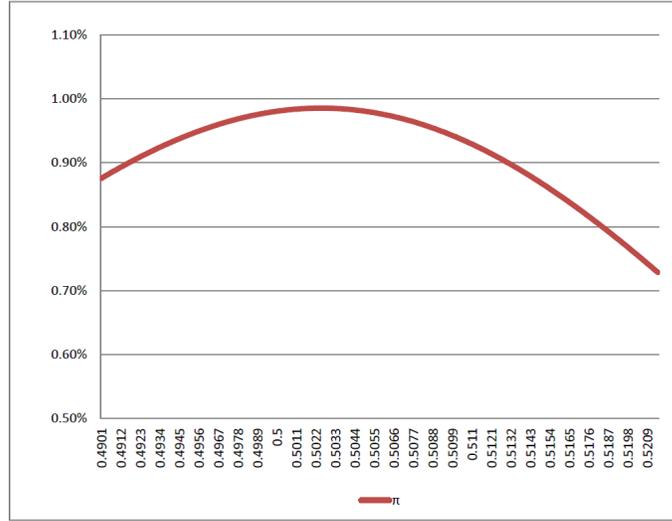
These results demonstrate that liquidity needs are endogenously determined by the fundamentals. Moreover, the optimal policy response in terms of liquidity provision requires a detailed knowledge about how shocks to the fundamentals affect different sectors, especially the distribution between the cash and the credit sectors, in the economy.

3.4 Alternative monetary policies

Here we consider alternative monetary policies and show that the EM policy dominates in terms of social welfare. We only allow policies that are consistent with our record-keeping technology. This has two implications. First, as there is no record of agents' money holdings, the mechanism cannot pay interest on money conditional on agents' money holdings.[19] Second, we do allow the mechanism to tax the agents, but such taxation has to be voluntary in the sense that the taxes, or fees, are tied to the use of the record-keeping technology and the only punishment is to give a bad credit record. Hence, we consider only two types of alternative policies: lump-sum transfers and deflationary policies that use taxes from credit meetings.

Lump-sum transfers of money

The most commonly discussed monetary policy in the literature is the lump-sum transfer of money. Here we assume that, as typically in the literature, that newly created money

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[19]Examples of such schemes include the interest-bearing money in Andolfatto (2010) and the progressive and regressive schemes in Wallace (2013). The mechanism specifies the amounts of money transferred to an agent as a function of the agent's money-holdings. As pointed out in Sanches and Gomis-Porqueras (2013), such schemes require some record-keeping. Indeed, the mechanism has to keep track of each agent's transfer to avoid double withdraw. Moreover, as pointed out in Andolfatto (2010), it is also crucial to avoid agents to pool their money holdings together to obtain higher transfers.

is given to all buyers with equal amount in a lump-sum fashion before the CM opens.[20] Let π be the net money growth rate and let M_t be the average money holdings at the beginning of period t. Then, the policy gives each buyer πM_t units of money at the beginning of period-t CM in a lump-sum fashion. The following theorem shows that such policy shrinks the set of implementable outcomes even against constant money supply.

Theorem 3.3 (Implementability under lump-sum transfers). *Let $\ell = 1$ and assume that net money growth rate is π with lump-sum transfers. Let $\zeta = (1 + \pi - \delta)/\delta \geq \rho$. Then, an allocation, $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$, is implementable under $\pi > 0$ only if it satisfies (3), (5), and*

$$-\rho z_1 - \zeta z_2 + [u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_2] \geq 0, \tag{14}$$

or

$$-\zeta z_1 - \rho z_2 + [u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_2] \geq 0. \tag{15}$$

Compared against Lemma 3.2, the conditions in Theorem 3.3 are more restrictive: (14) and (15) are more restrictive than (2) while (3) and (5) are the same. As in Lemma 3.2, (4) is necessary under $C = \{1\}$ but may not be necessary under $C = \{2\}$. Because (4) is never binding even for constrained-efficient allocations subject to constant money supply, these results imply that such inflation is never optimal even against a constant money supply.

Deflationary monetary policy

Now we turn to interventions that use taxes or fees. To be consistent with our environment, we assume that the mechanism may tax the agent only if the agent is in a credit meeting and decides to engage in a credit trade. Such taxes can be thought of as a fee to use the credit system. In particular, this implies that the lump-sum taxes are not feasible. The only punishment for not paying the fees is to give the individual a bad credit record. We consider interventions that use the fee revenue to buy back money and hence provides interest on money holdings. We call such interventions *deflationary monetary policy* (DMP).

Consider a mechanism where round-i DM has credit meetings. Then buyers may issue debts that are recorded for meetings where the technology is available. The DMP sets a fee (in terms of the CM good), η, on the use of the record-keeping technology and then buy back money with those revenues. Therefore, if a buyer b chooses to accept a credit trade at period t, the buyer has to pay extra η to keep his good record. Let τ be the net money contraction rate. Thus, for each t, $M_{t+1} = (1 - \tau)M_t$ and we focus only on

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[20]Because of the lump-sum nature, the assumption that only buyers receive the transfers and the assumption that every buyer receive the same amount are not crucial. For example, the result would be exactly the same if the money transfer is given randomly in a lump-sum fashion. In fact, this random transfer would be more consistent with our environment because it does not require the money to be sent to each agent, which may require some monitoring on agents' money holdings.

proposals with constant real balances, that is, $\phi_{t+1} = \phi_t/(1-\tau)$. Then, if β measure of buyers use the credit trades, feasibility requires a corresponding deflation rate τ such that

$$\beta\eta = \tau\phi_t M_{t-1}. \tag{16}$$

An allocation is implementable with (η, τ) that satisfies (16) if there exists some proposal $\langle \mathcal{P}, (\eta, \tau) \rangle$ such that the allocation is consistent with the simple equilibrium outcome under such proposal. Note that if an outcome is implementable under a constant money supply, it is implementable with DMP. We have the following theorem.

