Optimal Monetary Interventions in Credit Markets*

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Abstract

We characterize optimal credit market interventions with respect to two fundamental frictions—limited commitment and limited monitoring. We consider two classes of interventions: an inflationary policy which uses inflation tax to forgive private debt, and a deflationary policy which uses credit tax to increase the real rate of return on money. We show that both money and debt are essential and that intervention is generically optimal. The nature of the optimal intervention depends on the fundamentals of the economy and we provide conditions under which the optimal intervention is inflationary and under is deflationary. Our results hold irrespective of whether we use a bargaining protocol or optimal trading mechanisms.

JEL Codes: E50; E51; E52

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1 Introduction

The use of both money and credit as means-of-payments is widespread. However, the effect of monetary interventions in economies where money and credit coexist remains largely an open issue. While there is a sizable literature on the impact of interventions in the monetary sector (see, for example, Lagos, Rocheteau, and Wright (2017) for recent developments), most literature on the impact of interventions in the credit sector consider economies without money (see, for example, Gertler and Kiyotaki (2010)). In this paper, we aim at bridging this gap by considering monetary interventions in the credit market.

In our view, a model where money and credit coexist and where monetary interventions in the credit market can be meaningfully analyzed, requires two elements. First, one needs a friction which renders credit imperfect, thus making liquidity (or its lack thereof) important. In pure credit economies, Kehoe and Levine (1993) and Alvarez and Jermann (2000) have shown that the inability of individuals to commit introduces an endogenous borrowing constraint which limits the possibility of implementing the first-best allocation. Second, one needs a friction which renders money essential. This requires some degree of imperfect monitoring, as pointed out by Wallace (2010, 2014). It turns out to be a demanding task to combine limitations in commitment and imperfections in monitoring in ways that make both money and credit essential. In particular, Gu, Mattesini, and Wright (2016) show that money and credit cannot be both relevant in the Lagos and Wright (2005) (LW henceforth) environment, which is extensively used to study interventions in the monetary sector.

In this paper, we first propose a model which builds on LW but that combines limited commitment and imperfect monitoring in a way that makes both money and credit essential as means of payment. We then use the model to study optimal monetary interventions in the credit market among policies that are consistent the underlying frictions of the environment. For instance, we do not allow taxation schemes, such as lump-sum taxes, that abscond resources from activities that are not monitored. Moreover, we only consider interventions that respect agents' voluntary participation. We identify two novel classes of monetary policies that can improve welfare. The first class, labeled inflationary policies, uses inflation tax to forgive private debt; the second class, labeled deflationary policies, uses taxes on the credit sector to increase the real rate of return on money. The first class of policies tightens liquidity in the monetary sector by reducing the real rate of return on money but it increases liquidity in the credit sector by relaxing the borrowing constraint, while the second class does the opposite. Thus, the determination of the optimal policy in our environment depends on the liquidity needs across the two sectors, a trade-off which does not exist in economies without money and which differs greatly from the usual approach of modeling monetary injections through lump-sum transfers.

Our model modifies LW in two ways. First, we introduce debt by assuming an imperfect monitoring technology which records promises-to-pay from buyers and through which sellers can access past records in a fraction of meetings. Second, we assume that agents interact in three rounds in each period: the first two rounds correspond to the decentralized market (DM) in LW, and the last round corresponds to the centralized market (CM). This last modification is motivated by findings in Gu, Mattesini, and Wright (2016). As mentioned above, they show that money and credit cannot both play meaningful transactional roles in the baseline LW, where agents alternate between one round of DM trade and one round of CM trade. We focus on the model with two DM rounds for simplicity, but most results generalize to models with more than two DM rounds.

We provide a complete characterization of implementable allocations in an economy with limited monitoring, captured by assuming that only sellers participating in one of the DM rounds has access to the monitoring technology. In order to provide useful benchmarks against which our results can be interpreted, we also provide a complete characterization of implementable allocations in two extreme scenarios: an economy without money and with unlimited monitoring, i.e., the monitoring technology is available in both DM rounds; and an economy with a constant money supply and no monitoring, i.e., neither DM round is monitored.

We begin with the results for the extreme scenarios. In the unlimited monitoring economy, implementability requires that buyers are willing to repay the debt accumulated in the two DM rounds, given the expected gains from trade in all future periods. This constraint endogenously determines the debt limit that buyers face and hence the level of production in the DM rounds. In the no monitoring economy with a constant money supply, trades depend on the trade-off between the cost of holding money across periods and the expected gains from trade. This trade-off imposes more stringent restrictions than the participation constraint in the unlimited monitoring economy, and the optimal allocation under unlimited monitoring delivers a higher welfare.

Consider, now, the limited monitoring economy. If we assume away interventions by the monetary authority, i.e., if the money supply is constant, the optimal allocation exhibits the highest debt limit that is consistent with the repayment constraint. Moreover, under this allocation, agents do not want to use money in monitored meetings. Finally, trades in non monitored meetings resemble those in the no monitoring economy. This gives rise to a welfare ranking among the three economies: the unlimited monitoring economy has the highest welfare and the no monitoring economy has the lowest. This implies that money and credit are both essential in the limited monitoring economy.

Our main result shows that active intervention by the monetary authority is optimal in the limited monitoring economy. The nature of the optimal intervention, however, depends on the details of the economy and on the bargaining protocol. For most of our analysis we focus on take it or leave it (TIOLI) offers but we also consider optimal trading mechanisms. We obtain that, if agents are very patient, it is feasible to implement a deflationary policy that corresponds to the Friedman rule (through taxes from credit trades) and achieve the first-best under TIOLI offers. By continuity, deflationary policies are also optimal under TIOLI offers for a range of lower discount factors. The first best can also be achieved with a constant money supply if agents are patient and use the optimal trading mechanism.

However, if agents are relatively impatient, intervention is generically optimal, but it can be either inflationary or deflationary, irrespective of whether agents use TIOLI offers or the optimal trading mechanism. Under TIOLI offers, we provide examples to illustrate the optimality of inflation. Under the optimal trading mechanism, we provide an explicit characterization of the region of parameters under which inflation is optimal and under which deflation is optimal. In particular, inflation is optimal when meetings that require the use of money are more frequent. Intuitively, while the inflation tax reduces the rate of return on money and hurts production in non-monitored meetings, it also redistributes liquidity from those meetings to monitored ones and increases production in the later. This maximal increase depends on the proportion of credit trades relative to monetary trades. A larger proportion of monetary trades allows for a larger debt forgiveness on each credit trade and that can be optimal. Interestingly, we also find that, thanks to its redistributive role, the inflationary intervention can achieve welfare that is strictly higher than the welfare under unlimited monitoring.

The result that intervention is generically optimal is reminiscent of the conjecture in Wallace (2014) in the context of pure currency economies. In both cases no lump-sum taxes are allowed and the monetary authority is required to balance the budget. A key difference, however, is that the intervention in our framework makes use of the monitoring technology, which is absent in Wallace (2014), to tax credit trades or to forgive private debts. Moreover, our results suggest that a fully anticipated debt purchase intervention by the monetary authority can mitigate limited commitment in the credit market and increase lending in the private sector. Because of this benefit, the optimal long-term inflation rate can be positive, and its precise level depends on the liquidity needs in the money and the credit sectors.

Finally we give a brief literature review. Kehoe and Levine (1993) and Alvarez and Jermann (2001) introduce endogenous borrowing constraints in pure credit economies, and this concept was applied to the LW setting by a few papers to obtain coexistence of money and credit. Sanches and Williamson (2010) obtain coexistence between money and credit by introducing an exogenous cost of using money, while Liu, Wang, and Wright (2015) assume perfect enforcement of debts but introduce an exogenous cost of using credit. Gomis-Porqueras and Sanches (2013) and Lotz and Zhang (2013) obtain coexistence under limited monitoring, but both consider only lump-sum transfers.[1] None of these papers consider the two classes of interventions considered here. There are other papers that also study the LW setting with two DM rounds. Guerrieri and Lorenzoni (2009) study the amplification mechanism and Telyukova and Wright (2008) explain the credit card debt puzzle. In contrast to ours, both papers assume perfect enforcement and hence have no endogenous borrowing constraints.[2]

[1]Some papers assume that debts need to be settled in cash, which also leads to the coexistence between money and credit. This coexistence, though, is driven by their complementarity, and not by their substitutability, as considered here and in the papers mentioned above. References are Berentsen, Camera and Waller (2007) and Ferraris and Watanabe (2008).

[2]Some papers uuse two DM rounds to study pure currency economies, including Berentsen, Camera, and Waller (2005) on the short-run neutrality of money, and Ennis (2009) on the hot potato effect.

2 Model

2.1 Environment

Time is discrete and the horizon is infinite. The economy is populated by three types of unit measure agents, labeled buyers, type-1 sellers, and type-2 sellers. Each period is divided into three rounds. Buyers randomly meet type-j sellers in round j, and the probability of a successful meeting is σ_j, where $j = 1, 2$. There are three goods, one for each round. At round $j = 1, 2$, a type-j seller can produce y_j units of good j for a buyer at a linear cost $\rho_j y_j$ and the buyer's utility is $u_j(y_j)$. We assume that both u_j's are strictly concave and increasing, continuously differentiable, and satisfy the Inada conditions: $u_j'(0) = \infty$, $u_j'(\infty) = 0$. Let y_j^* be the solution to $u_j'(y) = \rho_j$, the quantity that maximizes the surplus. In the last round, agents meet in a centralized market. In this market, they can all consume and produce, and the (dis)utility is linear. Agents maximize their life-time expected utility with discount factor β. We let $r = \frac{1-\beta}{\beta}$.

There exists a technology which keeps track of buyers' trading histories and is available to a fraction of sellers. We call a meeting a *monitored meeting* if the seller has access to this technology, and a *non-monitored meeting* otherwise. The technology works as follows. For each buyer b, a recorded history at period t is a triple, $h = (h_1, h_2, h_3) \in H$, such that for $j = 1, 2$, $h_j = (b, s(j), d_j)$ keeps track of the buyer's round j DM promise to the seller, where b is the identity of the buyer, $s(j)$ is the identity of the seller, and d_j is the promise-to-pay in terms of CM good (debt); and $h_3 \in \mathbb{R}_+$ keeps track of the total repayment. For $j = 1, 2$, if the buyer does not meet a seller in round-j, or if the meeting is non-monitored, h_j is empty.

This monitoring technology is accompanied by a record-keeping technology for a given *debt limit* D. In our analysis D is endogenously determined subject to incentive compatibility constraints, and the choice of D is formulated as a mechanism design problem to maximize the social welfare. The set of records is given by $R = \{G, B\}$, which is updated by the function $\omega : \{G, B\} \times H \to \{G, B\}$ described below. The records are only accessible to sellers in monitored meetings. If the monitoring technology is used in both rounds, in the second DM round the seller also observes the buyer's debt d_1. We restrict attention to an updating function ω in which all buyers start with a record G (good), and a buyer keeps this record if and only if he pays his debt up to D at the end of the period; and acquires a record B (bad) otherwise. Precisely, $\omega(r, \emptyset) = r$ for $r \in \{G, B\}$, $\omega(B, h) = B$ for all $h \in H$, and $\omega(G, h) = G$ iff $h_3 \geq \min\{D, d_1 + d_2\}$. Finally, we assume that only buyers with a good record can use the monitoring technology to issue debts.[3]

Lastly, there is an intrinsically useless, divisible, and storable object, called money. The money supply at the end of period t is denoted by M_t. We assume that agents' money holdings are observable in a match. The price of money in the CM is given by ϕ_t. In what follows, we focus on symmetric stationary equilibria where real balances $Z_t = \phi_t M_t$ do not change over time.