Theorem 3.4 (Deflationary Monetary Policy). *Suppose that $\ell = 1$. An allocation, $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$, is implementable with DMP only if either*

(i) it satisfies (2), (3), and (5), or

(ii) it satisfies (3) and

$$\frac{\sigma_2 + \rho}{(1+\rho)\sigma_2}\{-\rho z_1 + [u_1(x_1) - z_1]\} + \{-\rho z_2 + \sigma_2[u_2(x_2) - z_2]\} \geq 0, \tag{17}$$

$$-\rho z_2 + \sigma_2[u_2(x_2) - z_2] \geq 0. \tag{18}$$

The necessary conditions in Theorem 3.4 are almost sufficient. What is missing is the pairwise core requirement, which may be required in some cases. In particular, to implement the proposed allocation under $C = \{1\}$ always requires (4), but not so under $C = \{2\}$. Because the DMP can use $C = \{2\}$ to implement an allocation that does not satisfy (5) while EM can only use $C = \{1\}$ for such allocations, there may be allocations that are not implementable with EM due to (4) but are implementable with DMP. However, (4) never binds for constrained-efficient allocations.[21] Ignoring the requirement (4), Theorem 3.4 shows that implementation with DMP is more restrictive than with EM. Indeed, any allocation satisfies (4) and conditions in Theorem 3.4 (i) can also be implemented with constant money supply, while any allocation that satisfies (4) and conditions in Theorem 3.4 (ii), according to Theorem 3.1 (i), can also be implemented with EM under $C = \{1\}$. Notice that the conditions (17) and (18) are more restrictive than (2). In fact, one can show that unless the DMP can implement the first-best, EM strictly dominates DMP generically.

4   Concluding Remarks

Here we discuss the robustness of our results to some assumptions and some possible extensions.

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[21]This result holds only for sufficiently high σ_1's. As a result, DMP may be useful for low σ_1's.

4.1 Alternative meeting patterns

Meeting probabilities: $\sigma_1 < 1$

Most of our results extend to the case where $\sigma_1 < 1$. First note that the first-best allocation is independent of σ_1. Hence, using the same logic of Theorem 3.2, we can show that for any $\sigma_1 \leq 1$, generically, there exists a $\rho^0 > \rho^M$ such that for all $\rho \in (\rho^M, \rho^0]$, the first-best is implementable only with EM and hence the optimal inflation rate is positive. Moreover, by continuity, we can show that the optimality of EM extends to lower ρ's as well: there exists $\rho^1 > \rho^0$ such that for all $\rho \in (\rho^0, \rho^1]$, the first-best is not implementable but the constrained-efficient allocation is only implementable with EM and hence the optimal inflation rate is positive. As a result, for any σ_1, unless money alone implements the first-best, the EM policy is generically optimal as long as the discount factor is not too low.

Meeting both seller types at both rounds

Our results are also robust to other meeting patterns as well. In particular, one special feature in our model is that buyers can only meet sellers from $\mathbb{S}_i$ at stage i. However, our results are robust to other arrangements as well. In particular, we can accommodate the following setting. In round-1 DM, a buyer may meet a seller from $\mathbb{S}_1$ or $\mathbb{S}_2$ or none. The probability of a successful meeting is $2\sigma_1 \leq 1$ and the probability of meeting a seller from $\mathbb{S}_j$ is σ_1 for both $j = 1, 2$. For simplicity we assume that only buyers with a successful meeting at round-1 has a chance to meet a seller from a different sector at round-2, which happens with probability $\sigma_2/\sigma_1 \leq 1$.[22] We also assume that a seller may meet at most one buyer at each period. Note that the ex ante probability of a buyer to meet a sector-j seller is $\sigma = \sigma_1 + \sigma_2$ for both $j = 1, 2$. We can show that under this setting the main result, Theorem 3.2 still hold.

4.2 Multi-stage DM's

We can also introduce many rounds of DM's, say, T rounds. It is then straightforward to derive conditions for implementation of the first-best allocations using our techniques. In particular, for $T \geq 3$, it can be the case that both money and credit are necessary to circulate in any optimal mechanism when $\ell < T$, and the EM policy may not be necessary for some parameter values. Nevertheless, EM policy with positive inflation will still be optimal for a large set of parameters (among all feasible policies).

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[22]This assumption allows us to focus on symmetric allocations across different meeting patterns for different buyers, and it plays a very similar role as $\sigma_1 = 1$ in previous sections. Results in this new setting are robust to this assumption in the same way as results in previous sections are robust to the assumption $\sigma_1 = 1$.

4.3 Other assets

In our model we only allow unsecured credit arrangements. In reality many credit arrangements involve both collateral and unsecured elements. It then may be fruitful to introduce other assets, such as capital or nominal bonds, and study optimal monetary policy. However, one difficulty in such a model is how to obtain coexistence. One possible route is provided by Hu and Rocheteau (2013, 2014), who show that coexistence of money and assets with higher rate-of-returns can coexist under the optimal monetary policy. We conjecture that, in the presence of an endogenous borrowing constraint, policy analogous to our EM may still be beneficial. However, the provision of public liquidity may come for other sources, such as the government bonds (as in Gertler and Karadi (2010)).

5 Appendix: Proofs

Proof of Theorem 2.1

($\Rightarrow$) First we prove necessity. We have proved the necessity of (2) and (3) in the text. Now consider (4). Suppose that it does not hold and hence

$$u_1(\hat{x}_1) - c_1(\hat{x}_1) > [u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_2] + [z_1 - c_1(x_1)]. \tag{19}$$

Note that the first two terms of the right side of (19) is the expected surpluses for the buyer (from the 2 DM's) and the last term is the surplus for the round-1 seller. By (19) there exists $\hat{z}_1 \in (0, z_1 + z_2]$ such that

$$u_1(\hat{x}_1) - \hat{z}_1 > [u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_2] \text{ and } \hat{z}_1 - c_1(\hat{x}_1) > z_1 - c_1(x_1),$$

and hence the buyer has a profitable deviation to offering $(\hat{x}_1, \hat{z}_1)$. Note that this deviation does not affect the buyer's credit record as long as he repays $\hat{z}_1$ in the CM.