2.2 Strategies and equilibrium

In every meeting in a DM round, terms of trade are determined by take it or leave it (TIOLI) offers. In a non-monitored meeting, the buyer makes the offer. In a monitored meeting, the buyer makes the offer if he has a good record and he chooses to use the monitoring technology. Otherwise, the seller makes the offer.[4]

We denote by s_b the strategy of a buyer. In each DM meeting, s_b maps the buyer's history up to that meeting, his real balance, and his record, to his choice of using the technology (if the choice is available), and to his offer (in case he makes the offer) or to his response to the seller's offer (otherwise). Note that a buyer's trading history is his private information, and his matched partner can only observe his real balance and, if available, his record and current debt. In the CM round s_b maps the buyer's trading history up to that round to his repayment decisions and to his final money holdings when leaving the CM. In turn, we denote by s_j the strategy of a type-j seller, where $j \in \{1, 2\}$. If the j^{th} DM round is monitored, the strategy s_j maps the seller's private history as well as the observables (buyer's real balance and, if available, his record and current debt) to her trading behavior: if the buyer uses the technology, the seller's response; otherwise, the seller's offer. If round-j DM round is not monitored, the strategy s_j maps the seller's private history as well as the observables (buyer's real balance and buyer's offer) to her response. We assume, without loss of generality, that

[3]Bethune, Hu, and Rocheteau (2017) show that this class of record-updating technologies can be used to characterize all possible equilibrium outcomes in a pure-credit economy that is similar to ours; their results can be easily generalized to our environment where there are two DM rounds and where money may be used. As a result, our record-updating technology is with no loss of generality.

[4]The buyer can use both money and debt in monitored meetings. Thus, absent intervention by the monetary authority, it is a weakly dominant strategy of a buyer with a good record to use the technology. However, it might not be so when intervention involves a tax on using the technology.

sellers do not carry money across periods.

We use stationary symmetric Perfect Bayesian Equilibrium as our solution concept, and denote a strategy profile by $s = (s_b, s_1, s_2)$, where s_b is the buyer strategy for all buyers, and s_j is the strategy for all sellers. Throughout the paper we restrict attention to equilibria where trade always takes place in the DM rounds, buyers with good record always repay their debts up to the debt limit D and they always choose to use the technology in monitored meetings, and the initial distribution of money across buyers is degenerate. We call such equilibria simple equilibria.

In what follows, to simplify the presentation and to convey our main insights, we restrict attention to the case where $\sigma_1 = 1$, and let $\sigma_2 \equiv \sigma$. Given this assumption, an equilibrium allocation can be characterized by the DM trades, (y_1, y_2), where y_j is the round-j DM consumption for buyers. The social welfare associated with such an allocation is given by

$$\mathcal{W}(y_1, y_2) = u_1(y_1) - \rho_1 y_1 + \sigma[u_2(y_2) - \rho_2 y_2]. \tag{1}$$

As we discuss below, the equilibrium allocation is determined by the debt limit we choose, and typically there is a continuum of debt limits that satisfy the incentive compatibility constraints for repayment. We deal with this multiplicity by choosing the constrained optimal debt limit and hence the constrained optimal allocation, i.e., the allocation that maximizes (1) among all allocations that are incentive compatible.

We finish this section by characterizing the constrained optimal allocation in two extreme economies: an economy with unlimited monitoring, where there is no money and all sellers have access to the monitoring technology; and an economy with no monitoring, where there is a constant money supply and no sellers have access to the monitoring technology. These scenarios serve as benchmark to the economy where monitoring is limited and there is intervention by the monetary authority, which will be analyzed in the next section.

2.3 Unlimited monitoring

In a simple equilibrium, the buyer keeps a good record if he repays his debt up to the limit D. Thus, a seller will not extend a loan to a buyer whose total obligation exceeds D in the coming CM. This implies that $d_1 + d_2 \leq D$, where d_j denotes the amount of debt the buyer issues in round-j DM. Now, the buyer issues debt to acquire goods and TIOLI offers imply that d_j is such that $\rho_j y_j = d_j$, i.e., the seller is indifferent between producing and not producing y_j. Thus, we can summarize the objects in a simple equilibrium by the triple (D, y_1, y_2).

Given D, the pair (y_1, y_2) is uniquely determined by the solution to the problem of the buyer with a good record, given by

$$\max_{y_1 \geq 0, y_2 \geq 0} \{u_1(y_1) - \rho_1 y_1 + \sigma[u_2(y_2) - \rho_2 y_2]\} \tag{2}$$

$$\text{s.t.} \quad \rho_1 y_1 + \rho_2 y_2 \leq D. \tag{3}$$

In turn, given (y_1, y_2), repaying D is incentive compatible if and only if

$$-D + \frac{\beta}{1-\beta} \{u_1(y_1) - \rho_1 y_1 + \sigma[u_2(y_2) - \rho_2 y_2]\} \geq 0,$$

that is, if and only if the temptation to renege, given by $-D$ is overcome by the discounted sum of all expected future gains from trade, $u_1(y_1) - \rho_1 y_1 + \sigma[u_2(y_2) - \rho_2 y_2]$. This can be reduced to

$$-rD + u_1(y_1) - \rho_1 y_1 + \sigma[u_2(y_2) - \rho_2 y_2] \geq 0. \tag{4}$$

A triple (D, y_1, y_2) constitutes a simple equilibrium if and only if it satisfies (2)-(4).

The constrained optimal allocation is then given by the triple (D, y_1, y_2) that maximizes (1) subject to (2)-(4). We first claim that, for any given (y_1, y_2), there exists D such that (D, y_1, y_2) satisfies (2)-(4) if and only if $y_1 \leq y_1^*$, $y_2 \leq y_2^*$,

$$\frac{u_1'(y_1)}{\rho_1} - 1 = \sigma \left[\frac{u_2'(y_2)}{\rho_2} - 1\right] \tag{5}$$

$$-r(\rho_1 y_1 + \rho_2 y_2) + u_1(y_1) - \rho_1 y_1 + \sigma[u_2(y_2) - \rho_2 y_2] \geq 0. \tag{6}$$

This implies that we can restrict attention to allocations (y_1, y_2) satisfying (5)-(6). We omit a complete proof of this claim, but we provide a sketch by considering two separate cases. First, let

$$r \leq r_c \equiv \frac{u_1(y_1^*) - \rho_1 y_1^* + \sigma[u_2(y_2^*) - \rho_2 y_2^*]}{\rho_1 y_1^* + \rho_2 y_2^*}. \tag{7}$$

In this case, (y_1^*, y_2^*) satisfies (5)-(6), and the solution to (2)-(3) is equal to (y_1^*, y_2^*) for any D such that $D \geq D^* \equiv \rho_1 y_1^* + \rho_2 y_2^*$. Now, let $D < D^*$. In this case, the pair (y_1, y_2) that solves (2)-(3) satisfies (5) and (6). The latter is true because the pair (y_1, y_2) that solves (2)-(3) satisfies (3) with equality, which implies that (4) is equivalent to (6). Conversely, for any pair (y_1, y_2) that satisfies (5)-(6), if we set $D = \rho_1 y_1 + \rho_2 y_2$, the triple (D, y_1, y_2) satisfies (2)-(4). The proof for the case where $r > r_c$ is similar. The following lemma fully characterizes the optimal allocation.

Lemma 1 *The optimal allocation maximizes (1) subject to the incentive compatibility condition (6). If $r \le r_c$, the first-best is the optimal allocation. If $r > r_c$, the optimal allocation is given by the unique solution, denoted by (y_1^c, y_2^c), to (5) and (6) at equality.*

The optimal allocation maximizes (1) subject to (5) and (6). Lemma 1 shows that we can ignore (5). The problem then becomes a standard maximization problem: the objective function (1) is strictly concave, and the constraint (6) gives a convex set, and hence it has a unique solution characterized by first-order conditions, which turn out to coincide with (5).

One can show that (6) is a necessary condition for any bargaining protocol (see the Supplemental Material for formal details). Thus, Lemma 1 also shows that TIOLI offers is an optimal trading mechanism under unlimited monitoring. Moreover, while we assume that there is no money, under the optimal debt limit, agents will choose to hold no money even if they are allowed to.

2.4 No monitoring

In the no monitoring economy, the only way to sustain trades is to use money. As standard in the LW framework, if $W(z)$ is the value function of a buyer with z real balances at the beginning of the CM round, then $W(z) = z + W_0$, where W_0 is a constant independent of z. Standard arguments also show that the buyer will simply bring enough real balances to finance his DM trades. Finally, TIOLI offers imply that all the surplus in each DM round goes to the buyer. Moreover, since $\sigma_1 = 1$, the buyer does not have to contemplate on the contingency where he does not meet a seller at round-1 DM. Thus, the buyer's problem in the CM can be written as (see also the proof of Lemma 2 below)

$$\max_{z_1 \ge 0, z_2 \ge 0} \left\{ -r(z_1 + z_2) + u_1\left(\frac{z_1}{\rho_1}\right) - z_1 + \sigma\left[u_2\left(\frac{z_2}{\rho_2}\right) - z_2\right]\right\}, \tag{8}$$

where z_j is the amount of real balances the buyer plans to spend in round-j. Under TIOLI offers, these real balances will allow him to consume $\frac{z_j}{\rho_j}$ units of round-j DM goods, $j \in \{1, 2\}$. The first term in (8) is the opportunity cost of carrying $z_1 + z_2$ real balances across periods, while the second and the third terms capture the surplus in each DM round. Equivalently, one may rewrite the problem in terms of planned consumptions (y_1, y_2) in the two DM rounds, with $y_j = \frac{z_j}{\rho_j}$. This gives rise to a characterization of the equilibrium allocation under no monitoring.

Lemma 2 *The unique equilibrium allocation under no monitoring, given by* (y_1^m, y_2^m)*, solves*

$$\frac{u_1'(y_1) - \rho_1}{\rho_1} = \frac{\sigma\left[u_2'(y_2) - \rho_2\right]}{\rho_2} = r. \tag{9}$$

Lemma 2 restricts attention to a constant money supply. If we allow for a growth rate of money equal to τ with lump-sum transfers, the unique equilibrium allocation solves a modified version of equation (9) where the real interest rate r is replaced with the nominal interest rate i, where $1 + i = (1 + r)(1 + \tau)$, using the Fisher equation.[5] If we then restrict attention to monetary interventions implemented by lump-sum transfers to all agents, and which respect agents' incentive constraints, we have $i \geq r$ and welfare is maximized under no intervention.