($\Leftarrow$) Here we prove sufficiency. First we formulate the proposed trades on the equilibrium path. The debt limit is given by $D = z_1 + z_2$. Because money has no value, a trade (x, z) only consists of output x and promise z and the proposed trade has no real balances as an argument; moreover, in state B the debt limit is 0 and is omitted. In state B, the buyer always gets no trade: $o_i(B) = (0,0)$ for $i = 1, 2$. In state G, $o_1(G, D) = (x_1, z_1)$ and $o_2(G, D - z_1) = (x_2, z_2)$. Because $u_i(x_i) \geq z_i \geq c_i(x_i)$, both agents are willing to accept the proposed trade against no trade for both rounds. However, it remains to show that the buyer has no profitable deviating offers at both DM rounds, and we need to specify $o_2(G, d)$ for any $d \in [0, D]$.

Let $\xi(d) = u_2(x_2) - z_2$ if $d \geq D - z_1 = z_2$ and let $\xi(d) = 0$ if $d < z_2$. The value $\xi(d)$ will be the buyer's surplus in round-2 DM if his available debt limit is d when entering that round. Then, $o_2(G, d)$ solves

$$\max_{(x,y)\in\mathbb{R}_+\times[0,d]} -c_2(x) + y \tag{20}$$

$$\text{s.t. } u_2(x) - y \geq \xi(d).$$

The solution to (20) exists for all $d \in [0, D]$ and is unique. We first show that $o_2(G, z_2) = (x_2, z_2)$ and then we show that there is no profitable deviating offer for the buyer at round 1. For the first claim, suppose, by contradiction, that $(x, y) \neq (x_2, z_2)$ is the solution to (20). Then, $y \leq z_2$ and (note that we assume that the buyer repays their debt up to D in the CM after the deviating offer, which is optimal because of (2))

$$-c_2(x) + y > -c_2(x_2) + z_2 \text{ and } u_2(x) - y \geq u_2(x_2) - z_2$$

and hence

$$u_2(x) - c_2(x) > u_2(x_2) - c_2(x_2).$$

Because $u_2(x_2^*) \geq u_2(x_2) \geq c_2(x_2)$, it follows that $x > x_2$ and hence $y > z_2$, a contradiction. Thus, $o_2(G, z_2) = (x_2, z_2)$. This also implies that the buyer has no profitable deviating offer at round-2 DM. Finally, we show that the buyer has no profitable deviating offer at round-1 DM. Suppose, by contradiction, there exists such a profitable deviating offer, (x, y), at round-1 DM. Then,

$$u_1(x) - y + \sigma_2 \xi(D - y) > u_1(x_1) - z_1 + \sigma_2 \xi(z_2) \text{ and } y - c_1(x) \geq z_1 - c_1(x_1),$$

and hence

$$u_1(x) - c_1(x) + \sigma_2 \xi(D - y) > u_1(x_1) - c_1(x_1) + \sigma_2[u_2(x_2) - z_2]. \tag{21}$$

Consider two cases.

(a) $y \leq z_1$. Then, $c_1(x) - c_1(x_1) \leq y - z_1 \leq 0$ and hence $x \leq x_1 \leq x_1^*$. Thus,

$$u_1(x) - c_1(x) + \sigma_2 \xi(D - y) \leq u_1(x_1) - c_1(x_1) + \sigma_2 \xi(D - z_1).$$

Note that because $y \leq z_1$, $\xi(D - y) = \xi(z_2)$, and this leads to a contradiction to (21).

(b) $y > z_1$. Then, $\xi(D - y) = 0$ and hence, by (21),

$$u_1(x) - c_1(x) > u_1(x_1) - c_1(x_1) + \sigma_2[u_2(x_2) - c_2(x_2)].$$

However, because $y \leq D$, it follows that $u_1(x) - c_1(x) \leq u_1(\hat{x}_1) - c_1(\hat{x}_1)$, and this leads to contradiction to (4).

Proof of Theorem 2.2

($\Rightarrow$) First we prove necessity. The necessity of (3) is clear. We proved the necessity for (5) in the text. Consider (2). Note that in equilibrium the buyer has to hold at least $z_1 + z_2$ units of real balances. Following the equilibrium path, the buyer's total payoff from CM to the next CM is at most

$$-(z_1+z_2)+\delta\{u_1(x_1)+\sigma_2 u_2(x_2)+(1-\sigma_2)z_2\} = \delta\{-\rho(z_1+z_2)+[u_1(x_1)-z_1]+\sigma_2[u_2(x_2)-z_2]\},$$

while deviating to holding zero balances in the CM the buyer can guarantees himself a zero payoff. Hence, (2) is necessary. The necessity of (4) follows exactly the same argument as in the proof of Theorem 2.1.

($\Leftarrow$) Here we prove sufficiency. First we formulate the proposed trades on the equilibrium path. The real balance buyers hold at the end of the CM is given by $z_1 + z_2$. Because there is no credit meetings, a trade (x, z) only consists of output x and money transfer z and the proposed trade has no debt limit or credit record as an argument. On equilibrium path, $o_1(z_1 + z_2) = (x_1, z_1)$ and $o_2(z_2) = (x_2, z_2)$. By standard arguments in the Lagos-Wright model, the CM value function is linear, that is, $W(m) = m + W(0)$, where $W(m)$ is the continuation value for a buyer entering the CM with m units of real balances. Because $u_i(x_i) \geq z_i \geq c_i(x_i)$, both agents are willing to accept the proposed trade against no trade for both rounds. However, it remains to show that the buyer has no profitable deviating offers at both DM rounds and the buyer is willing to hold $z_1 + z_2$ real balances to leave the CM, and we need to specify $o_1(m)$ and $o_2(m)$ for all m.

Let $\epsilon \in (0, z_2)$ be so small that

$$\epsilon < \frac{1}{2} \min \left\{ \frac{c_1'(x_1)\sigma_2[u_2(x_2) - z_2]}{u_1'(x_1) - c_1'(x_1)}, \frac{\sigma_2[u_2(x_2) - z_2]}{\rho} \right\}. \tag{22}$$

Let

$$\xi(m) = \begin{cases} u_2(x_2) - z_2 & \text{if } m \geq z_2; \\ 0 & \text{if } z \leq z_2 - \epsilon; \\ \left[1 - \frac{z_2 - m}{\epsilon}\right] [u_2(x_2) - z_2] & \text{if } m \in (z_2 - \epsilon, z_2). \end{cases}$$

Note that ξ is a piecewise linear continuous function. Then, $o_2(m)$ solves

$$\max_{(x,y)\in\mathbb{R}_+ \times [0,m]} -c_2(x) + y \tag{23}$$
$$\text{s.t. } u_2(x) - y \geq \xi(m).$$

The solution to (23) exists for all m and is unique with the constraint binding. Moreover, following exactly the same arguments as in the proof of Theorem 2.1, $o_2(z_2) = (x_2, z_2)$. This also implies that the buyer has no profitable deviating offer at round-2 DM.

Now we formulate $o_1(m)$. Let $\eta(m) = u_1(x_1) - z_1 + \sigma_2\xi(z_2)$ if $m \geq z_1 + z_2$ and let $\eta(m) = \sigma_2\xi(m)$ otherwise. For each $m \in \mathbb{R}_+$, let $o_1(m)$ be a solution to

$$\max_{(x,y)\in\mathbb{R}_+ \times [0,m]} -c_1(x) + y \tag{24}$$
$$\text{s.t. } u_1(x) - y + \sigma_2\xi(m - y) \geq \eta(m).$$

Because ξ is continuous, a solution to (24) exists.

For any solution to the problem, the constraint is binding. If not, then either $y < m$ and we can increase y slightly to increase the seller surplus without violating the constraint, or $y = m$ and hence $x > 0$ and we can decrease x slightly to increase the seller surplus without violating the constraint. Thus, although there may be multiple solutions, we can pick any one of them and we know that under o_1, the expected buyer surpluses from the two DM rounds for a buyer leaving the CM with m units of real balances is $\eta(m)$.

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Now we show that when $m = z_1 + z_2$, (x_1, z_1) is a solution. Suppose, by contradiction, (x, y) gives seller a higher surplus without violating the constraint. Hence,

$$u_1(x) - y + \sigma_2 \xi(z_1 + z_2 - y) \geq u_1(x_1) - z_1 + \sigma_2 \xi(z_2) \text{ and } y - c_1(x) > z_1 - c_1(x_1),$$

and hence

$$u_1(x) - c_1(x) + \sigma_2 \xi(z_1 + z_2 - y) > u_1(x_1) - c_1(x_1) + \sigma_2[u_2(x_2) - z_2]. \qquad (25)$$

Consider three cases.

(a) $y \leq z_1$. Then, the argument is exactly the same as in the proof of Theorem 2.1.

(b) $y \geq z_1 + \epsilon$. Then, $\xi(z_1 + z_2 - y) = 0$ and we can obtain a contradiction to (4) as in the proof of Theorem 2.1.

(c) $y \in (z_1, z_1 + \epsilon)$ and let $\epsilon' = y - z_1$. Then,

$$\xi(z_1 + z_2 - y) = \left(1 - \frac{\epsilon'}{\epsilon}\right)[u_2(x_2) - z_2].$$

However, because $y - c_1(x) > z_1 - c_1(x_1)$,

$$c_1'(x_1)[x - x_1] \leq c_1(x) - c_1(x_1) < y - z_1 = \epsilon'.$$

From the above two conditions and the definition of ϵ, (22),

$$[u_1(x) - c_1(x)] - [u_1(x_1) - c_1(x_1)] \leq [u_1'(x_1) - c_1'(x_1)](x - x_1) < \frac{[u_1'(x_1) - c_1'(x_1)]\epsilon'}{c_1'(x_1)}$$

$$< \frac{\epsilon'}{\epsilon}\sigma_2[u_2(x_2) - z_2] = \sigma_2[\xi(z_2) - \xi(z_1 + z_2 - y)],$$

a contradiction to (25). This also implies that the buyer has no profitable deviating offers at round-1 DM.

Finally, we show that at the CM it is optimal for the buyer to leave with $z_1 + z_2$ units of real balances. Now, a buyer who leaves with m units of real balances has the expected payoff

$$-m + \delta[\eta(m) + m + W(0)] = \delta\{-\rho m + \eta(m) + W(0)\}.$$

Recall that $\eta(m) = u_1(x_1) - z_1 + \sigma_2 \xi(z_2)$ if $m \geq z_1 + z_2$ and $\eta(m) = \sigma_2 \xi(m)$ otherwise. We distinguish three cases.

(a) $m \geq z_1 + z_2$. Because $\eta(m)$ is constant for all $m \geq z_1 + z_2$ but the cost of holding money increases with m, any $m > z_1 + z_2$ is strictly dominated by $m = z_1 + z_2$.

(b) $m \in (z_2, z_1 + z_2]$. Because $\xi(m)$ is constant for $m \geq z_2$, any $m \in (z_2, z_1 + z_2]$ is strictly dominated by z_2.

(c) $m < z_2$. Here we show that for any $\epsilon' \in (0, \epsilon]$, $z_2 - \epsilon'$ is strictly dominated by z_2. This will be the case if

$$-\rho(z_2 - \epsilon') + \sigma_2 \xi(z_2 - \epsilon') < -\rho z_2 + \sigma_2 \xi(z_2),$$

which is equivalent to

$$\rho\epsilon' < \sigma_2[\xi(z_2) - \xi(z_2 - \epsilon')] = \sigma_2\frac{\epsilon'}{\epsilon}[u_2(x_2) - z_2],$$

which holds by (22). Moreover, for any $m \leq z_2 - \epsilon$, it is strictly dominated by zero as $\xi(m)$ is constant below $z_2 - \epsilon$.