3 Limited monitoring

We now consider the limited monitoring economy. We first deal with the case where the second DM round is monitored. At the end of this section, we consider the case where the first DM round is monitored.

3.1 No intervention

As a benchmark, let us first assume a constant money supply. Irrespective of his record, the problem of the buyer in the non-monitored meeting is exactly as in the no monitoring economy. Instead, in the monitored round, a buyer with a good record can access the monitoring technology and use both money and debt. The decision on how much money to use in the monitored round is similar to the one in the no monitoring economy. The main difference is that the buyer has additional liquidity since he can issue debt up to the limit D.[6] Assuming that he repays his debt D , the choice of real balances by the buyer in the CM is then determined by

$$\max_{z_1 \geq 0, z_2 \geq 0} \left\{ -r(z_1 + z_2) + u_1\left(\frac{z_1}{\rho_1}\right) - z_1 + \sigma\left[u_2\left(\frac{z_2 + D}{\rho_2}\right) - z_2 - D\right]\right\}, \tag{10}$$

[5] The Fisher equation gives i as the nominal interest rate of an illiquid bond that is issued in one centralized round that matures in the next centralized round.

[6] We assume that $D \leq \rho_2 y_2^*$ so the buyer has enough debt limit to acquire the efficient quantity but not more than that. This assumption is without loss of generality as it is never optimal for the buyer to consume more than y_2^* in the second DM round.

where TIOLI offers imply that $\frac{z_1}{\rho_1}$ is the consumption of the buyer if he brings z_1 real balances to use in the first DM round, and $\frac{z_2+D}{\rho_2}$ is the consumption of the buyer if he issues an amount of debt equal to D and brings z_2 real balances to use in the second DM round. Note that it is never optimal for the buyer to replace debt with money, i.e., he always uses all the debt and he may choose to bring additional money if he wants to consume more than what the debt limit allows. This is so because debt is cheaper than money for every $r > 0$: debt can be issued at the spot, while money must be carried across periods.

As in the no monitoring economy, we may rewrite the choice of real balances with the choice of DM consumptions by letting $y_1 = \frac{z_1}{\rho_1}$ and $y_2 = \frac{z_2+D}{\rho_2}$. This also implies that we can use (D, y_1, y_2) as the equilibrium objects, and the real balances (z_1, z_2) can be inferred from those objects. The following FOCs determine the solution to the buyer's problem:

$$\frac{u_1'(y_1) - \rho_1}{\rho_1} = r, \tag{11}$$

and

$$\begin{cases} \rho_2 y_2 = D & \text{if } \sigma \frac{u_2'(D/\rho_2)-\rho_2}{\rho_2} < r, \\ \sigma \frac{u_2'(y_2)-\rho_2}{\rho_2} = r & \text{otherwise.} \end{cases} \tag{12}$$

According to (11), $y_1 = y_1^m$, and hence round-1 DM consumption is identical to that in the no monitoring economy. According to (12), if D is relatively high and hence the expected marginal surplus using debt alone is lower than the real interest rate, the buyer has no incentive to bring money to use in round-2 DM. In this case, he consumes up to the available debt limit and $y_2 > y_2^m$. Otherwise, the buyer supplements the debt with money so that the expected marginal surplus is equal to the real interest rate, and $y_2 = y_2^m$. In this case, the amount D is irrelevant for the equilibrium allocation; a lower D will simply be substituted by higher real balances. This result is related to Gu, Mattesini and Wright (2016), who obtain that if credit is too tight, then the debt limit is irrelevant for the equilibrium allocation for a wide class of bargaining protocols.

While (11)-(12) determine (y_1, y_2) from D, we need to check incentive compatibility for buyers to repay D. Here we claim that a triple (D, y_1, y_2) constitutes a simple equilibrium if and only if (y_1, y_2) solves (11)-(12) under D and

$$-r\rho_2 y_2 + \sigma \left[u_2(y_2) - \rho_2 y_2\right] \geq 0. \tag{13}$$

To prove this claim, we need to show that (13) is necessary and sufficient for buyers to repay the debt limit. Since the only punishment for not repaying the debt is to give the buyer a bad record, we first discuss the optimal strategy for a buyer with a bad record. If a buyer has bad record, he cannot use the monitoring technology and the seller makes a TIOLI offer in the second DM round. This implies that, although his surplus from round-1 DM does not depend on his record, he receives no surplus from round-2 DM. As a result, a buyer with a bad record only brings money to finance his round-1 DM trade. Thus, repayment is better than default if and only if

$$-r(D + z_2) + \sigma\left[u_2(y_2) - D - z_2\right] \geq 0, \tag{14}$$

where y_2 is given by (12) and $z_2 = \rho_2 y_2 - D$. Note that we leave out the cost of holding money and the trade surpluses from round-1 DM on both sides as they are not affected by the bad record. The constraint (13) follows from (14) by simple algebra. Lemma 3 fully characterizes the optimal allocation, denoted by (y_1^{cm}, y_2^{cm}).

Lemma 3 *Let*

$$r_{cm} \equiv \frac{\sigma[u_2(y_2^*) - \rho_2 y_2^*]}{\rho_2 y_2^*}. \tag{15}$$

If $r \leq r_{cm}$, the optimal allocation without intervention is given by $(y_1^{cm}, y_2^{cm}) = (y_1^m, y_2^*)$. Otherwise, the optimal allocation is given by $y_1^{cm} = y_1^m$, and y_2^{cm} is the unique positive solution to (13) at equality. In both cases, buyers do not use money in the monitored round.

If $r \leq r_{cm}$ then y_2^* satisfies (13), and hence $(D, y_1, y_2) = (\rho_2 y_2^*, y_1^m, y_2^*)$ constitutes a simple equilibrium. Consider now $r > r_{cm}$. Note that the choice of D does not affect equilibrium in the first DM round, which always equals y_1^m. Moreover, for D small such that the second case in (12) holds, equilibrium in the second DM round is also not affected by D—it always equals y_2^m. But at y_2^m the constraint (13) is slack, and the surplus $u_2(y_2) - \rho_2 y_2$ is maximized by increasing y_2 until it hits y_2^{cm}, which is the unique optimal allocation.

Our first main result shows that the optimal allocation in the limited monitoring economy without intervention is strictly better than the optimal allocation in the no monitoring economy, but worse than the optimal allocation in the unlimited monitoring economy.

Theorem 1 *Under no intervention by the monetary authority, we have the following ranking of welfare:*

$$\mathcal{W}(y_1^m, y_2^m) < \mathcal{W}(y_1^{cm}, y_2^{cm}) < \mathcal{W}(y_1^c, y_2^c). \tag{16}$$

Theorem 1 shows that limited monitoring delivers a strictly higher welfare than no monitoring and hence the use of debt is essential. This is so because credit is easy at the optimal allocation in the monitored round, and this allocation cannot be replicated by a constant money supply under TIOLI offers, as shown by Lemma 3. The use of money is also essential because credit is not available in the non-monitored round.

3.2 Intervention

We consider two classes of interventions: either the monetary authority uses inflation tax to forgive private debt or uses credit tax to increase the real rate of return on money.[7] We label interventions in the former category *inflationary policies* and interventions in the latter category *deflationary policies.* These policies are quite different from lump-sum interventions usually considered in the literature, and the analysis of their implications is the main contribution of this paper.

We first characterize the optimal allocation for any given deflationary policy and the optimal allocation for any given inflationary policy. We then allow the monetary authority to choose among all policies, including the passive policy of no intervention, and we show that intervention is always optimal. In particular, we show that the optimal intervention can be inflationary.

In what follows, it will be convenient to work with the nominal interest rate implied by the Fisher equation

$$i = (1 + r)(1 + \tau) - 1, \tag{17}$$

where τ is the growth rate of the money supply (note that $\tau < 0$ indicates shrinking money supply).

3.2.1 Deflationary policies

We assume that interventions by the monetary authority must be consistent with the underlying frictions of the environment. In the case of deflationary policies, this implies that only buyers with a good record who want to have future access to the monitoring technology are

[7]The literature also considers interventions which use the inflation tax to increase the real rate of return on money (Andolfatto (2010), Wallace (2014)). These policies require the observability of individual money holdings by the monetary authority, which we do not assume here. Besides, we show that these policies are not useful in our environment when the optimal trading mechanism is used. In contrast, the optimality of the interventions considered here is robust to the optimal trading mechanism.

required to pay the credit tax in the end of the CM each period. It further implies that the only punishment in case the buyer refuses to pay the credit tax is the assignment of a bad record and the exclusion of future access to the monitoring technology.

Let χ denote the credit tax (in terms of the CM good). When solving his problem, each buyer takes as given the nominal interest rate i, the debt limit D, and the credit tax χ. As in the case of no intervention, the buyer always issues D units of debt in the monitored meeting and he may use money to finance some additional consumption.[8] Assuming that the buyer repays his debt D, the problem on how much money to bring into the DM rounds is given by

$$\max_{z_1 \geq 0, z_2 \geq 0} \left\{ -i(z_1 + z_2) + u_1\left(\frac{z_1}{\rho_1}\right) - z_1 + \sigma\left[u_2\left(\frac{z_2 + D}{\rho_2}\right) - z_2 - D\right]\right\}, \tag{18}$$

and the corresponding FOCs are (where we let $y_1 = \frac{z_1}{\rho_1}$ and $y_2 = \frac{z_2+D}{\rho_2}$)

$$\frac{u_1'(y_1) - \rho_1}{\rho_1} = i, \tag{19}$$

and

$$\begin{cases} \rho_2 y_2 = D & \text{if } \sigma\frac{u_2'(D/\rho_2)-\rho_2}{\rho_2} < i, \\ \sigma\frac{u_2'(y_2)-\rho_2}{\rho_2} = i & \text{otherwise.} \end{cases} \tag{20}$$

The key difference with the case of a constant money supply is the opportunity cost of money, now given by i. Note also that, while y_1 only depends on i, y_2 depends both on i and D. To emphasize this dependence, we use $[y_1(i), y_2(i, D)]$ to denote the solution to (19)-(20).

Each buyer takes as given the credit tax χ. However, the equilibrium value of χ depends on the growth rate of the money supply and on the real balances that buyers carry out of the CM. For instance, if real balances are given by z, and the monetary authority wants to increase the real rate of return on money by taking a fraction τ of these balances out of circulation, it needs to raise τz in credit taxes. In our case, since equilibrium real balances are given by $\rho_1 y_1(i) + \rho_2 y_2(i, D) - D$, the tax required from each buyer with a good record is given by

$$\chi(i, D) = -\tau\left[\rho_1 y_1(i) + \rho_2 y_2(i, D) - D\right] = (r - i)\left[\rho_1 y_1(i) + \rho_2 y_2(i, D) - D\right]/(1 + r), \tag{21}$$

[8] As in the case of no intervention, we are assuming $D \leq \rho_2 y_2^*$ so the buyer has enough debt limit to acquire the efficient quantity, and he wants to use all the debt limit because debt is cheaper than money for every $i > 0$. We break the tie in favor of debt if $i = 0$.

where we use the Fisher equation with $1+i=(1+r)(1+\tau)$; note that under a deflationary policy we have $\tau<0$ and hence $i<r$.

Henceforth, the pair (i,D) denotes the policy of the monetary authority. We now turn to the buyer's incentive to keep a good record and thus participate in monitored meetings in future periods. It is sufficient to consider the case of a buyer who participated in a monitored meeting in the current period, since, in addition to paying the credit tax and possibly acquiring enough real balances to participate in the monitored meeting, this buyer has an outstanding debt equal to D, which must be paid if he wants to keep the good record. As a result, it is optimal for a buyer with a good record to repay his debt if and only if

$$-(1-\tau)z_2-D-\chi(i,D)+\frac{\beta}{1-\beta}\left\{\sigma[u(y_2(i,D))-\rho_2y_2(i,D)]+\tau z_2-\chi(i,D)\right\}\geq 0,$$

where $z_2=\rho_2y_2(i,D)-D$, and this can be reduced to

$$-r\left[D+\chi(i,D)\right]-i[\rho_2y_2(i,D)-D]+\sigma\left\{u_2[y_2(i,D)]-\rho_2y_2(i,D)\right\}-\chi(i,D)\geq 0. \tag{22}$$

The left hand side of (22) is equal to zero because the buyer with a bad record cannot access the monitoring technology, in which case the seller makes the TIOLI offer and appropriates all the surplus. Note that, since the buyer may carry real balances to use in the monitored round, we included the cost of carrying those balances across periods. Note also that, for i small relative to r, there may not be any D for which (22) is satisfied, which sets a limit on how much taxation can be imposed on credit trades. Lemma 4 characterizes the optimal allocation for any feasible deflationary policy.

Lemma 4 *There exists $\underline{i}<r$ such that for all $i\in[\underline{i},r]$, the set of debt limits satisfying (22) is nonempty. Moreover, for such i's, the optimal allocation is characterized as follows:*

$$\chi(i)=\frac{(r-i)\,\rho_1y_1(i)}{(1+r)}, \tag{23}$$

$y_2(i)$ is the largest $y_2\leq y_2^*$ such that

$$-r\left[\rho_2y_2+\chi(i)\right]+\sigma\left[u_2(y_2)-\rho_2y_2\right]-\chi(i)\geq 0. \tag{24}$$

In particular, under the optimal allocation, the buyer does not use money in the monitored round.

We obtain (24) from (22) by using the fact that money is not used in the monitored round and the fact that the constrained optimal debt limit is equal to $\rho_2y_2(i)$. Note that deflationary policies include as a particular case the scenario of no intervention, where $i = r$. In this case, there is no credit tax.

3.2.2 Inflationary policies

Under an inflationary policy, the monetary authority pays a maximum of private debt, denoted by η (in terms of CM goods), for each buyer. Thus, a buyer with debt d_2 entering CM only needs to pay $\max\{0, d_2 - \eta\}$ to keep his good record and the rest is paid by the monetary authority. Apart from the fact that nominal interest rates are higher under inflationary policies, the problem of the buyer on how much real balances to bring out of the CM is set up in exactly the same way as in the case of deflationary policies. Precisely, the buyer solves (18) and the corresponding FOCs are given by (19) and (20).

As in the case of deflationary policies, the equilibrium value of η depends on the growth rate of the money supply and on the real balances that buyers carry out of the CM. For instance, if real balances are given by z, the monetary authority needs to increase these balances by $\tau z = \sigma\eta$ in order to pay for an amount η of private debt, and this gives a lower bound on the money-creation rate, τ. Note that, unlike in the case of deflationary policies, the fraction σ of buyers participating in monitored meetings matter because only these buyers will have incurred some debt. It turns out, as it will be shown below, that this feature of inflationary policies can make them quite efficient when σ is small.

In our case, since equilibrium real balances are given by $\rho_1 y_1(i) + \rho_2 y_2(i, D) - D$, the maximum amount of debt paid for by the monetary authority is given by[9]

$$\eta(i, D) = (i - r)\left[\rho_1 y_1(i) + \rho_2 y_2(i, D) - D\right]/\sigma(1 + r). \tag{25}$$

It remains to check the incentive of a buyer with a good record to participate in monitored meetings. As in the case of deflationary policies, it suffices to consider the case of a buyer with an outstanding debt equal to D. If this buyer chooses not to pay his part of the debt, he gets a bad record and receives zero surplus in all future monitored meetings. Thus, he chooses to keep the good record if and only if

$$-r\left[D - \eta(i, D)\right] - i[\rho_2 y_2(i, D) - D] + \sigma\left\{u_2[y_2(i, D)] - \rho_2 y_2(i, D) + \eta(i, D)\right\} \geq 0. \tag{26}$$

[9]We are implicitly assuming that the monetary authority never pays for more debt than what the buyer actually incurred. This is without loss of generality because such policy only hurts non-monitored trades with no benefit to monitored trades.

Note that the monetary authority can always pay some of the debt incurred by the buyer. This is because $u_1'(0) = \infty$ implies $y_1(i) > 0$ for any $i > r$, and hence $\eta(i, D) > 0$ for any D. Lemma 5 characterizes the optimal allocation for any inflationary policy.

Lemma 5 *For any given inflationary policy with $i \geq r$, the set of debt limits satisfying (26) is nonempty, and, under the optimal allocation, the buyer does not use money in the monitored round. The optimal allocation is characterized as follows:*

$$\eta(i) = \frac{(i-r)\,\rho_1 y_1(i)}{(1+r)\sigma}, \tag{27}$$

$y_2(i)$ is the largest $y_2 \leq y_2^*$ such that

$$-r\left[\rho_2 y_2 - \eta(i)\right] + \sigma\left\{u_2(y_2) - \rho_2 y_2 + \eta(i)\right\} \geq 0. \tag{28}$$

Note that inflationary policies include as a particular case the scenario of no intervention, where $i = r$. In this case, there is no inflation tax and the monetary authority does not fund any portion of the buyer's private debt.

3.2.3 Optimal policies

The monetary authority solves

$$\max_{D,i} \mathcal{W}\left[y_1(i), y_2(D,i)\right]$$

choosing among the set of all incentive feasible deflationary and inflationary allocations, which amounts to the set that satisfies (22) in the case of deflationary policies ($i \leq r$) and the set that satisfies (26) in the case of inflationary policies ($i \geq r$). The monetary authority can also choose not to intervene by setting the nominal interest rate equal to r. Note that, in the latter case, the allocation is the same, irrespective of whether we use Lemma 4 or Lemma 5.[10]

Theorem 2 *The optimal policy (i, D) is such that $i \neq r$. Thus, intervention is always optimal.*

The proof is given by the following two lemmas.

[10] There always exist a solution to the maximization problem because the welfare function is continuous on i and D, and the set of incentive feasible deflationary policies and inflationary policies is compact.

Lemma 6 *If $r \leq r_{cm}$, then the optimal intervention is always deflationary. In particular, for r sufficiently small, the first-best is implementable.*

Lemma 6 shows that, if agents are very patient, setting the nominal interest rate equal to zero, i.e., the Friedman rule, is feasible and it implements the first-best. The Friedman rule is not feasible if agents are not too patient, but the optimal policy is still deflationary if agents are patient enough. Intuitively, patient agents are willing to pay a large credit tax in order to participate in monitored meetings, which allows to retire a large fraction of real balances and substantially increase the real rate of return on money.

Now let $[y_1(i), y_2(i)]$ be given by Lemma 4 (for $i \in [\underline{i}, r]$) and Lemma 5 (for $i \geq r$), and let

$$\bar{\mathcal{W}}(i) \equiv \mathcal{W}[y_1(i), y_2(i)].$$

Lemma 7 *Suppose that $r > r_{cm}$. Then, $\bar{\mathcal{W}}(i)$ is continuously differentiable for all $i \in [\underline{i}, \infty) - \{r\}$, and either*

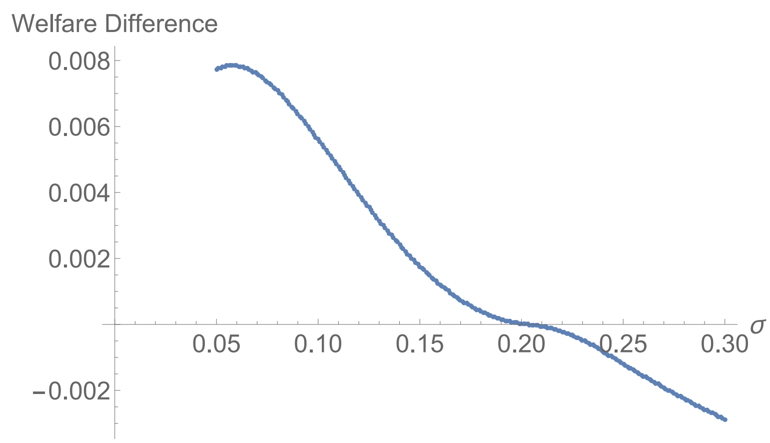
$$\lim_{i \downarrow r} \frac{\mathrm{d}}{\mathrm{d}i} \bar{\mathcal{W}}(i) > 0 \text{ or } \lim_{i \uparrow r} \frac{\mathrm{d}}{\mathrm{d}i} \bar{\mathcal{W}}(i) < 0.$$

Now, to prove Theorem 2, Lemma 6 shows that we only need to consider $r > r_{cm}$. Lemma 4 and 5 show that it is with no loss of generality to consider $\bar{\mathcal{W}}(i)$, at least when we look for optimum around $i = r$. According to Lemma 7, in this case, although there may be a kink, $\bar{\mathcal{W}}(r)$ cannot be a local maximum. This proves Theorem 2.

Theorem 2 shows that intervention is always optimal. Moreover, depending on parameters, the optimal policy may involve inflation or deflation. Figure 1 illustrates this point, by showing the welfare difference between the optimal inflationary and the optimal deflationary policy, as a function of σ. For each $j = 1, 2$, we let $u_j(y_j) = y_j^{1-\theta_j}/(1-\theta_j)$. We use $r = 0.08$, $\theta_1 = 0.5$, $\theta_2 = 0.25$, and $\rho_1 = \rho_2 = 1$. We obtain that deflation is optimal if σ is relatively large, while inflation is optimal if σ is relatively small. Interestingly, note that, at $\sigma = 0.203$, where deflation overtakes inflation as the optimal policy, the optimal deflation rate and the optimal inflation rate are both nonzero.

The result that inflation is optimal when σ is relatively small is particularly surprising because a reduction in σ increases the relative weight of the non-monitored trade, which would call for deflation, not inflation, as the natural intervention. The intuition is that,

Figure 1: Welfare Difference between Optimal Inflationary and Deflationary Policy



when σ is small, a given inflation tax allows the monetary authority to pay for a large amount of debt to each buyer participating in a monitored trade.