Given the above disucssion, to show that holding $z_1 + z_2$ is optimal, it is sufficient to show that it is better than z_2 and 0; the first follows from (5) and the second follows from (2).

Proof of Lemma 3.2

We show the necessity of (5). Suppose first that $C = \{1\}$. Let $z_{1,m}$ and $z_{1,c}$ be the transfer of real balances and promise to pay in round-1 DM. Let z be the real balances the buyer has to hold when leaving CM. Then, the buyer may deviate to repaying nothing and holding only $z - z_{1,m}$ units of real balances but still accept equilibrium trades at round-2 DM's in all future periods. This deviation is profitable if

$$-(z - z_{1,m}) + \frac{1}{\rho}\sigma_2[u_2(x_2) - z_2] > -(z + z_{1,c}) + \frac{1}{\rho}\{[u_1(x_2) - z_1] + \sigma_2[u_2(x_2) - z_2]\},$$

that is, if

$$-\rho(z_{1,c} + z_{1,m}) + [u_1(x_2) - z_1] < 0.$$

Hence, (5) is necessary to prevent this profitable deviation. Now suppose that $C = \{2\}$. Then, the round-1 DM trade has to be financed by transfer of real balances. (5) is then necessary for buyers not to skip round-1 DM trades.

Proof of Theorem 3.1

Proof of (i) ($\Rightarrow$) We now prove necessity. The necessity of (3) and (5) follows previous arguments. To prove the necessity of (2), consider an arbitrary EM policy, (k, π) that satisfies (6). This implies (2).

($\Leftarrow$) Here we prove sufficiency. First we formulate the EM policy. If $-\rho z_1 + [u_1(x_1) - z_1] \geq 0$, then we set $k = \pi = 0$. Otherwise, the EM policy is such that

$$-\rho(z_1 - k) + [u_1(x_1) - (z_1 - k)] = 0,$$

that is,

$$k = z_1 - \frac{u_1(x_1)}{1 + \rho} \in (0, z_1). \tag{26}$$

We will set buyers to hold z_2 real balances. Feasibility then implies $k = \pi z_2$, or $\pi = k/z_2$. The proposal is given by: $C = \{1\}$, $\phi_t M_{t-1} = z_2$ for all t, and the debt limit is given by $D = z_1 - k$. It remains to specify the proposed trades.

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We start with o_2. Because the second round is a non-credit meeting, o_2 only depends on the buyer's announcement of real balances, m. Let $\epsilon \in (0, z_2)$ be so small that

$$\epsilon < \frac{1}{2} \min \left\{ \frac{c_1'(x_1)\sigma_2[u_2(x_2) - z_2]}{u_1'(x_1) - c_1'(x_1)}, \frac{\sigma_2[u_2(x_2) - z_2]}{\rho + (1 + \rho)\pi} \right\}. \tag{27}$$

Note that condition (27) is exactly the same as (22) except the term ρ, which is replaced by $[\rho + (1 + \rho)\pi]$, reflecting the difference in the cost of holding money. Let

$$\xi(m) = \begin{cases} u_2(x_2) - z_2 & \text{if } m \geq z_2; \\ 0 & \text{if } m \leq z_2 - \epsilon; \\ \left[1 - \frac{z_2 - m}{\epsilon}\right][u_2(x_2) - z_2] & \text{if } m \in (z_2 - \epsilon, z_2). \end{cases}$$

Note that ξ is a piecewise linear continuous function. Then, $o_2(m)$ solves

$$\max_{(x,y) \in \mathbb{R}_+ \times [0,m]} -c_2(x) + y \tag{28}$$
$$\text{s.t. } u_2(x) - y \geq \xi(m).$$

The solution to (28) exists for all m and is unique with the constraint binding. Moreover, following exactly the same arguments as in the proof of Theorem 2.1, $o_2(z_2) = (x_2, z_2)$. This also implies that the buyer has no profitable deviating offer at round-2 DM.

Now we formulate $o_1(m, r, d)$. Note that d can only take two values: $d = D$ when $r = G$ and $d = 0$ when $r = B$. Let $\eta(m, G) = u_1(x_1) - (z_1 - k) + \sigma_2\xi(m)$ and let $\eta(m, B) = \sigma_2\xi(m)$. η is continuous as ξ is. When $r = G$, for each $m \in \mathbb{R}_+$, let $o_1(m, G, D)$ be a solution to

$$\max_{(x,y_c,y_m) \in \mathbb{R}_+ \times [0,D+k] \times [0,m]} -c_1(x) + y_c + y_m \tag{29}$$
$$\text{s.t. } u_1(x) - \max(y_c - k, 0) - y_m + \sigma_2\xi(m - y_m) \geq \eta(m, G).$$

When $r = B$, for each $m \in \mathbb{R}_+$, let $o_1(m, B, 0)$ be a solution to

$$\max_{(x,y_m) \in \mathbb{R}_+ \times [0,m]} -c_1(x) + y_m \tag{30}$$
$$\text{s.t. } u_1(x) - y_m + \sigma_2\xi(m - y_m) \geq \eta(m, B).$$

Because ξ is continuous, a solution to (29) and (30) exists. For any solution to the problem, the constraint is binding, following exactly the same arguments as in the proof of Theorem 2.1. Thus, although there may be multiple solutions, we can pick any one of them and we know that under o_1, the expected buyer surpluses from the two DM rounds for a buyer leaving the CM with m units of real balances is $\eta(m, r)$ for both $r = G, B$. Note that, under $r = B$, the EM does not purchase any IOU from the buyer.