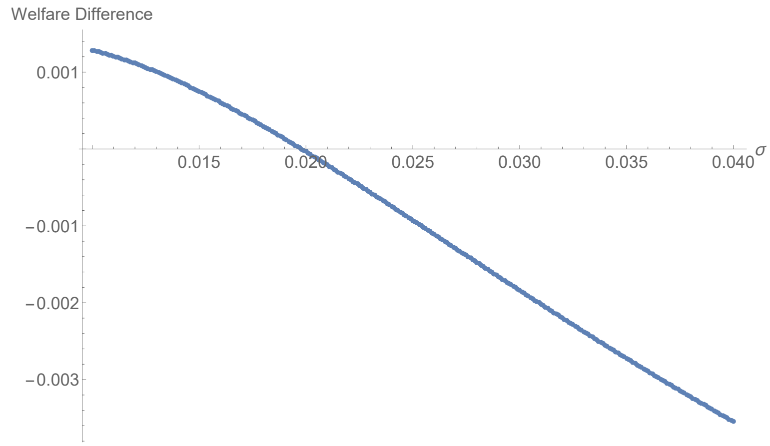
In fact, interventions by the monetary authority can upset the welfare ranking given by Theorem 1, which says that the optimal allocation in the unlimited monitoring economy always dominates the one in the limited monitoring economy. The following example illustrates this. Let $r = 0.2$, $u_1(y_1) = \sigma \frac{y_1^{0.65}}{0.65}$, $u_2(y_2) = \frac{y_2^{0.4}}{0.4}$, $\rho_1 = \sigma$ and $\rho_2 = 1$. Figure 2 looks at the difference between welfare under optimal intervention with limited monitoring and welfare in the optimal allocation under unlimited monitoring, as a function of σ. As seen in the figure, the optimal intervention can dominate unlimited monitoring for some parameters (in this case, for sufficiently small σ). Moreover, for such parameters, an inflationary policy is optimal. As said above, this is because the inflation tax consists of a relatively efficient way to raise revenue when σ is small and relatively more agents participate in non-monitored trades.

3.3 Monitored meetings in the first DM round

Suppose now that the first DM round is monitored. In this case, it is straightforward to adapt Lemma 3 and obtain that, if $r \leq \widehat{r}_{cm}$, the optimal allocation without intervention is given by $(y_1^{cm}, y_2^{cm}) = (y_1^*, y_2^m)$, where

$$\widehat{r}_{cm} = \frac{u_1(y_1^*) - \rho_1 y_1^*}{\rho_1 y_1^*} \tag{29}$$

Figure 2: Welfare Difference between Optimal Welfare under Intervention and Unlimited Monitoring



becomes the new threshold. If, instead, $r > \widehat{r}_{cm}$, the optimal allocation is given by $y_2^{cm} = y_2^m$, and y_1^{cm} is the unique positive solution to

$$-r\rho_1 y_1 + u_1(y_1) - \rho_1 y_1 = 0.$$

Conditional on this adaptation, Theorem 1 holds without any change.

In turn, Theorem 2 still holds but it requires a genericity condition:

$$\frac{u_2'(y_2^{cm}) - \rho_2}{u_2''(y_2^{cm})y_2^{cm}} + \frac{u_1'(y_1^{cm}) - \rho_1}{(1+r)\rho_1 - u_1'(y_1^{cm})}\frac{1}{1+r} \neq 0. \tag{30}$$

This condition is generic in the following sense. Suppose that (30) is violated for some u_2. Perturb u_2 by

$$v_2(y_2; \epsilon) = u_2(y_2) + \epsilon\frac{y^{1-\theta}}{1-\theta},$$

where $\theta \in (0,1)$. Then, except for possibly one knife-edge θ, for all $\theta \in (0,1)$, (30) holds under $v_2(\cdot; \epsilon)$ for an open interval of ϵ around zero.

The only key difference is that, although intervention is generically optimal and it can be inflationary, it is always the case that welfare under unlimited monitoring is weakly better than welfare under limited monitoring. The key reason for this to happen is that all buyers participate in monitored meetings when round-1 DM is monitored. As a result, under an inflationary policy, all buyers benefit from the debt forgiveness. In contrast, when round-2 DM is monitored and $\sigma < 1$, only those who participate in the monitored meeting benefit from debt forgiveness. Thus, in the latter case, the inflation tax required by the monetary authority to pay for the private debt is relatively small. We conjecture that, if we allow the meeting probability in round-1 DM to be smaller than one, inflationary policies can also achieve a strictly higher welfare than unlimited monitoring.

4 Mechanism Design

In this section, we check the robustness of our results by allowing arbitrary trading mechanisms that are incentive compatible subject to the frictions in the environment. We first characterize the set of incentive feasible allocations and then look for the allocation that maximizes social welfare within this set. An allocation is denoted by

$$\mathcal{L} = [(y_1, y_2), (z_1, z_2)],$$

where y_j denotes a buyer's consumption in round-j DM and z_j denotes the CM consumption of a round-j seller, $j \in \{1, 2\}$. We restrict our attention to allocations that satisfy $z_1 \leq u_1(y_1) \leq u_1(y_1^*)$ and $z_2 \leq u_2(y_2) \leq u_2(y_2^*)$. This restriction is without loss of generality as far as constrained efficient allocations are concerned, but it avoids many uninteresting cases.

We start with the extreme case of the unlimited monitoring economy. Using the same arguments as in Section 2.1, it is straightforward to verify that an allocation $[(y_1, y_2), (z_1, z_2)]$ is consistent with the repayment constraint of buyers and the participation constraint of sellers if and only if

$$-r\,(z_1 + z_2) + u_1(y_1) - z_1 + \sigma[u_2(y_2) - z_2] \geq 0, \tag{31}$$

$$\rho_1 y_1 \leq z_1, \;\; \rho_2 y_2 \leq z_2. \tag{32}$$

It is also straightforward to verify that, if $r \leq r_c$, the first-best satisfies (31)-(32); while, if $r > r_c$, $\mathcal{L}^c = [(y_1^c, y_2^c), (\rho_1 y_1^c, \rho_2 y_2^c)]$ maximizes (1) subject to (31)-(32). For our later analysis, we also note that, generically, for $r > r_c$, either

$$-r\rho_1 y_1^c + [u_1(y_1^c) - \rho_1 y_1^c] < 0, \text{ or} \tag{33}$$

$$-r\rho_2 y_2^c + \sigma[u_2(y_2^c) - \rho_2 y_2^c] < 0. \tag{34}$$

Since for $r > r_c$, the allocation $\mathcal{L}^c$ satisfies (31) at equality, violation of both (33) and (34) only happens in the knife-edge case. Thus, generically, either round-1 DM has a "tight" liquidity constraint (when (33) happens), or round-2 DM has a "tight" liquidity constraint (when (34) happens).

Now we turn to the limited monitoring economy, focusing on the case where the second DM round is monitored. As in section 2.1, the choice of the debt limit D is formulated as a mechanism design problem to maximize the social welfare subject to incentive compatibility constraints. We also maintain the same set of records $R = \{G, B\}$, and the same function $\omega : \{G, B\} \times H \to \{G, B\}$.

The key difference is that terms of trades are now determined by the mechanism designer. Precisely, we have a pair of functions (o_1, o_2) defined as follows:

$$o_1(m) = (y_1, z_{1,m}) \text{ and } o_2(m, r, d) = (y_2, z_{2,c}, z_{2,m}),$$

where m is the buyer's real balance holdings, r is his record, d is his debt limit, $(y_1, z_{1,m})$ is the proposed trade in a non-monitored meeting and $(y_2, z_{2,c}, z_{2,m})$ is the proposed trade in a monitored meeting – $z_{j,m}$ is the transfer of real balances in round j, and $z_{2,c}$ is the promise of the buyer in the monitored round. Proposals are implemented as follows. First, both the buyer and the seller respond with *yes* or *no* to the corresponding proposed trade. If both respond with *yes* then they move to the next stage; otherwise, the meeting is autarkic. If they move to the next stage, the buyer makes a TIOLI offer, which is implemented if the seller responds with *yes* while the originally proposed trade by the mechanism is carried out otherwise. This trading protocol is in the spirit of a direct mechanism. In particular, it allows arbitrary ways to split the trading surpluses only subject to individual rationality and coalition-proofness.[11] Finally, in the CM, each buyer chooses how much to repay of his promises, and agents trade competitively against ϕ_t to rebalance their money holdings.

We restrict attention to symmetric equilibria where real balances $Z_t = \phi_t M_t$ do not change over time, and proposals are stationary. Strategies are as in section 2.2, with the difference that we replace TIOLI offers by the proposal and the trading protocol described above. The equilibrium concept is symmetric Perfect Bayesian Equilibrium, and we restrict attention to simple equilibria: buyers with good record always repay their debts up to the debt limit D and they always choose to use the technology in monitored meetings, buyers

[11] This trading mechanism generalizes the trading protocols considered in Zhu (2008) and Hu, Kennan, and Wallace (2009) to our setting with monitored meetings. As in those papers, the first stage ensures that the mechanism satisfies individual rationality, and the second stage ensures that it satisfies the pairwise core requirement and hence is coalition-proof.

and sellers respond with *yes* in all DM meetings and buyers always offer the trades proposed by the mechanism, and the initial distribution of money across buyers is degenerate.

First we give some necessary conditions for implementation under limited monitoring without intervention by the monetary authority.

Lemma 8 *An allocation $L = [(y_1, y_2), (z_1, z_2)]$ is implementable under a constant money supply only if it satisfies (31)-(32) and*

$$-rz_1 + u_1(y_1) - z_1 \geq 0. \tag{35}$$

As compared to the case of unlimited monitoring, Lemma 8 requires an additional condition, (35). This additional constraint reflects the fact that in the limited monitoring economy, since the first DM round is not monitored, buyers with different private histories who enter the second DM round with the same money holdings must be treated equally. In particular, all buyers who enter the monitored round with the equilibrium amount of real balances must obtain the equilibrium surplus in that round.

Lemma 8 suggests that money is used in a much more efficient way than under TIOLI offers. In particular, one can show that if $\mathcal{L}^* = [(y_1^*, y_2^*), (\rho_1 y_1^*, \rho_2 y_2^*)]$ satisfies (31), (32), and (35), then it is implementable with a constant money supply. Thus, for all $r \leq \min\{r_c, \widehat{r}_{cm}\}$, where r_c is given by (7) and $\widehat{r}_{cm}$ is given by (29), the first-best allocation is implementable under the optimal mechanism, which cannot be done with TIOLI offers. However, Lemma 8 also suggests that, for higher r's, there is still room for intervention, even under the optimal mechanism. As in Section 3.2, we consider either inflationary policies that use the inflation tax to forgive private debt, or deflationary policies that use credit taxes to increase the real rate of return on money.

Theorem 3 characterizes the set of implementable allocations under limited monitoring with monetary interventions in the credit sector.

Theorem 3 *(i) An allocation, $\mathcal{L} = [(y_1, y_2), (z_1, z_2)]$, is implementable with an inflationary policy if and only if it satisfies (32), (35), and*

$$-r(z_1 + z_2) + u_1(y_1) - z_1 + \sigma[u_2(y_2) - z_2] \geq \frac{r(1-\sigma)}{r+\sigma}\{-rz_2 + \sigma[u_2(y_2) - z_2]\}. \tag{36}$$

(ii) An allocation, $\mathcal{L} = [(y_1, y_2), (z_1, z_2)]$, is implementable with a deflationary policy if and only if it satisfies (31) and (32), and

$$-rz_2 + \sigma[u_2(y_2) - z_2] \geq 0. \tag{37}$$

Note that deflationary interventions under limited monitoring cannot improve upon the set of implementable allocations under unlimited monitoring. However, if (32) and (35) hold, we can have allocations that satisfy (36) but fail to satisfy (31). These allocations can be achieved under limited monitoring with an inflationary policy but they cannot be achieved under unlimited monitoring.

Theorem 4 characterizes the optimal monetary intervention in the credit sector when $r > \min\{r_c, \widehat{r}_{cm}\}$ and the first best cannot be achieved under limited monitoring and no intervention. In particular, it shows that, whenever the optimal policy is inflationary, its implied welfare dominates the welfare achieved by (y_1^c, y_2^c), the constrained optimal allocation under unlimited monitoring.

Theorem 4 *Suppose that* $r > \min\{r_c, \widehat{r}_{cm}\}$*. Then intervention is generically optimal. If (33) holds, the optimal intervention is deflationary. If, instead, (34) holds, the optimal intervention is inflationary. In the latter case, the optimal allocation achieves a strictly higher welfare than* (y_1^c, y_2^c)*.*

Theorem 4 shows that, unless the first-best allocation is implementable with a constant money supply, intervention is generically optimal. This result then extends the result in Theorem 2 to optimal mechanisms and shows that the need for intervention is not driven by assuming a suboptimal trading mechanism.[12]

Theorem 4 also gives a precise condition for the optimal policy to call for inflation or deflation. Indeed, when the first DM round has a "tight" liquidity in the sense that (33) holds, it would be useful to increase the monetary trades there and hence deflation is optimal. In contrast, when the second DM round has a "tight" liquidity in the sense that (34) holds, it would be useful to increase the credit trades there and hence inflation is optimal. In the latter, we also obtain that intervention by the monetary authority always achieves a strictly higher welfare than in the case of unlimited monitoring. As in the case of TIOLI offers, the inflation tax efficiently raises revenue when σ is small and relatively more agents participate in non-monitored trades.

[12] Bjaj et al. (2016) show that one can use inflation and the scheme considered in Andoffatto (2010) under TIOLI offers to obtain the constrained efficient allocation that is obtained in Hu, Kennan, and Wallace (2009) in the LW model with one DM round. Such schemes could help in our previous section with TIOLI offers, but not under the optimal trading mechanisms considered here.

5 Concluding remarks

In this paper we propose a model of monetary interventions in credit markets in which both money and debt play essential liquidity roles. We show that the interventions considered here are indeed optimal ones, even under the optimal trading mechanism. The interventions are monetary in the sense that money is the only instrument through which the monetary authority intervenes. Our model features heterogenous liquidity needs in monetary and credit sectors, and the optimal monetary intervention balances these needs by redistributing liquidity across these two sectors. Specifically, inflation would be optimal when the credit sector has a tight borrowing constraint absent intervention relative to the amount of money agents are willing to hold. In contrast, when agents are not willing to hold sufficiently amount of money and the credit sector has a relatively loose borrowing constraint, deflation would be optimal. Since the liquidity needs in the two sectors are endogenously determined, the precise shape of intervention would depend on the details of the economy, such as discount factor, matching probability, and the curvature of the utilities.

We made a few special assumptions to obtain our results. The linear cost of production in the DM can be easily generalized. We assume that round-1 DM has no matching uncertainty and there are only two DM round at each period. Under the optimal trading mechanism, we can extend Theorems 3 and 4 to more general trading patterns, including more than two rounds of DM trades and matching uncertainty in all rounds, as long as we assume that a buyer can have a successful meeting in a later round only if he had one earlier. This excludes heterogenous trading histories for active buyers. Otherwise, there would be endogenous distribution of different debt limits and money holdings. While we suspect in such a case both our credit market intervention and the interventions considered in Wallace (2014) would have nontrivial and interesting interactions, it would require techniques beyond the scope of this paper.

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Appendix: Proofs of Lemmas and Theorems

Proof of Lemma 1: The Lagrangian of maximizing (1) subject to (6) is given by

$$u_1'(y_1) - \rho_1 + \nu[-r\rho_1 + u_1'(y_1) - \rho_1] \;=\; 0, \tag{38}$$

$$\sigma[u_2'(y_2) - \rho_2] + \nu\left\{-r\rho_1 + \sigma[u_2'(y_2) - \rho_2]\right\} \;=\; 0, \tag{39}$$

where $\nu \geq 0$ is the Lagrange multiplier. When $r \leq r_c$, the solution is (y_1^*, y_2^*), and (5) is satisfied. When $r > r_c$ the constraint (6) is binding and hence $\nu > 0$. This implies that

$$\frac{u_1'(y_1)}{\rho_1} - 1 = \frac{r\nu}{1+\nu} = \sigma\left[\frac{u_2'(y_2)}{\rho_2} - 1\right],$$

which implies (5).

Proof of Lemma 2: Let $V_j(z)$ be the value function of a buyer with z real balances at the beginning of the j^{th} DM round, where $j \in \{1, 2\}$. We start with V_2:

$$V_2(\rho_2 z_2) = z_2 + W_0 + \sigma \max_{0<y_2\leq z_2} \left\{u_2(y_2) - \rho_2 y_2\right\}, \tag{40}$$

where TIOLI offers implies that his consumption is equal to y_2 if he spend $\rho_2 y_2$ real balances. If $z_2 \geq y_2^*$, then $y_2 = y_2^*$; otherwise, $y_2 = z_2$. Now consider V_1:

$$V_1(z) = \max_{0\leq y_1\leq \frac{z}{\rho_1}} \left[u_1(y_1) + V_2(z - \rho_1 y_1)\right]. \tag{41}$$

The Inada conditions imply $y_1 > 0$ and $y_2 > 0$. There are two cases to consider. If $z \geq \rho_1 y_1^* + \rho_2 y_2^*$, then the optimal choice is $y_1 = y_1^*$ and $y_2 = y_2^*$; otherwise, the buyer anticipates that he will spend $z - \rho_1 y_1$ in the second DM round, and hence (41) becomes

$$V_1(z) = (1-\sigma)z + \max_{0\leq y_1\leq \frac{z}{\rho_1}} \left[u_1(y_1) + \sigma u_2\left(\frac{z - \rho_1 y_1}{\rho_2}\right) - (1-\sigma)\rho_1 y_1\right]. \tag{42}$$

The first-order condition is

$$u_1'(y_1) = (1-\sigma)\rho_1 + \sigma u_2'\left(\frac{z-\rho_1 y_1}{\rho_2}\right)\frac{\rho_1}{\rho_2}. \tag{43}$$

The buyer equates the marginal benefit of spending one unit of real balances in the first DM round, with the marginal benefit of bringing one unit of real balances into the second DM round. Thus, the buyer's CM problem is

$$\max_{z\geq 0} \left[-(1+r)z + V_1(z)\right], \tag{44}$$

and standard arguments show that it is never optimal to choose more real balances than the necessary to purchase the first-best in both DM rounds. Using the Envelope Theorem, the FOC for (44) becomes

$$\frac{u_1'(y_1)}{\rho_1} - 1 = \frac{(1-\sigma)\,\rho + \sigma u_2'(y_2)}{\rho} = 1 + r. \tag{45}$$

where $\rho_2 y_2 = z - \rho_1 y_1$.

Proof of Lemma 3: If $r \leq r_{cm}$, the optimal allocation is then (y_1^m, y_2^*), which can be implemented by $D = \rho y_2^*$. Suppose now that $r > r_{cm}$. If D is so small that $\sigma[u_2'(D/\rho_2)/\rho_2 - 1] < r$, then in equilibrium y_2 solves

$$\sigma \frac{u_2'(y_2) - \rho_2}{\rho_2} = r,$$

and hence this $y_2 = y_2^m$. Now, since for y_2^m (13) is satisfied with a strict inequality by concavity of u_2, we can get a strictly higher y_2 by increasing D to be above $\rho_2 y_2^m$ without violating (13), which delivers a strictly better welfare. So we only need to consider $D \geq \rho_2 y_2^m$. In this case in equilibrium $y_2 = D/\rho_2$, and there exists a largest $y_2 > y_2^m$ such that (13) holds (at which it holds with equality); this is our y_2^{cm} that delivers optimal allocation. It can be implemented with $D^{cm} = \rho_2 y_2^{cm}$.

Proof of Theorem 1: First we show that $\mathcal{W}(y_1^c, y_2^c) > \mathcal{W}(y_1^{cm}, y_2^{cm})$. We consider two cases.

(1.a) Suppose that $r \leq r_c$. Then, $(y_1^c, y_2^c) = (y_1^*, y_2^*)$, and $y_1^{cm} = y_1^m < y_1^*$, $y_2^{cm} \leq y_2^*$. Thus, $\mathcal{W}(y_1^c, y_2^c) > \mathcal{W}(y_1^{cm}, y_2^{cm})$.

(1.b) Suppose that $r < r_c$. Then, at (y_1^c, y_2^c), (6) holds at equality. The strict concavity of u_1 implies that

$$-r\rho_1 y_1^{cm} + [u_1(y_1^{cm}) - \rho_1 y_1^{cm}] > 0,$$

and Lemma 2 implies that

$$-r\rho_2 y_2^{cm} + \sigma[u_2(y_2^{cm}) - \rho_2 y_2^{cm}] \geq 0.$$

Thus, at (y_1^{cm}, y_2^{cm}), (6) holds at strict inequality, and hence it is feasible when maximizing (1) subject to (6). This implies that $\mathcal{W}(y_1^c, y_2^c) > \mathcal{W}(y_1^{cm}, y_2^{cm})$.

Now we show that $\mathcal{W}(y_1^{cm}, y_2^{cm}) > \mathcal{W}(y_1^m, y_2^m)$. First note that $y_1^{cm} = y_1^m$, and hence we only need to compare y_2^{cm} against y_2^m. Now, by Lemma 3, either y_2^{cm} satisfies (13) at equality, or $y_2^{cm} = y_2^*$. However, strict concavity of u_2 implies that $y_2^m < y_2^{cm}$. Thus, $\mathcal{W}(y_1^{cm}, y_2^{cm}) > \mathcal{W}(y_1^m, y_2^m)$.

Proof of Lemma 4: We may assume that $i < r$. First we show that, there exists $\underline{i} < r$ such that for all $i \geq \underline{i}$, it is never optimal to set a debt limit so low that buyers use money in the monitored meeting.