Now we show that when $m = z_2$, $(x_1, z_1, 0)$ is a solution to (29). Suppose, by contradiction, (x, y_c, y_m) gives seller a higher surplus without violating the constraint. We may assume that $y_c \geq k$, for otherwise we can increase y_c to give the seller even a higher surplus without changing the buyer's. Hence,

$$u_1(x) - (y_c - k) - y_m + \sigma_2\xi(z_1 + z_2 - y) \geq u_1(x_1) - (z_1 - k) + \sigma_2\xi(z_2), \quad y_m + y_c - c_1(x) > z_1 - c_1(x_1),$$

and hence

$$u_1(x) - c_1(x) + \sigma_2\xi(z_1 + z_2 - y) > u_1(x_1) - c_1(x_1) + \sigma_2[u_2(x_2) - z_2]. \qquad (31)$$

Note that (31) is exactly the same as (25), and following exactly the same reasoning as in the proof of Theorem 2.2, we can obtain a contradiction from the construction of ϵ, (27), and (4).

Now we show that the following strategies form a simple equilibrium. All agents respond with *yes* to the proposed trades and buyers offer the proposed trades, on both equilibrium and off-equilibrium paths. Buyers under state G always repay their IOU's up to D, and buyers under state B never repay anything. All buyers leave the CM with z_2 units of real balances.

By construction, the outcome functions $o_1(m, r, d)$ and $o_2(m)$ ensure that buyers and sellers always prefer to say *yes*, buyers are willing to offer the proposed trade, and buyers always announce their money holdings truthfully. Moreover, because $\eta(m, G) - \eta(m, B) = u_1(x_1) - (z_1 - k)$ is independent of m, we can write the continuation values as follows. Let $V_B^c = 0$,

$$V_G^c = \frac{1}{1 - \delta}[u_1(x_1) - (z_1 - k)],$$

$$V(m) = \sigma_2\xi(m) + W(0),$$

and

$$W(0) = -(1 + \pi)z_2 + \frac{\delta}{1 - \delta}\left\{\sigma_2 u_2(x_2) + (1 - \sigma_2)z_2 - (1 + \pi)z_2\right\}.$$

Then, the continuation value for a buyer entering DM with credit record r and real balances m is $V_r^c + V(m)$. This implies that the choice of real balances in the CM and the repayment decision for the debts are independent from each other.

Here we show that buyers are willing to leave the CM with z_2 units of real balances. Now, a buyer who leaves with m units of real balances has the expected payoff (regardless of the amount of repayment to his debts)

$$-(1 + \pi)m + \delta\left[\eta(m, r) + m + W(0)\right] = \delta\left\{-[\rho + (1 + \rho)\pi]m + \eta(m, r) + W(0)\right\}.$$

Recall that

$$\eta(m, r) = \begin{cases} \mathbf{1}_{r=G}[u_1(x_1) - z_1 + k] + \sigma_2\xi(z_2) & \text{if } m \geq z_2, \\ \mathbf{1}_{r=G}[u_1(x_1) - z_1 + k] + \sigma_2\xi(m) & \text{otherwise.} \end{cases}$$

Because $\eta(m, r)$ is constant for all $m \geq z_2$ but the cost of holding money increases with m, any $m > z_2$ is strictly dominated by $m = z_2$.

Here we show that for any $\epsilon' \in (0, \epsilon]$, $z_2 - \epsilon'$ is strictly dominated by z_2. This will be the case if

$$-[\rho + (1 + \rho)\pi](z_2 - \epsilon') + \sigma_2\xi(z_2 - \epsilon') < -[\rho + (1 + \rho)\pi]z_2 + \sigma_2\xi(z_2),$$

which is equivalent to

$$[\rho + (1 + \rho)\pi]\epsilon' < \sigma_2[\xi(z_2) - \xi(z_2 - \epsilon')] = \sigma_2\frac{\epsilon'}{\epsilon}[u_2(x_2) - z_2],$$

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which holds by (27). Moreover, for any $m \leq z_2 - \epsilon$, it is strictly dominated by zero as $\xi(m)$ is constant below $z_2 - \epsilon$. Thus, to show that holding z_2 is optimal, it is sufficient to show that it is better than 0, and this will be the case if and only if

$$-[\rho + (1+\rho)\pi]z_2 + \sigma_2[u_2(x_2) - z_2] \geq 0.$$

Using $\pi z_2 = k = z_1 - \frac{u_1(x_1)}{1+\rho}$, we can rewrite this inequality as

$$-\rho(z_1 + z_2) + [u_1(x_1) - z_1] + \sigma_2[u_2(z_2) - z_2] \geq 0,$$

which corresponds to (2).

Finally, we show that a buyer under state G has incentive to repay $D = z_1 - k$ whenever his IOU is at least z_1 (where the EM pays k for him). Because the buyer's payoff are affected by his record only through η and the buyer holds z_2 units of real balances when leaving the CM regardless of his records, he has incentive to repay D if and only if

$$-(z_1 - k) + \frac{\delta}{1-\delta}\eta(z_2, G) \geq \frac{\delta}{1-\delta}\eta(z_2, B),$$

which is equivalent to

$$-\rho(z_1 - k) + u_1(x_1) - (z_1 - k) \geq 0.$$

This holds by (26).

Proof of (ii) ($\Leftarrow$) We start sufficiency. First we formulate the EM policy. If $-\rho z_2 + [u_2(x_2) - z_2] \geq 0$, then set $k = \pi = 0$. Otherwise, let

$$k = z_2 - \frac{\sigma_2}{\sigma_2 + \rho}u_2(x_2) \in (0, z_2). \tag{32}$$

We will buyers to hold z_1 real balances. Feasibility then implies $\sigma_2 k = \pi z_1$, or $\pi = \sigma_2 k / z_1$. The proposal is given by: $C = \{2\}$, $\phi_t M_{t-1} = z_1$ for all t. The debt limit is given by $D = z_2 - k$.