Let $y_1(i)$ be the solution to (19) and let $\chi(i) = \frac{(r-i)\rho_1 y_1(i)}{(1+r)}$. Recall that by Lemma 3, y_2^{cm} is the largest y_2 for which (13) holds (and it holds with equality). Since $\chi(r) = 0$ and χ is continuously differentiable, by the Implicit Function Theorem, there exist $\underline{i}' < r$ such that for each $i \in [\underline{i}', r]$, $y_2(i)$ exists such that

$$-\rho_2 y_2 - \chi(i) + \frac{\beta}{1-\beta}\sigma\left[u_2(y_2) - \rho_2 y_2\right] - \chi(i)] = 0, \tag{46}$$

with $y_2(r) = y_2^{cm}$. Moreover, since we have

$$\sigma \frac{u_2'(y_2(r)) - \rho_2}{\rho_2} < r,$$

where exists $\underline{i} < r$ such that for all $i \in [\underline{i}, r]$,

$$\sigma \frac{u_2'(y_2(i)) - \rho_2 y_2(i)}{\rho_2} \leq i,$$

and hence buyers do not spend money in round-2 DM. Moreover, for such i's $[y_1(i), y_2(i)]$ is better than any other allocation in which the buyer uses money in round-2 DM.

Proof of Lemma 5: We may assume that $i > r$. If D is so small that $\sigma[u_2'(D/\rho_2)/\rho_2 - 1] < i$, then in equilibrium y_2 solves

$$\sigma \frac{u_2'(y_2) - \rho_2}{\rho_2} = i.$$

Now, since $i > r$, this $y_2 < y_2^{cm}$, and, since at $D_{cm} = \rho_2 y_2^{cm}$, (13) and hence (26) are satisfied, it delivers a strictly better welfare. So we only need to consider $D \geq D_{cm}$. In this case in equilibrium $y_2 = D/\rho_2$, and (26) can be rewritten as

$$-r\left[\rho_2 y_2 + \eta(i)\right] + \sigma\left\{u_2(y_2) - \left[\rho_2 y_2 + \eta(i)\right]\right\} \geq 0, \tag{47}$$

where $\eta(i) = -\tau\rho_1 y_1(i)/\sigma$ and is independent of D. There then exists a largest $y_2 > y_2^{cm}$ such that (47) holds (at which it holds with equality), denoted by $y_2(i)$. Then, $[y_1(i), y_2(i)]$ is the optimal allocation and the debt limit to implement it is $D(i) = \rho_2 y_2(i)$.

Proof of Lemma 6: Since $r \leq r_{cm}$, the first-best y_2^* is implementable without any intervention, that is, $y_2^{cm} = y_2^*$. Since $y_1^{cm} = y_1^m < y_1^*$, it is optimal to increase y_1 and to decrease

y_2. A deflationary policy will achieve that and hence will increase welfare relative to no intervention. A inflationary policy cannot increase y_1.

Proof of Lemma 7: First we give some derivatives. For $y_1(i)$, it does not depend on whether $i > r$ or $i < r$ and

$$y_1'(i) = \frac{\rho_1}{u_1''[y_1(i)]}. \tag{48}$$

For $y_2(i)$, we need to distinguish the two cases:

$$y_2'(i) = \frac{\sigma + r}{\sigma(1+r)} \frac{\rho_1 y_1(i) + (i-r)\rho_1 y_1'(i)}{(r+\sigma)\rho_2 - \sigma u_2'[y_2(i)]} \text{ if } i > r. \tag{49}$$

$$y_2'(i) = \frac{\rho_1 y_1(i) + (i-r)\rho_1 y_1'(i)}{(r+\sigma)\rho_2 - \sigma u_2'[y_2(i)]} \text{ if } i < r; \tag{50}$$

Suppose, by contradiction, that

$$\lim_{i \downarrow r} \frac{\mathrm{d}}{\mathrm{d}i}\bar{\mathcal{W}}(i) \leq 0 \text{ and } \lim_{i \uparrow r} \frac{\mathrm{d}}{\mathrm{d}i}\bar{\mathcal{W}}(i) \geq 0.$$

Then, we have

$$\begin{aligned} \frac{\rho_1[u_1'(y_1^{cm}) - \rho_1]}{u_1''(y_1^{cm})} + \frac{\rho_1 y_1^{cm}[u_2'(y_2^{cm}) - \rho_2]}{(r+\sigma)\rho_2 - \sigma u_2'(y_2^{cm})} &\geq 0, \\ \frac{\rho_1[u_1'(y_1^{cm}) - \rho_1]}{u_1''(y_1^{cm})} + \frac{\sigma + r}{\sigma(1+r)} \frac{\rho_1 y_1^{cm}[u_2'(y_2^{cm}) - \rho_2]}{(r+\sigma)\rho_2 - \sigma u_2'(y_2^{cm})} &\leq 0. \end{aligned}$$

This then implies that

$$-\frac{(1-\sigma)r}{\sigma(1+r)} \frac{[u_2'(y_2^{cm}) - \rho_2]}{(r+\sigma)\rho_2 - \sigma u_2'(y_2^{cm})} \geq 0,$$

but, since $0 < y_2^{cm} < y^*$ and $(r+\sigma)\rho_2 y_2^{cm} - \sigma u_2(y_2^{cm}) = 0$,

$$u_2'(y_2^{cm}) - \rho_2 > 0 \text{ and } (r+\sigma)\rho_2 - \sigma u_2'(y_2^{cm}) > 0,$$

a contradiction.

Proof of Lemma 8: It is straightforward to verify that (32) is necessary and sufficient to ensure participation of sellers. Here we show that (31) is necessary to ensure participation of buyers. By following the equilibrium behavior, the continuation value upon entering DM at any given period is

$$V = \frac{1}{1-\beta}\left\{[u_1(x_1) - z_1] + \sigma[u_2(x_2) - z_2]\right\}.$$

Consider the CM decision for a buyer who has the largest amount of debt and the smallest amount of real balances along the equilibrium path. To follow the equilibrium behavior, he needs to produce $z_1 + z_2$ in order to repay his debt or to rebalance his money holdings. However, he has a deviation to produce nothing in the CM and receive no trade afterwards. This deviation is not profitable only if

$$-(z_1 + z_2) + \beta V \geq 0,$$

which is equivalent to (31).

To show the necessity of (35), note that the buyer can choose to bring just sufficient real balance for his round-2 DM trade (which can be, of course, zero). To deter this deviation, it must be the case that

$$-z_1 + \beta \{u_1(y_1) + z_1]\} \geq 0,$$

that is,

$$-(1-\beta)z_1 + \beta[u_1(y_1) - z_1] \geq 0,$$

which is equivalent to (35) and hence (35) is necessary.

Proof of Theorem 3: (i) ($\Rightarrow$) First, we prove necessity. To finance consumption in the first round the buyer has to bring at least z_1 units of real balances. Moreover, the buyer may deviate to bringing only sufficient real balances for the round-2 DM (which could be zero in this case). Hence, (35) is necessary as in the case without intervention. In fact, for the same reason, if the inflation rate is τ, then it is necessary that

$$-(1+\tau)z_1 + \beta u_1(y_1) \geq 0,$$

that is,

$$-(1+r)\tau z_1 - rz_1 + [u_1(y_1) - z_1] \geq 0. \tag{51}$$

To prove the necessity of (36), consider an arbitrary inflationary policy (η, τ) with $\eta < 0$. Consider a buyer with z_2 units of debts in the CM. To follow the equilibrium behavior, the buyer in the CM needs to repay $\min\{z_2 + \eta, 0\}$ and buy at least $(1+\tau)z_1$ units of real balances. Note that it has to be case that $z_2 + \eta \geq 0$; for otherwise they can deviate to a bigger trade without any cost. Alternatively, he can deviate to repay nothing and hold zero real balances. Thus, for him to follow the equilibrium behavior, it must be the case that

$$-(1+\tau)z_1 - (z_2 + \eta) + \frac{\beta}{1-\beta}\{[u_1(y_1) - z_1] + \sigma[u_2(y_2) - (z_2 + \eta)] - \tau z_1\} \geq 0, \tag{52}$$

that is,

$$-rz_1 - rz_2 + [u_1(y_1) - z_1] + \sigma[u_2(y_2) - z_2] - (1+r)\tau z_1 - (r+\sigma)\eta \geq 0.$$

Since budget-balancedness implies that $\sigma\eta = -\tau z_1$, the above inequality, together with (51), implies (36). As usual, (32) is necessary, otherwise sellers would not be willing to participate.

($\Leftarrow$) We now prove sufficiency. First we formulate the policy. If $-rz_2 + \sigma[u_2(y_2) - z_2] \geq 0$, then we set $\eta = \tau = 0$. Otherwise, set

$$\eta = -\left[z_2 - \sigma\frac{u_2(y_2)}{\sigma + r}\right] \text{ and } \tau = \sigma\frac{\eta}{z_1}, \tag{53}$$

the debt limit is $D = z_2$ if the buyer has a record G, and is equal to $D = 0$ if the buyer has a record B. In equilibrium the buyers always remain with record G. Standard arguments in the LW environment shows that the continuation value for a buyer with record G entering CM with m units of real balances and with x_c units of debt is given by

$$W_G(m, x_c) = m - \min\{D + \eta, \max\{x_c + \eta, 0\}\} + W_G(0, 0). \tag{54}$$

It remains to specify the proposed trades $o_1(m)$ and $o_2(m, R, D)$ for all $m \geq 0$ and $R \in \{G, B\}$.

We start with $o_2(m, R, D)$. Let $o_2(m, G, D)$ be a solution to

$$\begin{aligned} \max_{(y,x_c,x_m)\in\mathbb{R}_+\times[0,D]\times[0,m]} \quad & -\rho_2 y + x_c + x_m \\ \text{s.t.} \quad & u_2(y) - \max(x_c + \eta, 0) - x_m \geq u_2(y_2) - (z_2 + \eta), \end{aligned} \tag{55}$$

and let $o_2(m, B, 0)$ be a solution to

$$\begin{aligned} \max_{(y,x_m)\in\mathbb{R}_+\times[0,m]} \quad & -\rho_2 y + x_m \\ \text{s.t.} \quad & u_2(y) - x_m \geq 0. \end{aligned} \tag{56}$$

The solutions to (55) and (56) exist and are unique, with the constraints binding at the optimum. Moreover, it can be verified that

$$o_2(0, G, D) = (y_2, z_2, 0).$$

It is also straightforward to see that, regardless of the buyer record, the buyer's surplus in round-2 DM is independent of m.

We now move to $o_1(m)$. Since the first DM round is non-monitored, the payoff of the buyer in this round cannot depend on his record or on the debt limit. Let $\zeta(m) = u_1(y_1) - z_1$ if $m \geq z_1$ and let $\zeta(m) = 0$ otherwise. In turn, let $o_1(m)$ be a solution to

$$\begin{aligned} \max_{(y,x)\in\mathbb{R}_+\times[0,m]} \quad & -\rho_1 y + x \\ \text{s.t.} \quad & u_1(y) - x \geq \zeta(m). \end{aligned} \tag{57}$$

The solution to (57) exist and is unique with the constraints binding at the optimum.