Now we formulate the proposed trades. We start with o_2. Since the second round is a credit meeting, o_2 depends on the buyer's announcement of real balances m and on his record r. Note that the available credit limit can only take two values: $d = D$ when $r = G$ and $d = 0$ when $r = B$. Let $o_2(m, G, D)$ be a solution to

$$\max_{(x,y_c,y_m)\in\mathbb{R}_+\times[0,D+k]\times[0,m]} \quad -c_2(x) + y_c + y_m \tag{33}$$
$$\text{s.t.} \quad u_2(x) - \max(y_c - k, 0) - y_m \geq u_2(x_2) - (z_2 - k).$$

Let $o_2(m, B, 0)$ be a solution to

$$\max_{(x,y_m)\in\mathbb{R}_+\times[0,m]} \quad -c_2(x) + y \tag{34}$$
$$\text{s.t.} \quad u_2(x) - y_m \geq 0.$$

The solutions to (33) and (34) exist and are unique. Moreover, the constraints are always binding. The fact that $o_2(0, G, D) = (x_2, z_2)$ follows from $x_2 \leq x_2^*$.

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We now move to o_1. Because the first round is a non-credit meeting, o_1 only depends on the buyer's announcement of real balances. Let $\eta(m) = u_1(x_1) - z_1$ if $m \geq z_1$ and let $\eta(m) = 0$ otherwise. Let $o_1(m)$ be a solution to

$$\max_{(x,y) \in \mathbb{R}_+ \times [0,m]} -c_1(x) + y \tag{35}$$
$$\text{s.t. } u_1(x) - y \geq \eta(m).$$

The solutions to (35) exist and are unique. Moreover, the constraint on the reservation utility of the buyer is always binding. The fact that $o_1(z_1) = (x_1, z_1)$ follows from $x_1 \leq x_1^*$.

Now we show that the following strategies form a simple equilibrium. All agents respond with *yes* to the proposed trades and buyers offer the proposed trades, on both equilibrium and off-equilibrium paths. Buyers under state G always repay their IOU's up to D, and buyers under state B never repay anything. All buyers leave the CM with z_1 units of real balances.

The proposed trades $o_1(m)$ and $o_2(r, m, d)$ ensure that buyers and sellers always prefer to say *yes*, buyers are willing to offer the prosed trades, and buyers always announce their money holdings truthfully. Moreover, we can write the continuation values as follows. Let $V_B^c = 0$,

$$V_G^c = \frac{1}{1 - \delta} \sigma_2[u_2(x_2) - (z_2 - k)] \text{ and } V_B^c = 0,$$
$$V(m) = \eta(m) + W(0),$$

and

$$W(0) = -(1 + \pi)z_1 + \frac{\delta}{1 - \delta}\{u_1(x_1) - (1 + \pi)z_1\}.$$

Then, the continuation value for a buyer entering DM with credit record r and real balances m is $V_r^c + V(m)$. This implies that the choice of real balances in the CM and the repayment decision for the IOU's are independent from each other.

Now, consider the incentive of a buyer to repay his debt up to D under state G. For a buyer in the CM under state G, he has incentive to repay D if and only if

$$-(z_2 - k) + \delta V_G^c \geq \delta V_B^c,$$

that is,

$$-\rho(z_2 - k) + \sigma_2[u_2(x_2) - (z_2 - k)] \geq 0,$$

which holds with equality by (32).

We now show that the buyer has incentive to carry z_1 real balances. A buyer who leaves CM with m units of real balances has the expected payoff (regardless of his records or the amount of repayment to his debts)

$$-(1 + \pi)m + \delta[\eta(m) + m + W(0)] = \delta\{-[\rho + (1 + \rho)\pi]m + \eta(m) + W(0)\}.$$

Note that $\eta(m)$ is constant for all $m \geq z_2$ and is constant for all $m \in [0, z_2)$. Thus, we only need to show that bringing z_2 is better than zero, which will be the case if and only if

$$-[\rho + (1 + \rho)\pi]z_1 + u_1(x_1) - z_1 \geq 0.$$

Using $\pi z_1 = \sigma_2 k = \sigma_2 z_2 - \sigma_2 \frac{\sigma_2}{\sigma_2 + \rho} u_2(x_2)$, we can rewrite this inequality as

$$\{-\rho z_1 + u_1(x_1) - z_1\} + \frac{(1 + \rho)\sigma_2}{\sigma_2 + \rho} \{-\rho z_2 + \sigma_2 [u_2(x_2) - z_2]\} \geq 0,$$

which corresponds to (7).

($\Rightarrow$) We now prove necessity. As in the case, of $C = \{1\}$, buyers are indifferent between repaying and not repaying the debt when k satisfies (32). Thus, we cannot implement $\mathcal{L}$ for lower values of k, as buyers would have no incentive to keep a record G. In turn, for higher values of k, we would relax the incentives of buyers to keep a record G but we reduce the incentives of buyers to participate in the first DM round. This implies that there is no EM policy which can do better than the chosen one. Since the chosen EM policy requires (7), this condition is necessary. In turn, (3) is necessary, otherwise sellers would not be willing to participate. Finally, (5) is necessary otherwise the buyer would never carry z_1 real balances.□

Proof of Theorem 3.2

(1) By definition, the allocation (x_1^{C2}, x_2^{C2}) is implementable with EM under $C = \{2\}$. Next, we show that the allocation (x_1^{C}, x_2^{C}) is implementable with EM under $C = \{1\}$. It suffices to show that it satisfies (4). Suppose, by contradiction, that for $\hat{x}_1 = \min\{x_1^*, c_1^{-1}(z_1 + z_2)\}$ with $z_i = c_i(x_i^{C})$, $i = 1, 2$, we have

$$u_1(x_1^{C}) - c_1(x_1^{C}) + \sigma_2[u_2(x_2^{C}) - z_2] < u_1(\hat{x}_1) - c_1(\hat{x}_1). \tag{36}$$

Because (x_1^{C}, x_2^{C}) satisfies (2), (36) implies that the pair $(\hat{x}_1, 0)$ satisfies (2) as well. Note that $c_1(\hat{x}_1) \leq z_1 + z_2$. But (36) implies that it generates a higher welfare than (x_1^{C}, x_2^{C}), a contradiction. Thus, (x_1^{C}, x_2^{C}) satisfies (4) and hence is implementable with $C = \{1\}$ and with EM.