Hence, for $o_1(m) = (y, x)$ and for a G-buyer with m units of real balances upon entering the DM, his continuation value by following the equilibrium is given by

$$\begin{aligned} & u_1(y) + \sigma[u_2(y_2) - (z_2 + \eta) + (m - x)] + (1 - \sigma)(m - y) + W_G(0, 0) \\ = \quad & \zeta(m) + m + \sigma_2[u_2(y_2) - (z_2 + \eta)] + W_G(0, 0), \end{aligned}$$

that is,

$$V_G(m) = \zeta(m) + m + \sigma[u_2(y_2) - (z_2 + \eta)] + W_G(0, 0). \tag{58}$$

Similarly, for $o_1(m) = (x, y)$ and for a B-buyer with m units of real balances upon entering the DM, his continuation value by following the equilibrium is given by

$$u_1(y) + (m - x) + W_B(0) = \zeta(m) + m + W_B(0),$$

that is,

$$V_B(m) = \zeta(m) + m + W_B(0). \tag{59}$$

Moreover, the construction of $o_1(m)$ ensures that there is no profitable deviating offer in the first DM. Now we show that $o_1(z_1) = (y_1, z_1)$. If this is not the case, there exists $(y, x) \neq (y_1, z_1)$, with $x < z_1$, which provide a higher surplus to the seller without violating the constraint. Hence

$$u_1(y) - x \geq u_1(y_1) - z_1 \text{ and } -\rho_1 y + x \geq -\rho_1 y_1 + z_1,$$

which implies

$$u_1(y_1) - \rho_1 y_1 \geq u_1(y_1) - \rho_1 y_1,$$

and $y > y_1$. Since $-\rho_1 y + x \geq -\rho_1 y_1 + z_1$, we have $x > z_1$, a contradiction.

Now we show that the following strategies form a simple equilibrium. All agents respond with *yes* to the proposed trades and buyers offer the proposed trades, on both equilibrium and off-equilibrium paths. Buyers under state G always repay their debts up to D, and buyers under state B never repay anything. All buyers leave the CM with z_1 units of real balances, on both equilibrium and off-equilibrium paths.

First we show that the buyer carry z_1 real balances to the DM, irrespective of his record $R \in \{G, B\}$. By linearity of W_R, the CM problem is given by

$$\begin{aligned} & \max_{m \geq 0} -(1+\tau)m + \beta V_R(m) \\ = \; & \beta \left\{ -[r + (1+r)\tau]m + \zeta(m) + \mathbf{1}_{R=G}\{\sigma[u_2(y_2) - (z_2+\eta)]\} + W_R(0,0) \right\}. \end{aligned}$$

Note that $\zeta(m)$ is constant for all $m \geq z_1$ and is constant for all $m \in [0, z_1)$. Thus, we only need to show that bringing z_1 is better than zero, which will be the case if and only if

$$-[r + (1+r)\tau]z_1 + u_1(y_1) - z_1 \geq 0.$$

Using $\tau z_1 = \sigma\eta = \sigma z_2 - \sigma \frac{\sigma}{\sigma+r} u_2(y_2)$, we can rewrite this inequality as

$$\{-rz_1 + u_1(y_1) - z_1\} + \frac{(1+r)\sigma}{\sigma + r} \{-rz_2 + \sigma\left[u_2(y_2) - z_2\right]\} \geq 0,$$

which corresponds to (36).

Now we show that buyers are willing to repay their debts. Note that, according to the equilibrium behavior,

$$\begin{aligned} W_G(0, z_2) &= -[(1+\tau)z_1 + (z_2+\eta)] + \beta V_G(z_1) \\ &= -[(1+\tau)z_1 + (z_2+\eta)] + \beta\{\zeta(z_1) + \sigma[u_2(y_2) - (z_2+\eta)] + z_1 + W_G(0, z_2)\}, \end{aligned}$$

and hence

$$W_G(0, z_2) = -[(1+\tau)z_1 + (z_2+\eta)] + \frac{\beta}{1-\beta} \{[u_1(y_1) - z_1] + \sigma[u_2(y_2) - (z_2+\eta)] - \tau z_1\}.$$

Similarly,

$$W_B(0) = -(1+\tau)z_1 + \frac{\beta}{1-\beta} \{[u_1(y_1) - z_1] - \tau z_1\}.$$

We need to show that the buyer wants to keep the record G. Recall that by (53) $\eta < 0$. He has incentive to repay $z_2 + \eta$ in order to keep the record if and only if

$$-(z_2+\eta) + \beta V_G(z_1) \geq \beta V_B(z_1),$$

and by (58) and (59), this is equivalent to

$$-(z_2+\eta)+\beta\left\{\sigma[u_2(y_2)-(z_2+\eta)]+W_G(0,0)-W_B(0)\right\}\geq 0,$$

that is,

$$-(z_2+\eta)+\frac{\beta}{1-\beta}\left\{\sigma[u_2(y_2)-(z_2+\eta)]\right\}\geq 0.$$

which holds with equality by (53).

(ii) ($\Rightarrow$) First, we prove necessity. To finance consumption in the first round the buyer has to bring at least z_1 units of real balances. Moreover, the buyer may deviate to bringing only sufficient real balances for the round-2 DM (which could be zero in this case). Now, (51) is still valid, although with deflation $\tau<0$.

To prove the necessity of (31), consider an arbitrary deflationary policy (η,τ) with $\tau<0$. Note that now budget-balancedness requires (as we charge fees in the end of CM)

$$\chi=-\tau z_1. \tag{60}$$

Consider a buyer with z_2 units of debts in the CM. To follow the equilibrium behavior, the buyer in the CM needs to repay $z_2+\chi$ and buy at least $(1+\tau)z_1$ units of real balances. Thus, for him to follow the equilibrium behavior, it must be the case that

$$-(1+\tau)z_1-(z_2+\chi)+\frac{\beta}{1-\beta}\{[u_1(y_1)-z_1]+\sigma[u_2(y_2)-z_2]-\chi-\tau z_1\}\geq 0. \tag{61}$$

Note that (61) differs from (52) as all buyers are required to pay χ here while in the later only those who traded get subsidized. (61) then implies

$$-rz_1-rz_2+[u_1(y_1)-z_1]+\sigma[u_2(y_2)-z_2]-(1+r)(\tau z_1+\chi)\geq 0.$$

Since budget-balancedness, (60), implies that $\chi=-\tau z_1$, the above inequality, together with (51), implies (31). As usual, (32) is necessary, otherwise sellers would not be willing to participate.

($\Leftarrow$) We now prove sufficiency. First we formulate the policy. If $-rz_1+[u_1(y_1)-z_1]\geq 0$, then we set $\chi=\tau=0$. Otherwise, set

$$\tau=-\frac{\left[z_1-\frac{u_1(y_1)}{1+r}\right]}{z_1}<0 \text{ and } \chi=-\tau z_1, \tag{62}$$

the debt limit is $D = z_2$ if the buyer has a record G, and is equal to $D = 0$ if the buyer has a record B. In equilibrium the buyers always remain with record G. We can then construct $o_1(m)$ and $o_2(m, R, D)$ as in (i), with the difference that, in those construction, we take $\chi = 0$; instead, here χ is paid in the CM. Indeed, because of this difference, we need then to change (54) into

$$\begin{aligned} W_G(m, x_c) &= m - \min\{D, x_c\} + W_G(0, 0), \\ W_B(m) &= m + W_B(0), \end{aligned}$$

and

$$\begin{aligned} W_G(0, 0) &= \max_{m,R} \{-\mathbf{1}_{R=G}\chi - m + \beta V_R(m)\}, \\ W_B(0) &= \max_{m} \{-m + \beta V_B(m)\}, \end{aligned}$$

where

$$\begin{aligned} V_G(m) &= \zeta(m) + m + \sigma[u_2(y_2) - z_2] + W_G(0, 0), \\ V_B(m) &= \zeta(m) + m + W_B(0). \end{aligned}$$

As in (i), we first show that the buyer carry z_1 real balances to the DM, irrespective of his record $R \in \{G, B\}$. By linearity of W_R, the CM problem (for choosing money holdings) is given by

$$\begin{aligned} & \max_{m \geq 0} -(1+\tau)m + \beta V_R(m) \\ = \; & \beta\{-[\rho + (1+\rho)\tau]m + \zeta(m) + \mathbf{1}_{R=G}\{\sigma[u_2(y_2) - z_2]\} + W_R(0, 0)\}. \end{aligned}$$

Note that $\zeta(m)$ is constant for all $m \geq z_1$ and is constant for all $m \in [0, z_1)$. Thus, we only need to show that bringing z_1 is better than zero, which will be the case if and only if

$$-[r + (1+r)\tau]z_1 + u_1(y_1) - z_1 \geq 0,$$

and this holds as $\tau z_1 = \frac{u_1(y_1)}{1+r} - z_1$.

Now we show that buyers are willing to repay their debts, together with the fee for accessing the monitoring technology, η. Now, following the equilibrium behavior,

$$\begin{aligned} W_G(0, z_2) &= -[(1+\tau)z_1 + z_2] - \eta + \frac{\beta}{1-\beta}\{[u_1(y_1) - z_1] + \sigma[u_2(y_2) - z_2] - \tau z_1 - \chi\}, \\ W_B(0) &= -(1+\tau)z_1 + \frac{\beta}{1-\beta}\{[u_1(y_1) - z_1] - \tau z_1\}. \end{aligned}$$

We need to show that the buyer wants to keep the record G. He has incentive to repay $z_2 + \chi$ in order to keep the record if and only if

$$-(z_2 + \chi) + \beta V_G(z_1) \geq \beta V_B(z_1),$$

and this is equivalent to

$$-(z_2 + \chi) + \beta \left\{ \sigma[u_2(y_2) - z_2] + W_G(0,0) - W_B(0) \right\} \geq 0,$$

that is,

$$-(z_2 + \chi) + \frac{\beta}{1-\beta} \left\{ \sigma[u_2(y_2) - z_2] - \chi \right\} \geq 0.$$

which holds with equality by (62) and (31).

Proof of Theorem 4: Suppose that (33) holds. Then, by Lemma 8, (y_1^c, y_2^c) is not implementable by a constant money supply, and it exceeds the maximum welfare under a constant money supply. Since (31) holds for $\mathcal{L}^c = [(y_1^c, y_2^c), (\rho_1 y_1^c, \rho_2 y_2^c)]$, by (33), (37) holds with strict inequality for $\mathcal{L}^c$. By Theorem 3 (ii), $\mathcal{L}^c$ is implementable with a deflationary policy.

Suppose that, instead, (34) holds. Then, (y_1^c, y_2^c) is implementable, and is the optimal allocation under a constant money supply. We show that we can find another allocation that is implementable with an inflationary monetary policy that does better than (y_1^c, y_2^c). First note that since $r > \min\{\hat{r}_{cm}, r_c\}$ we cannot have $(y_1^c, y_2^c) = (y_1^*, y_2^*)$. Now, (34) implies that both (35) and (36) hold with strict inequality for $\mathcal{L}^c$. Since $y_1^c < y_1^*$ and $y_2^c < y_2^*$, we can strictly improve welfare by either increasing y_1 or y_2 or both without violating (35) or (36) (and, of course, maintain (32) at the same time). Theorem 3 then implies that an inflationary policy can do better than a constant money supply. Since deflationary policy cannot do better than (y_1^c, y_2^c) as it also subjects to (31), inflationary policy is optimal.