Finally, because, by Lemma 3.2 and Theorem 3.1, the set of implementable allocations with EM includes all allocations implementable under constant money supply, the constrained-efficient allocation is either (x_1^{C}, x_2^{C}) or (x_1^{C2}, x_2^{C2}).

(2) We consider two cases.

(i) Suppose that (x_1^{C}, x_2^{C}) does not satisfy (5), and hence is not implementable with constant money supply. Then, the constrained efficient allocation can only be implemented with EM but not with constant money supply.

(ii) Suppose that (x_1^{C}, x_2^{C}) is implementable with constant money supply, and hence, by Theorem 2.2, $-\rho c_1(x_1^{C}) + [u_1(x_1^{C}) - c_1(x_1^{C})] \geq 0$. Also, $\rho > \rho_M$ implies $(x_1^{C}, x_2^{C}) \neq (x_1^*, x_2^*)$. This also implies that constraint (2) is binding at (x_1^{C}, x_2^{C}). Thus, if $-\rho c_1(x_1^{C}) + [u_1(x_1^{C}) - c_1(x_1^{C})] = 0$, then $-\rho c_2(x_2^{C}) + \sigma_2[u_2(x_2^{C}) - c_2(x_2^{C})] = 0$. That is, $(x_1^{C}, x_2^{C}) = (\bar{x}_1, \bar{x}_2)$, a contradiction. Therefore,

$$u_1(x_1^{C}) - (1 + \rho)c_1(x_1^{C}) > 0.$$

This inequality, together with $\sigma_2 < 1$ and (2) for (x_1^C, x_2^C), implies

$$[u_1(x_1^C) - (1+\rho)c_1(x_1^C)] + \frac{(\rho+1)\sigma_2}{\rho+\sigma_2}[\sigma_2 u_2(x_2^C) - (\sigma_2 + \rho)c_2(x_2^C)] > 0.$$

Because $x_1^C < x_1^*$, there exists $\epsilon > 0$ such that $x_1' = x_1^C + \epsilon < x_1^*$ and

$$u_1(x_1') - (1+\rho)c_1(x_1') > 0,$$
$$[u_1(x_1') - (1+\rho)c_1(x_1')] + \frac{(\rho+1)\sigma_2}{\rho+\sigma_2}[\sigma_2 u_2(x_2^C) - (\sigma_2 + \rho)c_2(x_2^C)] > 0.$$

By Theorem 3.1 (ii), (x_1', x_2^C) is implementable with EM, but it has a strictly higher welfare than (x_1^C, x_2^C). $\square$

Proof of Theorem 3.3

Suppose that $\mathcal{L} = [(x_1, x_2), (z_1, z_2)]$ is implementable under $\pi > 0$. We consider two cases.

(i) $C = \{1\}$. Because the buyer can always skip round-1 DM, the necessity of (5) follows exactly the same proof as in Lemma 3.2. Because only money can be used to finance trades in round-2 DM and hence buyers have to hold at least z_2 units of real balances in equilibrium and repay z_1, (14) is necessary to avoid buyers from not participating the whole scheme. Note that buyers can obtain the lump-sum transfer independent of his behavior.

(ii) $C = \{2\}$. Because the buyer can always skip round-1 DM and because only money can be used to finance the round-1 consumption, (5) with r replaced by $\zeta > r$ is necessary. Because buyers have to hold at least z_1 units of real balances in equilibrium and repay z_2, (15) is necessary to avoid buyers from not participating the whole scheme. Note that buyers can obtain the lump-sum transfer independent of his behavior.

Proof of Theorem 3.4

Suppose that $\mathcal{L}$ is implementable with DMP. We consider two cases.

(i) $C = \{1\}$. Because the buyer can always skip the round-1 DM, (5) is necessary. Let $(z_{1,c}, z_{1,m})$ be the composition of payments by buyers in round-1 DM with $z_1 = z_{1,c} + z_{1,m}$. In order to avoid the buyer staying at the autarky, the equilibrium continuation payoff for a buyer at the beginning of the CM has to be at least zero, that is,

$$-z_{1,c} - \eta - (1-\tau)(z_{1,m} + z_2) + \frac{1}{\rho}\{u_1(x_1) - z_{1,c} - (1-\tau)z_1 + \sigma_2[u_2(x_2) - z_2] + \tau z_2\} \geq 0.$$

By (16), $\tau(z_{1,m} + z_2) = \eta$, and the above inequality reduces to (2). Finally, (4) is still necessary.

(ii) $C = \{2\}$. Let $(z_{2,c}, z_{2,m})$ be the composition of payments by buyers in round-1 DM with $z_2 = z_{2,c} + z_{2,m}$. In order to avoid the buyer staying at the autarky, the equilibrium

continuation payoff for a buyer at the beginning of the CM has to be at least zero, that is,

$$-z_{2,c} - \eta - (1-\tau)(z_{2,m} + z_1) + \frac{1}{\rho}\left\{u_1(x_1) - z_1 + \sigma_2[u_2(x_2) - z_2 - \eta] + \tau(z_{2,m} + z_1)\right\} \geq 0.$$

By (16), $\tau(z_{2,m} + z_1) = \sigma_2\eta$, and the above inequality reduces to

$$-z_2 - z_1 - (1-\sigma_2)\eta + \frac{1}{\rho}\left\{u_1(x_1) - z_1 + \sigma_2[u_2(x_2) - z_2]\right\} \geq 0. \tag{37}$$

Because $\eta \geq 0$, this implies (2). If (5) holds, then we have case (i) in the theorem. Suppose that (5) does not hold. Then, by (37), (18) holds with strict inequality. Moreover, buyers do not skip round-1 meetings only if

$$-\rho z_1 + (1+\rho)\tau z_1 + [u_1(x_1) - z_1] \geq 0,$$

that is, only if

$$\tau z_1 \geq \frac{1}{1+\rho}\{\rho z_1 - [u_1(x_1) - z_1]\}.$$

This gives a lower bound on τ and hence on η, and plug in this lower bound into (37) we obtain (17). $\square$

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